OCR
Oxford Cambridge and RSA

# AS Level Further Mathematics B (MEI)

**Y410/01** Core Pure

# Practice Paper – Set 2

## Time allowed: 1 hour 15 minutes

**You must have:**
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

**You may use:**
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

**1** **In this question you must show detailed reasoning.**

**(i)** Solve the equation $z^2 - 2z + 4 = 0$. **[2]**

**(ii)** Express the roots of the equation in part **(i)** in exact modulus-argument form. **[4]**

**2** The matrix **M** is such that $\mathbf{M}\begin{pmatrix} 1 & 0 & k \\ 2 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \end{pmatrix}$.

Find

- the matrix **M**,
- the value of the constant $k$. **[6]**

**3** **In this question you must show detailed reasoning.**

Given that $\text{Im}[(2+b\text{i})^4] = 0$, where $b$ is real, find the possible values of $b$. **[5]**

**4** Prove by induction that the sum of the first $n$ cube numbers is $\frac{1}{4}n^2(n+1)^2$. **[5]**

**5** **In this question you must show detailed reasoning.**

The equation $x^3 - 2x^2 + 3x + 1 = 0$ has roots $\alpha$, $\beta$ and $\gamma$.

**(i)** Find $\alpha^2 + \beta^2 + \gamma^2$. **[3]**

**(ii)** Find a cubic equation with integer coefficients whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. **[4]**

**6** In the quartic equation $z^4 + az^3 + bz^2 + cz + d = 0$, the coefficients $a$, $b$, $c$ and $d$ are real. Two of the roots of the equation are i and $2 - \text{i}$.

Find the value of $a$, $b$, $c$ and $d$. **[6]**

**7** The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

**(i)** The unit square shown in the printed answer booklet has vertices O (0, 0), A (1, 0), B (1, 1) and C (0, 1). Plot the image of the unit square under the linear transformation represented by the matrix $\mathbf{M}$. **[2]**

**(ii)** (*A*) Calculate $\det \mathbf{M}$. **[1]**

(*B*) Comment on the significance of the value of $\det \mathbf{M}$ for the linear transformation represented by $\mathbf{M}$. **[1]**

Matrices $\mathbf{S}$ and $\mathbf{T}$ are given by $\mathbf{S} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{T} = \mathbf{S}^{-1}\mathbf{M}$.

**(iii)** (*A*) Calculate $\mathbf{T}$. **[2]**

(*B*) Deduce a sequence of two linear transformations equivalent to the transformation represented by $\mathbf{M}$. **[3]**

(*C*) How does this sequence of transformations relate to the answers in part **(ii)**? **[1]**

**8** Fig. 8 shows a half line and a circle drawn in an Argand diagram. The centre of the circle is at the point representing 3i. The half line passes through the point representing 4i.

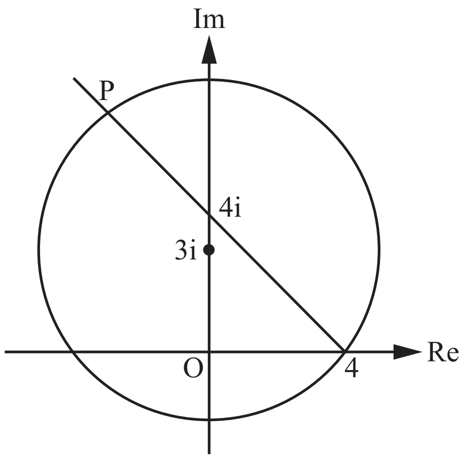


**Fig. 8**

**(i)** Find the conditions, involving a complex number $z$, that define

(*A*) the circle, **[2]**

(*B*) the half line. **[1]**

**(ii)** Find, in the form $a + b\mathrm{i}$, the complex number represented by the point P. **[5]**

**9** The plane $\Pi$ has normal vector $a\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, where $a$ is a positive constant, and the point (0, 1, 2) lies in $\Pi$. The vector $\mathbf{j} + 7\mathbf{k}$ makes an angle of $30°$ with $\Pi$.

Find the cartesian equation of $\Pi$. **[7]**

**END OF QUESTION PAPER**