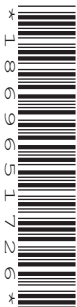


Friday 6 June 2025 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 The method of interval bisection is used to find a sequence of approximations to one of the roots of the equation $e^x - x^2 - 3x = 0$.

The table shows the associated spreadsheet output.

| | A | B | C | D | E | F |
|---|------|----------|-------|----------|--------|----------|
| 1 | a | $f(a)$ | b | $f(b)$ | c | $f(c)$ |
| 2 | 2 | -2.61094 | 3 | 2.085537 | 2.5 | -1.56751 |
| 3 | 2.5 | -1.56751 | 3 | 2.085537 | 2.75 | -0.16987 |
| 4 | 2.75 | -0.16987 | 3 | 2.085537 | 2.875 | 0.834799 |
| 5 | 2.75 | -0.16987 | 2.875 | 0.834799 | 2.8125 | 0.303839 |
| 6 | | | | | | |

- (a) Write down a suitable formula for cell E2. [1]

The formula in cell A3 is $= \text{IF}(F2 < 0, E2, A2)$.

- (b) Write down a similar formula for cell C3. [1]

- (c) Complete row 6 of the table in the **Printed Answer Booklet**. [2]

- (d) **Without** doing any more calculations, write down the value of the root correct to the maximum number of decimal places which seems justified. You must explain the precision quoted. [1]

- 2 The table gives three values of x and the associated values of y .

| | | | |
|-----|-------|------|------|
| x | -1 | 2 | 3 |
| y | -3.39 | 0.18 | 0.45 |

Use Lagrange's form of the interpolating polynomial to construct a polynomial of degree 2 for the values in the table. Give your answer in the form

$$y = ax^2 + bx + c,$$

where a , b and c are constants to be determined. [4]

- 3 (a) Find the relative error when π is **chopped** to 3 decimal places. [2]
- (b) Find the relative error when π is **rounded** to 3 decimal places. [2]

You are given that $y = \pi^2 - 5$ and $z = (\pi - 5)^2$.

You are also given the following information.

- Y is an approximation to y .
- Z is an approximation to z .
- Y and Z are found by using $\pi = 3.14$.

A student states that the relative error in using Y to approximate y is exactly the same as the relative error in using Z to approximate z , because in each case the calculation involves squaring and subtracting 5.

- (c) **Without** doing any calculations, explain whether the student is correct. [1]

- 4 A student is using a spreadsheet to investigate the curve $y = f(x)$ at the point where $x = 2$. Some of the spreadsheet output is shown below.

Table 4.1 shows some values of x and the associated values of $f(x)$.

Table 4.2 shows **two** approximations to $f'(2)$ found using the same method with $h = 0.001$ and $h = 0.01$.

| | E | F | G | H | I | J |
|----|---|------------------|---------|---------|--------|---|
| 2 | | | | | | |
| 3 | | Table 4.1 | | | | |
| 4 | | x | 2 | 2.001 | 2.01 | |
| 5 | | $f(x)$ | 0.30103 | 0.30125 | 0.3032 | |
| 6 | | | | | | |
| 7 | | | | | | |
| 8 | | Table 4.2 | | | | |
| 9 | | h | 0.001 | 0.01 | | |
| 10 | | $f'(2)$ | 0.21709 | 0.21661 | | |
| 11 | | | | | | |

(a) The formula in cell G10 is $\boxed{= (H5 - G5)/G9}$.

The formula in cell H10 is $\boxed{= (I5 - G5)/H9}$.

State the method being used to find these approximations to $f'(2)$. [1]

(b) • Calculate $\frac{0.30125 - 0.30103}{0.001}$.

• Explain why your answer is not the same as the value displayed in cell G10. [3]

(c) State the value of $f'(2)$ as accurately as you can. You must justify the precision quoted. [1]

(d) Hence determine an estimate for the error when $f(2)$ is used as an approximation to $f(2.1)$. [2]

(e) Explain whether your answer to part (d) is likely to be an over-estimate or an under-estimate. [1]

- 5 The spreadsheet output shows a sequence of approximations to $\int_{0.5}^{1.5} f(x) dx$ using the trapezium rule and Simpson's rule, together with some further analysis of the approximations found using Simpson's rule.

| | F | G | H | I | J |
|----|-----|------------|------------|------------|-------|
| 21 | n | T_n | S_n | difference | ratio |
| 22 | 1 | 0.84557616 | | | |
| 23 | 2 | 0.88144672 | 0.89340357 | | |
| 24 | 4 | 0.89066651 | 0.89373977 | 0.00034 | |
| 25 | 8 | 0.89299453 | 0.89377054 | 3.1E-05 | 0.092 |
| 26 | 16 | 0.89357822 | 0.89377279 | 2.2E-06 | 0.073 |
| 27 | 32 | 0.89372426 | 0.89377294 | 1.5E-07 | 0.066 |
| 28 | 64 | 0.89376077 | 0.89377295 | 9.4E-09 | 0.063 |
| 29 | 128 | 0.8937699 | 0.89377295 | 5.9E-10 | 0.063 |

- (a) Write down the value displayed in cell I25 using standard mathematical notation. [1]
- (b) State the value of $\int_{0.5}^{1.5} f(x) dx$ as accurately as you can. You must justify the precision quoted. [1]
- (c) Write down a suitable formula for cell H23. [2]
- (d) (i) Explain what the values in column J tell you about the order of Simpson's rule in this case. [2]
- (ii) Explain whether this is usual. [1]

- 6 The equation $0.5 \ln x + x^2 - x - 2 = 0$ has a root α , such that $1 < \alpha < 2$.

A student attempts to find α using the iterative formula

$$x_{n+1} = g(x_n) \text{ with } x_0 = 1, \text{ where } g(x) = 0.5 \ln x + x^2 - 2.$$

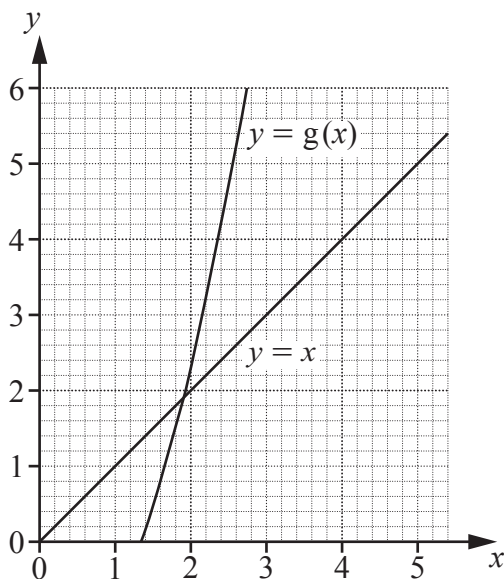
The associated spreadsheet output is shown in the table.

| | K | L |
|---|-----|-------|
| 1 | n | x_n |
| 2 | 0 | 1 |
| 3 | 1 | -1 |
| 4 | 2 | #NUM! |

- (a) Explain why #NUM! is displayed in cell L4.

[1]

The graph shows part of the graphs of $y = x$ and $y = g(x)$.



- (b) On the copy of the graph in the **Printed Answer Booklet** show how the iterative formula $x_{n+1} = g(x_n)$ with $x_0 = 2$ finds x_1 and x_2 .

[1]

- (c) Explain why the iterative formula $x_{n+1} = g(x_n)$ may **not** be used to successfully find α , whatever the value of x_0 .

[1]

- (d) Use the relaxed iteration

$$x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n),$$

with $\lambda = -0.328$ and $x_0 = 1$, to determine the value of α correct to 6 decimal places.

[3]

- 7 A student makes a cup of tea and investigates how long it takes to reach room temperature. The student places the cup of tea on a table in the kitchen and records the temperature $T^{\circ}\text{C}$ of the tea t minutes after it was made, at 5 minute intervals starting with $t = 5$. The data collected are shown in the table.

| | | | | |
|---|------|------|------|------|
| Time after tea was made t (minutes) | 5 | 10 | 15 | 20 |
| Temperature of tea T ($^{\circ}\text{C}$) | 78.5 | 68.1 | 58.7 | 50.3 |

- (a) Show that a quadratic polynomial would be a reasonable model for these data. [3]
- (b) Use Newton's forward difference interpolation method to determine a suitable quadratic polynomial for these data. [4]
- (c) The temperature in the kitchen is 19°C . It takes one hour for the tea to cool to room temperature.
- (i) Use the model to find an estimate of the temperature of the tea after 1 hour. [1]
- (ii) Use the model to find an estimate of the temperature of the tea after an hour and a half. [1]
- (iii) Hence identify a limitation of the model. [1]
- 8 Table 8.1 shows some values of x and the associated values of $f(x)$.

Table 8.1

| | | | |
|--------|---------|---------|---------|
| x | 0.5 | 0.9 | 1.3 |
| $f(x)$ | 1.41421 | 1.09947 | 0.71101 |

- (a) Use the central difference method to determine an approximation to $f'(0.9)$. [2]

Table 8.2 shows a sequence of approximations to $f'(0.9)$ for different values of the step length, h , together with some further analysis.

Table 8.2

| | | | | | |
|------------|-----------|-----------|-----------|-----------|-----------|
| h | 0.2 | 0.1 | 0.05 | 0.025 | 0.0125 |
| $f'(0.9)$ | -0.957844 | -0.977203 | -0.982021 | -0.983225 | -0.983525 |
| difference | | -0.019360 | -0.004818 | -0.001203 | -0.000301 |
| ratio | | | 0.248882 | 0.249719 | 0.249930 |

- (b) Use extrapolation to determine the value of $f'(0.9)$ as accurately as you can. You must justify the precision quoted. [5]

Turn over for question 9

9 In this question you must show detailed reasoning

The equation $0.2x^4 + 0.8x - 0.2 = 0$ has a root α such that $0 < \alpha < 1$.

- (a) Use the Newton-Raphson method with $x_0 = 1$ to determine the value of α correct to 6 decimal places. [5]

The equation $0.2x^4 + 0.8x - 0.2 = 0$ has another root β such that $-2 < \beta < -1$.

The Newton-Raphson method is used to find a sequence of approximations to β . The associated spreadsheet output is shown in the table below, together with some further analysis.

| A | B | C | D |
|-----|-----------|------------|-----------|
| r | x_r | difference | ratio |
| 0 | -2 | | |
| 1 | -1.75 | 0.25 | |
| 2 | -1.670923 | 0.079077 | 0.3163082 |
| 3 | -1.663319 | 0.007604 | 0.096159 |
| 4 | -1.663252 | 6.7E-05 | 0.0088147 |
| 5 | -1.663252 | 5.18E-09 | 7.724E-05 |

- (b) (i) Explain what the values in column D suggest about the order of convergence of this sequence of approximations. [2]
- (ii) Explain whether this is usual. [1]

END OF QUESTION PAPER

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