

## Monday 12 May 2025 – Afternoon

### AS Level Further Mathematics B (MEI)

#### Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **4** pages.

### ADVICE

- Read each question carefully before you start your answer.

1 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}.$$

(a) Use **A** and **B** to show that matrix multiplication is **not**, in general, commutative. [2]

(b) Verify that **A** and **B** satisfy  $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ . [3]

2 In this question you must show detailed reasoning.

Find the acute angle between the vector  $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and the normal vector to the plane  $2x + 3y + z = 6$ . [4]

3 The matrices **M** and **N** are given by

$$\mathbf{M} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix} b & -a \\ a & b \end{pmatrix} \text{ where } a \text{ and } b \text{ are positive constants.}$$

(a) Given that  $\mathbf{M}^2 = \mathbf{N}$ , determine the exact values of  $a$  and  $b$ . [4]

(b) Hence state the transformations of the plane associated with matrices **M** and **N**. [3]

- 4 (a) The transformation  $T$  is represented by the matrix  $\mathbf{M} = \begin{pmatrix} 1 & -2 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{pmatrix}$ .

A shape  $S_1$  is mapped to a shape  $S_2$  by the transformation  $T$ .

Show that volume of  $S_1$  is the same as the volume of  $S_2$ . [2]

- (b) Three planes have equations

$$\begin{aligned} x - 2y + 2z &= \lambda, \\ 2x + y &= 2, \\ x + 2y - z &= 0, \end{aligned}$$

where  $\lambda$  is a constant.

(i) Explain why the three planes intersect at a point for any value of  $\lambda$ . [2]

(ii) Use a matrix method to determine, in terms of  $\lambda$ , the coordinates of this point. [4]

5 In this question you must show detailed reasoning.

The complex number  $w$  is given by  $w = -4\sqrt{2} + (4\sqrt{2})i$ .

(a) (i) Find  $|w|$ . [2]

(ii) Find  $\arg(w)$ . [2]

The complex numbers  $z_1$  and  $z_2$  are given by  $z_1 = a + i$  and  $z_2 = 4(\cos \theta + i \sin \theta)$ , where  $a$  is a positive real constant and  $-\pi < \theta \leq \pi$ .

(b) You are given that  $z_1 z_2 = w$ .

(i) Find the exact value of  $a$ . [3]

(ii) Find the value of  $\theta$ . Give your answer as an exact multiple of  $\pi$ . [3]

6 (a) Express  $\frac{1}{(r-1)^2} - \frac{1}{(r+1)^2}$  as a single simplified fraction. [2]

(b) Hence determine the limit which  $\sum_{r=2}^n \frac{r}{(r-1)^2(r+1)^2}$  converges to as  $n \rightarrow \infty$ . [5]

7 The region R of the Argand diagram consists of the set of points representing complex numbers  $z$  which satisfy the following inequalities.

$$\operatorname{Im}(z) \geq 0 \qquad \arg(z+2) \leq \frac{1}{4}\pi \qquad |z| \leq |z-4-2i|$$

(a) Sketch and clearly label the region R on an Argand diagram. [4]

(b) **In this question you must show detailed reasoning.**

Find the largest value of  $\operatorname{Im}(z)$  in the region R. [7]

8 The three distinct roots of the equation  $z^3 - 4z^2 + pz + q = 0$ , where  $p$  and  $q$  are real, are drawn on an Argand diagram. The three points which represent these roots do not lie on a straight line but instead form a triangle T.

(a) Show that T is isosceles. [3]

(b) **In this question you must show detailed reasoning.**

You are given the following information.

- The area of T is 10 square units.
- One of the roots of the equation  $z^3 - 4z^2 + pz + q = 0$  is  $z = -2$ .

Find the other roots of the equation. [5]

**END OF QUESTION PAPER**

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