OCR
Oxford Cambridge and RSA

Friday 19 May 2017 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} a & 4 \\ 7 & a+3 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find

(i) $2\mathbf{A} - \mathbf{B} + 3\mathbf{I}$, giving your answer in terms of a, **[3]**

(ii) the value of a for which $\mathbf{AB} = \begin{pmatrix} 3 & -11 \\ 19 & 17 \end{pmatrix}$, **[2]**

(iii) the values of a for which **B** is singular. **[3]**

2 The complex number $2-3\mathrm{j}$ is denoted by z.

(i) Find $|z|$ and $\arg z$. **[2]**

(ii) You are given that $2az + 3z^* = 5 - b\mathrm{j}$, where a and b are real. Find the values of a and b. **[4]**

3 **(i)** Using the standard summation formulae, find an expression for $\sum_{r=1}^{n} (1-2r)^2$ in terms of n. Give your answer in a fully factorised form. **[6]**

(ii) Hence evaluate $\sum_{r=25}^{75} (1-2r)^2$. **[2]**

4 The Argand diagram in Fig. 4 shows a half-line l and a circle C. The circle has centre $3+4\text{j}$.

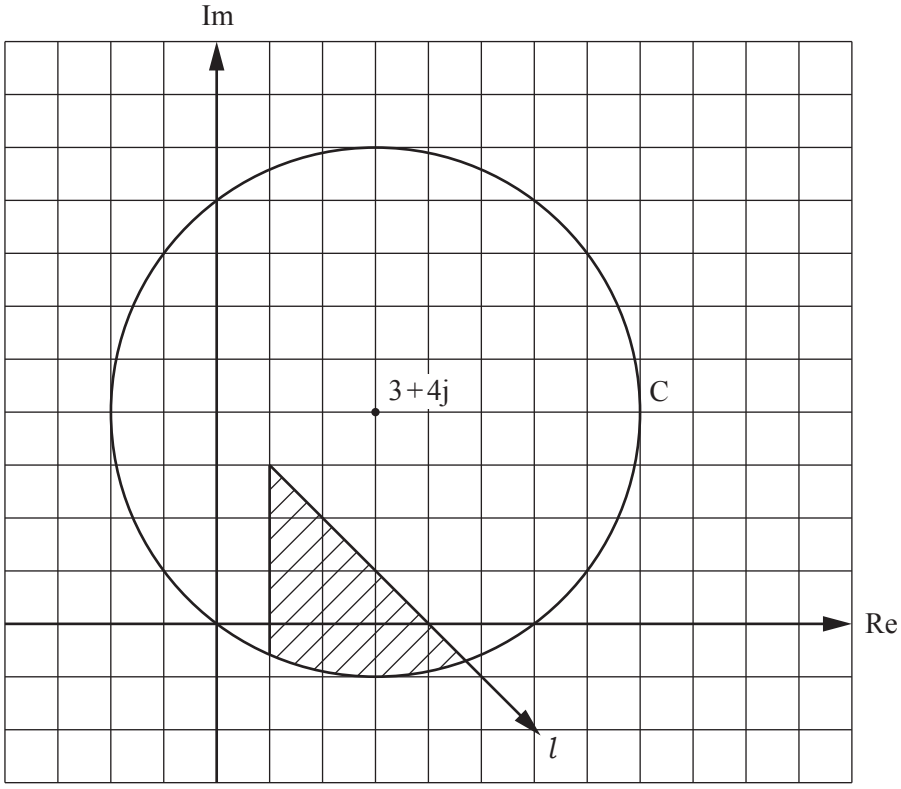

Fig. 4

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the shaded region indicated in Fig. 4, together with its boundaries. **[3]**

5 Prove by induction that $\sum_{r=1}^{n} \frac{r}{2^r} = 2 - \left(\frac{1}{2}\right)^n (2+n)$. **[7]**

Section B (36 marks)

6 A curve has equation $y = \dfrac{ax^2 - 12}{4x^2 + bx - 6}$, where a and b are constants.

(i) Find the coordinates of the point where the curve crosses the y-axis. **[1]**

(ii) You are given that the curve has a vertical asymptote at $x = 2$. Find the value of b and the equation of the other vertical asymptote. **[3]**

(iii) You are given that the curve crosses the x-axis when $x = \pm\sqrt{6}$. Find the value of a and the equation of the horizontal asymptote. **[2]**

(iv) Sketch the curve. **[3]**

(v) Find the set of values for which $y \geqslant 0$. **[3]**

7 **(a)** The roots of the cubic equation $2x^3 - x^2 + 4x + 2 = 0$ are α, β and γ. Find the cubic equation whose roots are 3α, 3β and 3γ, expressing your answer in a form with integer coefficients. **[5]**

(b) A second cubic equation $x^3 + px^2 + qx + r = 0$, where p, q and r are real, has roots that may be written as $a - \lambda$, a and $a + \lambda$.

(i) By considering the sum of the roots show that $2p^3 - 9pq + 27r = 0$. **[4]**

(ii) Given that $p = -6$ and $q = 37$ find the roots of this second cubic equation. **[4]**

8 **(i)** The matrix **P** is given by $\mathbf{P} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by **P**. **[2]**

The matrix **R** is given by $\mathbf{R} = \begin{pmatrix} 2 - 3\sqrt{3} & 3 + 2\sqrt{3} \\ 1 + \sqrt{3} & -1 + \sqrt{3} \end{pmatrix}$.

(ii) Show that the multiplication of **P** and **R** is not commutative. **[2]**

(iii) The transformation represented by **R** is equivalent to the transformation represented by **P** followed by another transformation represented by the matrix **Q**. Find **Q**. **[5]**

(iv) The transformation represented by **Q** is applied to a figure of area 4 square units. Find the area of the transformed figure. **[2]**

END OF QUESTION PAPER