

Pearson Edexcel Level 3

**Advanced Subsidiary and Advanced GCE Mathematics and
Further Mathematics**

MATHEMATICAL FORMULAE AND STATISTICAL TABLES

FIRST CERTIFICATION FROM 2018

Advanced Subsidiary GCE in Mathematics (8MA0)

Advanced GCE in Mathematics (9MA0)

Advanced Subsidiary GCE in Further Mathematics (8FM0)

FIRST CERTIFICATION FROM 2019

Advanced GCE in Further Mathematics (9FM0)

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INTRODUCTION

The formulae in this booklet have been arranged by qualification. Students sitting AS or A Level Further Mathematics papers may be required to use the formulae that were introduced in AS or A Level Mathematics papers.

It may also be the case that students sitting Mechanics and Statistics papers will need to use formulae introduced in the appropriate Pure Mathematics papers for the qualification they are sitting.

1 AS Mathematics

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PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Binomial series

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Differentiation

First Principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

STATISTICS

Probability

$$P(A') = 1 - P(A)$$

Standard deviation

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

$$\text{Interquartile range} = \text{IQR} = Q_3 - Q_1$$

For a set of n values $x_1, x_2, \dots, x_i, \dots, x_n$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

Standard deviation (continued)

$$\text{Standard deviation} = \sqrt{\frac{S_{xx}}{n}} \text{ or } \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Statistical tables

The following statistical tables are required for A Level

Mathematics:

Binominal Cumulative Distribution Function (see pages 101 to 165)

Random Numbers (see pages 205 to 206)

MECHANICS

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

2. A Level Mathematics formulae

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PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$$

Binomial series

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r} x^r +$$

$$\dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad A \pm B \neq (k + \frac{1}{2})\pi$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

Trigonometric identities (continued)

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Small angle approximations

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

where θ is measured in radians

Differentiation

First Principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f(x)$	$f'(x)$
--------	---------

$\tan kx$	$k \sec^2 kx$
-----------	---------------

$\sec kx$	$k \sec kx \tan kx$
-----------	---------------------

$\cot kx$	$-k \operatorname{cosec}^2 kx$
-----------	--------------------------------

$\operatorname{cosec} kx$	$-k \operatorname{cosec} kx \cot kx$
---------------------------	--------------------------------------

First Principles (continued)

$$\frac{f(x)}{g(x)} \quad \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Integration (+ constant)

$$f(x) \quad \int f(x) dx$$

$$\sec^2 kx \quad \frac{1}{k} \tan kx$$

$$\tan kx \quad \frac{1}{k} \ln |\sec kx|$$

Integration (+ constant) (continued)

$$\cot kx \quad \frac{1}{k} \ln |\sin kx|$$

$$\operatorname{cosec} kx \quad -\frac{1}{k} \ln |\operatorname{cosec} kx + \cot kx|, \quad \frac{1}{k} \ln \left| \tan \left(\frac{1}{2} kx \right) \right|$$

$$\sec kx \quad \frac{1}{k} \ln |\sec kx + \tan kx|, \quad \frac{1}{k} \ln \left| \tan \left(\frac{1}{2} kx + \frac{1}{4} \pi \right) \right|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Numerical methods

The trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\},$$

where $h = \frac{b - a}{n}$

The Newton-Raphson iteration for solving

$$f(x) = 0 : x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

STATISTICS

Probability

$$P(A') = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

Probability (continued)

For independent events A and B,

$$P(B | A) = P(B)$$

$$P(A | B) = P(A)$$

$$P(A \cap B) = P(A)P(B)$$

Standard deviation

Standard deviation = $\sqrt{\text{Variance}}$

Interquartile range = IQR = $Q_3 - Q_1$

For a set of n values $x_1, x_2, \dots, x_i, \dots, x_n$

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$\text{Standard deviation} = \sqrt{\frac{S_{xx}}{n}} \quad \text{or} \quad \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

Discrete distributions

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

Statistical tables

The following statistical tables are required for A Level

Mathematics:

Binominal Cumulative Distribution Function (see pages 101 to 165)

Percentage Points of The Normal Distribution (see pages 166 to 167)

Critical Values for Correlation Coefficients: Product Moment Coefficient (see pages 192 to 204)

Random Numbers (see pages 205 to 206)

MECHANICS

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

3. AS Further Mathematics formulae

Students sitting an AS Level Further Mathematics paper may also require those formulae listed for A Level Mathematics in Section 2.

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PURE MATHEMATICS

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \text{ for all } x$$

Maclaurin's and Taylor's Series (continued)

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

Vectors

$$\text{Vector product: } \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

Vectors (continued)

If **A** is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through **A** with direction vector **b** has cartesian

equation
$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through **A** with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a} \cdot \mathbf{n}$

Vectors (continued)

The plane through non-collinear points **A**, **B** and **C** has

vector equation $r = a + \lambda(b - a) + \mu(c - a) = (1 - \lambda - \mu)a + \lambda b + \mu c$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $r = a + sb + tc$

The perpendicular distance of (α, β, γ) from

$$n_1x + n_2y + n_3z + d = 0 \text{ is } \frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\operatorname{arcosh} x = \ln \{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\operatorname{arsinh} x = \ln \{x + \sqrt{x^2 + 1}\}$$

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Differentiation

$f(x)$	$f'(x)$
--------------------------	---------------------------

$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
-------------------------------	--------------------------

$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
-------------------------------	---------------------------

$\arctan x$	$\frac{1}{1+x^2}$
-------------------------------	-------------------

$\sinh x$	$\cosh x$
-----------------------------	-----------------------------

$\cosh x$	$\sinh x$
-----------------------------	-----------------------------

Differentiation (continued)

$f(x)$	$f'(x)$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{arsinh} x$	$\frac{1}{\sqrt{1+x^2}}$
$\operatorname{arcosh} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{artanh} x$	$\frac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$$f(x) \quad \int f(x) dx$$

$$\sinh x \quad \cosh x$$

$$\cosh x \quad \sinh x$$

$$\tanh x \quad \operatorname{Incosh} x$$

$$\frac{1}{\sqrt{a^2 - x^2}} \quad \arcsin \left(\frac{x}{a} \right) \quad (|x| < a)$$

$$\frac{1}{a^2 + x^2} \quad \frac{1}{a} \arctan \left(\frac{x}{a} \right)$$

Integration (+ constant; $a > 0$ where relevant) (continued)

$$\frac{1}{\sqrt{x^2 - a^2}} \quad \operatorname{arcosh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2 - a^2}\} \quad (x > a)$$

$$\frac{1}{\sqrt{a^2 + x^2}} \quad \operatorname{arsinh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2 + a^2}\}$$

$$\frac{1}{a^2 - x^2} \quad \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| = \frac{1}{a} \operatorname{artanh} \left(\frac{x}{a} \right) \quad (|x| < a)$$

$$\frac{1}{x^2 - a^2} \quad \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

STATISTICS

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $P(X = x_i)$

Expectation (mean): $E(X) = \mu = \sum x_i P(X = x_i)$

Variance: $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 P(X = x_i) = \sum x_i^2 P(X = x_i) - \mu^2$

Discrete distributions (continued)**Standard discrete distributions:**

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1 - p)^{n-x}$	np	$np(1 - p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $E(X) = \mu = \int x f(x) dx$

Variance: $\text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x) f(x) dx$

Cumulative distribution function: $F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(t) dt$

Continuous distributions (continued)

Standard continuous distribution

Distributions of X	P.D.F.	Mean	Variance
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

Correlation and regression (continued)

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}}$$

$$= \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

Correlation and regression (continued)

The regression coefficient of y on X is

$$b = \frac{S_{xy}}{S_{xx}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

Least squares regression line of y on X is $y = a + bx$

where $a = \bar{y} - b\bar{x}$

$$\text{Residual Sum of Squares (RSS)} = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = S_{yy} (1 - r^2)$$

Correlation and regression (continued)

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_v$

Statistical tables

The following statistical tables are required for AS Level

Further Mathematics:

**Binominal Cumulative Distribution Function (see pages
101 to 165)**

**Poisson Cumulative Distribution Function (see pages
168 to 180)**

**Percentage Points of the χ^2 Distribution (see pages
181 to 191)**

Statistical tables (continued)

Critical Values for Correlation Coefficients: Product Moment Coefficient and Spearman's Coefficient (see pages 192 to 204)

Random Numbers (see pages 205 to 206)

MECHANICS

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r , angle at centre 2α : $\frac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r , angle at centre 2α : $\frac{2r \sin \alpha}{3\alpha}$

from centre

4. A Level Further Mathematics

Students sitting an A Level Further Mathematics paper may also require those formulae listed for A Level Mathematics in Section 2.

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PURE MATHEMATICS

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

Complex numbers

$$\{r(\cos\theta + i\sin\theta)\}^n = r^n (\cos n\theta + i\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Maclaurin's and Taylor's Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \text{ for all } x$$

Maclaurin's and Taylor's Series (continued)

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

Vectors

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\hat{\mathbf{n}}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{vmatrix} = \begin{pmatrix} \mathbf{a}_2\mathbf{b}_3 - \mathbf{a}_3\mathbf{b}_2 \\ \mathbf{a}_3\mathbf{b}_1 - \mathbf{a}_1\mathbf{b}_3 \\ \mathbf{a}_1\mathbf{b}_2 - \mathbf{a}_2\mathbf{b}_1 \end{pmatrix}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \\ \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{vmatrix} = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

Vectors (continued)

If **A** is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector \mathbf{b} is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through **A** with direction vector \mathbf{b} has cartesian

equation
$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through **A** with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a} \cdot \mathbf{n}$

Vectors (continued)

The plane through non-collinear points **A**, **B** and **C** has vector equation $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from

$$n_1x + n_2y + n_3z + d = 0 \text{ is } \frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\operatorname{arcosh} x = \ln \{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\operatorname{arsinh} x = \ln \{x + \sqrt{x^2 + 1}\}$$

$$\operatorname{artanh} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$
Parametric Form	$(a \cos \theta, b \sin \theta)$	$(at^2, 2at)$
Eccentricity	$e < 1$ $b^2 = a^2 (1 - e^2)$	$e = 1$
Foci	$(\pm ae, 0)$	$(a, 0)$
Directrices	$x = \pm \frac{a}{e}$	$x = -a$
Asymptotes	none	none

Conics (continued)

	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a \sec \theta, b \tan \theta)$ $(\pm a \cosh \theta, b \sinh \theta)$	$\left(ct, \frac{c}{t} \right)$
Eccentricity	$e > 1$ $b^2 = a^2 (e^2 - 1)$	$e = \sqrt{2}$
Foci	$(\pm ae, 0)$	$(\pm \sqrt{2}c, \pm \sqrt{2}c)$
Directrices	$x = \pm \frac{a}{e}$	$x + y = \pm \sqrt{2}c$
Asymptotes	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

Differentiation

f(x) **f'(x)**

arcsin x $\frac{1}{\sqrt{1-x^2}}$

arccos x $-\frac{1}{\sqrt{1-x^2}}$

arctan x $\frac{1}{1+x^2}$

sinh x **cosh x**

cosh x **sinh x**

Differentiation (continued)

f(x)	f'(x)
tanh x	sech² x
arsinh x	$\frac{1}{\sqrt{1+x^2}}$
arcosh x	$\frac{1}{\sqrt{x^2-1}}$
artanh x	$\frac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$$f(x) \quad \int f(x) dx$$

$$\sinh x \quad \cosh x$$

$$\cosh x \quad \sinh x$$

$$\tanh x \quad \operatorname{Incosh} x$$

$$\frac{1}{\sqrt{a^2 - x^2}} \quad \arcsin \left(\frac{x}{a} \right) \quad (|x| < a)$$

$$\frac{1}{a^2 + x^2} \quad \frac{1}{a} \arctan \left(\frac{x}{a} \right)$$

Integration (+ constant; $a > 0$ where relevant) (continued)

$$f(x) \quad \int f(x) dx$$

$$\frac{1}{\sqrt{x^2 - a^2}} \quad \operatorname{arcosh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2 - a^2}\} \quad (x > a)$$

$$\frac{1}{\sqrt{a^2 + x^2}} \quad \operatorname{arsinh} \left(\frac{x}{a} \right), \quad \ln \{x + \sqrt{x^2 + a^2}\}$$

$$\frac{1}{a^2 - x^2} \quad \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| = \frac{1}{a} \operatorname{artanh} \left(\frac{x}{a} \right) \quad (|x| < a)$$

$$\frac{1}{x^2 - a^2} \quad \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \text{(cartesian coordinates)}$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad \text{(parametric form)}$$

$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \quad \text{(polar form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$S_x = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

$$S_x = 2\pi \int r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \quad (\text{polar form})$$

STATISTICS

Discrete distributions

For a discrete random variable X taking values x_i with probabilities $P(X = x_i)$

Expectation (mean): $E(X) = \mu = \sum x_i P(X = x_i)$

Variance: $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 P(X = x_i) = \sum x_i^2 P(X = x_i) - \mu^2$

For a function $g(X)$: $E(g(X)) = \sum g(x_i) P(X = x_i)$

The probability generating function of X is $G_X(t) = E(t^X)$ and

$E(X) = G'_X(1)$ and $\text{Var}(X) = G''_X(1) + G'_X(1) - [G'_X(1)]^2$

Discrete distributions (continued)

For $Z = X + Y$, where X and Y are independent:

$$\mathbf{G_Z(t) = G_X(t) \times G_Y(t)}$$

Discrete distributions

Standard discrete distributions:

Distribution of X	$P(X = x)$	Mean	Variance	P.G.F.
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pt)^n$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ	$e^{\lambda(t-1)}$
Geometric Geo (p) on $1, 2, \dots$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pt}{1-(1-p)t}$
Negative binomial on $r, r+1, \dots$	$\binom{x-1}{r-1} p^r (1-p)^{x-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pt}{1-(1-p)t} \right)^r$

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $E(X) = \mu = \int x f(x) dx$

Variance: $\text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x) f(x) dx$

Cumulative distribution function: $F(x_0) = P(X \leq x_0) = \int_{-\infty}^{x_0} f(t) dt$

Standard continuous distribution:

Distribution of X	P.D.F.	Mean	Variance
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

Correlation and regression (continued)

The product moment correlation coefficient is

$$\begin{aligned} r &= \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} \\ &= \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}} \end{aligned}$$

Correlation and regression (continued)

The regression coefficient of y on X is

$$b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

Least squares regression line of y on X is $y = a + bx$

where $a = \bar{y} - b\bar{x}$

$$\text{Residual Sum of Squares (RSS)} = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = S_{yy} (1 - r^2)$$

Correlation and regression (continued)

Spearman's rank correlation coefficient is $r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$

Expectation algebra

For independent random variables X and Y

$$E(XY) = E(X)E(Y), \text{ Var}(aX \pm bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$$

Sampling distributions

(i) Tests for mean when σ is known

For a random sample X_1, X_2, \dots, X_n of n independent observations from a distribution having mean μ and variance σ^2 :

\bar{X} is an unbiased estimator of μ , with $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2 , where $S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$

Sampling distributions (continued)

For a random sample of n observations from $N(\mu, \sigma^2)$,

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$

and, independently, a random sample of n_y observations

from, $N(\mu_y, \sigma_y^2)$,

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

Sampling distributions (continued)

(ii) Tests for variance and mean when σ is not known

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$$

$$\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{n-1} \text{ (also valid in matched-pairs situations)}$$

Sampling distributions (continued)

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$
and, independently, a random sample of n_y observations
from $N(\mu_y, \sigma_y^2)$

$$\frac{S_x^2 / \sigma_x^2}{S_y^2 / \sigma_y^2} \sim F_{n_x-1, n_y-1}$$

Sampling distributions (continued)

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}} \sim t_{n_x + n_y - 2}$$

where $S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$

Non-parametric tests

Goodness-of-fit test and contingency tables:

$$\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_v^2$$

Statistical tables

The following statistical tables are required for A Level

Further Mathematics:

Binominal Cumulative Distribution Function (see pages 101 to 165)

Poisson Cumulative Distribution Function (see pages 168 to 180)

Percentage Points of the χ^2 Distribution (see pages 181 to 191)

Statistical tables (continued)

Critical Values for Correlation Coefficients: Product Moment Coefficient and Spearman's Coefficient (see pages 192 to 204)

Random Numbers (see pages 205 to 206)

Percentage points of Student's t Distribution (see pages 207 to 213)

Percentage points of the F Distribution (see pages 214 to 238)

MECHANICS

Centres of mass

For uniform bodies:

Triangular lamina: $\frac{2}{3}$ along median from vertex

Circular arc, radius r , angle at centre 2α : $\frac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r , angle at centre 2α : $\frac{2r \sin \alpha}{3\alpha}$ from centre

Solid hemisphere, radius r : $\frac{3}{8}r$ from centre

Centres of mass (continued)

Hemispherical shell, radius r : $\frac{1}{2}r$ from centre

**Solid cone or pyramid of height h : $\frac{1}{4}h$ above the base on
the line from centre of base to vertex**

**Conical shell of height h : $\frac{1}{3}h$ above the base on the line
from centre of base to vertex**

Motion in a circle

Transverse velocity: $v = r \dot{\theta}$

Transverse acceleration: $\dot{v} = r \ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

5. STATISTICAL TABLES

BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a binomial distribution with index n and parameter p .

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 5, x = 0	0.7738	0.5905	0.4437	0.3277	0.2373
1	0.9774	0.9185	0.8352	0.7373	0.6328
2	0.9988	0.9914	0.9734	0.9421	0.8965
3	1.0000	0.9995	0.9978	0.9933	0.9844
4	1.0000	1.0000	0.9999	0.9997	0.9990

Binomial Cumulative Distribution Function (continued)

n = 5 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 5, x = 0	0.1681	0.1160	0.0778	0.0503	0.0312
1	0.5282	0.4284	0.3370	0.2562	0.1875
2	0.8369	0.7648	0.6826	0.5931	0.5000
3	0.9692	0.9460	0.9130	0.8688	0.8125
4	0.9976	0.9947	0.9898	0.9815	0.9688

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 6, x = 0	0.7351	0.5314	0.3771	0.2621	0.1780
1	0.9672	0.8857	0.7765	0.6554	0.5339
2	0.9978	0.9842	0.9527	0.9011	0.8306
3	0.9999	0.9987	0.9941	0.9830	0.9624
4	1.0000	0.9999	0.9996	0.9984	0.9954
5	1.0000	1.0000	1.0000	0.9999	0.9998

Binomial Cumulative Distribution Function (continued)

	n = 6 (continued)				
p =	0.30	0.35	0.40	0.45	0.50
n = 6, x = 0	0.1176	0.0754	0.0467	0.0277	0.0156
1	0.4202	0.3191	0.2333	0.1636	0.1094
2	0.7443	0.6471	0.5443	0.4415	0.3438
3	0.9295	0.8826	0.8208	0.7447	0.6563
4	0.9891	0.9777	0.9590	0.9308	0.8906
5	0.9993	0.9982	0.9959	0.9917	0.9844

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 7, x = 0	0.6983	0.4783	0.3206	0.2097	0.1335
1	0.9556	0.8503	0.7166	0.5767	0.4449
2	0.9962	0.9743	0.9262	0.8520	0.7564
3	0.9998	0.9973	0.9879	0.9667	0.9294
4	1.0000	0.9998	0.9988	0.9953	0.9871
5	1.0000	1.0000	0.9999	0.9996	0.9987
6	1.0000	1.0000	1.0000	1.0000	0.9999

Binomial Cumulative Distribution Function (continued)

	n = 7 (continued)				
p =	0.30	0.35	0.40	0.45	0.50
n = 7, x = 0	0.0824	0.0490	0.0280	0.0152	0.0078
1	0.3294	0.2338	0.1586	0.1024	0.0625
2	0.6471	0.5323	0.4199	0.3164	0.2266
3	0.8740	0.8002	0.7102	0.6083	0.5000
4	0.9712	0.9444	0.9037	0.8471	0.7734
5	0.9962	0.9910	0.9812	0.9643	0.9375
6	0.9998	0.9994	0.9984	0.9963	0.9922

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 8, x = 0	0.6634	0.4305	0.2725	0.1678	0.1001
1	0.9428	0.8131	0.6572	0.5033	0.3671
2	0.9942	0.9619	0.8948	0.7969	0.6785
3	0.9996	0.9950	0.9786	0.9437	0.8862
4	1.0000	0.9996	0.9971	0.9896	0.9727
5	1.0000	1.0000	0.9998	0.9988	0.9958
6	1.0000	1.0000	1.0000	0.9999	0.9996
7	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

	n = 8 (continued)				
p =	0.30	0.35	0.40	0.45	0.50
n = 8, x = 0	0.0576	0.0319	0.0168	0.0084	0.0039
1	0.2553	0.1691	0.1064	0.0632	0.0352
2	0.5518	0.4278	0.3154	0.2201	0.1445
3	0.8059	0.7064	0.5941	0.4770	0.3633
4	0.9420	0.8939	0.8263	0.7396	0.6367
5	0.9887	0.9747	0.9502	0.9115	0.8555
6	0.9987	0.9964	0.9915	0.9819	0.9648
7	0.9999	0.9998	0.9993	0.9983	0.9961

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 9, x = 0	0.6302	0.3874	0.2316	0.1342	0.0751
1	0.9288	0.7748	0.5995	0.4362	0.3003
2	0.9916	0.9470	0.8591	0.7382	0.6007
3	0.9994	0.9917	0.9661	0.9144	0.8343
4	1.0000	0.9991	0.9944	0.9804	0.9511
5	1.0000	0.9999	0.9994	0.9969	0.9900

Binomial Cumulative Distribution Function (continued)**n = 9 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 9, x = 6	1.0000	1.0000	1.0000	0.9997	0.9987
7	1.0000	1.0000	1.0000	1.0000	0.9999
8	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 9 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 9, x = 0	0.0404	0.0207	0.0101	0.0046	0.0020
1	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.7297	0.6089	0.4826	0.3614	0.2539
4	0.9012	0.8283	0.7334	0.6214	0.5000
5	0.9747	0.9464	0.9006	0.8342	0.7461

Binomial Cumulative Distribution Function (continued)**n = 9 (continued)**

p =	0.30	0.35	0.40	0.45	0.50
n = 9, x = 6	0.9957	0.9888	0.9750	0.9502	0.9102
7	0.9996	0.9986	0.9962	0.9909	0.9805
8	1.0000	0.9999	0.9997	0.9992	0.9980

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 10, x = 0	0.5987	0.3487	0.1969	0.1074	0.0563
1	0.9139	0.7361	0.5443	0.3758	0.2440
2	0.9885	0.9298	0.8202	0.6778	0.5256
3	0.9990	0.9872	0.9500	0.8791	0.7759
4	0.9999	0.9984	0.9901	0.9672	0.9219
5	1.0000	0.9999	0.9986	0.9936	0.9803

Binomial Cumulative Distribution Function (continued)

n = 10 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 10, x = 6	1.0000	1.0000	0.9999	0.9991	0.9965
7	1.0000	1.0000	1.0000	0.9999	0.9996
8	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 10 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 10, x = 0	0.0282	0.0135	0.0060	0.0025	0.0010
1	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.8497	0.7515	0.6331	0.5044	0.3770
5	0.9527	0.9051	0.8338	0.7384	0.6230

Binomial Cumulative Distribution Function (continued)

n = 10 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 10, x = 6	0.9894	0.9740	0.9452	0.8980	0.8281
7	0.9984	0.9952	0.9877	0.9726	0.9453
8	0.9999	0.9995	0.9983	0.9955	0.9893
9	1.0000	1.0000	0.9999	0.9997	0.9990

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 12, x = 0	0.5404	0.2824	0.1422	0.0687	0.0317
1	0.8816	0.6590	0.4435	0.2749	0.1584
2	0.9804	0.8891	0.7358	0.5583	0.3907
3	0.9978	0.9744	0.9078	0.7946	0.6488
4	0.9998	0.9957	0.9761	0.9274	0.8424
5	1.0000	0.9995	0.9954	0.9806	0.9456

Binomial Cumulative Distribution Function (continued)**n = 12 (continued)**

p =	0.30	0.35	0.40	0.45	0.50
n = 12, x = 0	0.0138	0.0057	0.0022	0.0008	0.0002
1	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.7237	0.5833	0.4382	0.3044	0.1938
5	0.8822	0.7873	0.6652	0.5269	0.3872

Binomial Cumulative Distribution Function (continued)

n = 12 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 12, x = 6	1.0000	0.9999	0.9993	0.9961	0.9857
7	1.0000	1.0000	0.9999	0.9994	0.9972
8	1.0000	1.0000	1.0000	0.9999	0.9996
9	1.0000	1.0000	1.0000	1.0000	1.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 12 (continued)**

p =	0.30	0.35	0.40	0.45	0.50
n = 12, x = 6	0.9614	0.9154	0.8418	0.7393	0.6128
7	0.9905	0.9745	0.9427	0.8883	0.8062
8	0.9983	0.9944	0.9847	0.9644	0.9270
9	0.9998	0.9992	0.9972	0.9921	0.9807
10	1.0000	0.9999	0.9997	0.9989	0.9968
11	1.0000	1.0000	1.0000	0.9999	0.9998

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 15, x = 0	0.4633	0.2059	0.0874	0.0352	0.0134
1	0.8290	0.5490	0.3186	0.1671	0.0802
2	0.9638	0.8159	0.6042	0.3980	0.2361
3	0.9945	0.9444	0.8227	0.6482	0.4613
4	0.9994	0.9873	0.9383	0.8358	0.6865
5	0.9999	0.9978	0.9832	0.9389	0.8516
6	1.0000	0.9997	0.9964	0.9819	0.9434
7	1.0000	1.0000	0.9994	0.9958	0.9827

Binomial Cumulative Distribution Function (continued)

n = 15 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 15, x = 8	1.0000	1.0000	0.9999	0.9992	0.9958
9	1.0000	1.0000	1.0000	0.9999	0.9992
10	1.0000	1.0000	1.0000	1.0000	0.9999
11	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 15 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 15, x = 0	0.0047	0.0016	0.0005	0.0001	0.0000
1	0.0353	0.0142	0.0052	0.0017	0.0005
2	0.1268	0.0617	0.0271	0.0107	0.0037
3	0.2969	0.1727	0.0905	0.0424	0.0176
4	0.5155	0.3519	0.2173	0.1204	0.0592
5	0.7216	0.5643	0.4032	0.2608	0.1509
6	0.8689	0.7548	0.6098	0.4522	0.3036
7	0.9500	0.8868	0.7869	0.6535	0.5000

Binomial Cumulative Distribution Function (continued)

n = 15 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 15, x = 8	0.9848	0.9578	0.9050	0.8182	0.6964
9	0.9963	0.9876	0.9662	0.9231	0.8491
10	0.9993	0.9972	0.9907	0.9745	0.9408
11	0.9999	0.9995	0.9981	0.9937	0.9824
12	1.0000	0.9999	0.9997	0.9989	0.9963
13	1.0000	1.0000	1.0000	0.9999	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 20, x = 0	0.3585	0.1216	0.0388	0.0115	0.0032
1	0.7358	0.3917	0.1756	0.0692	0.0243
2	0.9245	0.6769	0.4049	0.2061	0.0913
3	0.9841	0.8670	0.6477	0.4114	0.2252
4	0.9974	0.9568	0.8298	0.6296	0.4148
5	0.9997	0.9887	0.9327	0.8042	0.6172
6	1.0000	0.9976	0.9781	0.9133	0.7858

Binomial Cumulative Distribution Function (continued)

n = 20 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 20, x = 7	1.0000	0.9996	0.9941	0.9679	0.8982
8	1.0000	0.9999	0.9987	0.9900	0.9591
9	1.0000	1.0000	0.9998	0.9974	0.9861
10	1.0000	1.0000	1.0000	0.9994	0.9961
11	1.0000	1.0000	1.0000	0.9999	0.9991
12	1.0000	1.0000	1.0000	1.0000	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 20 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 20, x = 15	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 20 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 20, x = 0	0.0008	0.0002	0.0000	0.0000	0.0000
1	0.0076	0.0021	0.0005	0.0001	0.0000
2	0.0355	0.0121	0.0036	0.0009	0.0002
3	0.1071	0.0444	0.0160	0.0049	0.0013
4	0.2375	0.1182	0.0510	0.0189	0.0059
5	0.4164	0.2454	0.1256	0.0553	0.0207
6	0.6080	0.4166	0.2500	0.1299	0.0577

Binomial Cumulative Distribution Function (continued)

n = 20 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 20, x = 7	0.7723	0.6010	0.4159	0.2520	0.1316
8	0.8867	0.7624	0.5956	0.4143	0.2517
9	0.9520	0.8782	0.7553	0.5914	0.4119
10	0.9829	0.9468	0.8725	0.7507	0.5881
11	0.9949	0.9804	0.9435	0.8692	0.7483
12	0.9987	0.9940	0.9790	0.9420	0.8684
13	0.9997	0.9985	0.9935	0.9786	0.9423
14	1.0000	0.9997	0.9984	0.9936	0.9793

Binomial Cumulative Distribution Function (continued)

n = 20 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 20, x = 15	1.0000	1.0000	0.9997	0.9985	0.9941
16	1.0000	1.0000	1.0000	0.9997	0.9987
17	1.0000	1.0000	1.0000	1.0000	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 25, x = 0	0.2774	0.0718	0.0172	0.0038	0.0008
1	0.6424	0.2712	0.0931	0.0274	0.0070
2	0.8729	0.5371	0.2537	0.0982	0.0321
3	0.9659	0.7636	0.4711	0.2340	0.0962
4	0.9928	0.9020	0.6821	0.4207	0.2137
5	0.9988	0.9666	0.8385	0.6167	0.3783
6	0.9998	0.9905	0.9305	0.7800	0.5611
7	1.0000	0.9977	0.9745	0.8909	0.7265

Binomial Cumulative Distribution Function (continued)

n = 25 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 25, x = 8	1.0000	0.9995	0.9920	0.9532	0.8506
9	1.0000	0.9999	0.9979	0.9827	0.9287
10	1.0000	1.0000	0.9995	0.9944	0.9703
11	1.0000	1.0000	0.9999	0.9985	0.9893
12	1.0000	1.0000	1.0000	0.9996	0.9966
13	1.0000	1.0000	1.0000	0.9999	0.9991
14	1.0000	1.0000	1.0000	1.0000	0.9998

Binomial Cumulative Distribution Function (continued)

n = 25 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 25, x = 15	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 25 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 25, x = 0	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.0016	0.0003	0.0001	0.0000	0.0000
2	0.0090	0.0021	0.0004	0.0001	0.0000
3	0.0332	0.0097	0.0024	0.0005	0.0001
4	0.0905	0.0320	0.0095	0.0023	0.0005
5	0.1935	0.0826	0.0294	0.0086	0.0020
6	0.3407	0.1734	0.0736	0.0258	0.0073
7	0.5118	0.3061	0.1536	0.0639	0.0216

Binomial Cumulative Distribution Function (continued)

n = 25 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 25, x = 8	0.6769	0.4668	0.2735	0.1340	0.0539
9	0.8106	0.6303	0.4246	0.2424	0.1148
10	0.9022	0.7712	0.5858	0.3843	0.2122
11	0.9558	0.8746	0.7323	0.5426	0.3450
12	0.9825	0.9396	0.8462	0.6937	0.5000
13	0.9940	0.9745	0.9222	0.8173	0.6550
14	0.9982	0.9907	0.9656	0.9040	0.7878

Binomial Cumulative Distribution Function (continued)

n = 25 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 25, x = 15	0.9995	0.9971	0.9868	0.9560	0.8852
16	0.9999	0.9992	0.9957	0.9826	0.9461
17	1.0000	0.9998	0.9988	0.9942	0.9784
18	1.0000	1.0000	0.9997	0.9984	0.9927
19	1.0000	1.0000	0.9999	0.9996	0.9980
20	1.0000	1.0000	1.0000	0.9999	0.9995
21	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 30, x = 0	0.2146	0.0424	0.0076	0.0012	0.0002
1	0.5535	0.1837	0.0480	0.0105	0.0020
2	0.8122	0.4114	0.1514	0.0442	0.0106
3	0.9392	0.6474	0.3217	0.1227	0.0374
4	0.9844	0.8245	0.5245	0.2552	0.0979
5	0.9967	0.9268	0.7106	0.4275	0.2026
6	0.9994	0.9742	0.8474	0.6070	0.3481
7	0.9999	0.9922	0.9302	0.7608	0.5143

Binomial Cumulative Distribution Function (continued)

	n = 30 (continued)				
p =	0.05	0.10	0.15	0.20	0.25
n = 30, x = 8	1.0000	0.9980	0.9722	0.8713	0.6736
9	1.0000	0.9995	0.9903	0.9389	0.8034
10	1.0000	0.9999	0.9971	0.9744	0.8943
11	1.0000	1.0000	0.9992	0.9905	0.9493
12	1.0000	1.0000	0.9998	0.9969	0.9784
13	1.0000	1.0000	1.0000	0.9991	0.9918
14	1.0000	1.0000	1.0000	0.9998	0.9973

Binomial Cumulative Distribution Function (continued)

n = 30 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 30, x = 15	1.0000	1.0000	1.0000	0.9999	0.9992
16	1.0000	1.0000	1.0000	1.0000	0.9998
17	1.0000	1.0000	1.0000	1.0000	0.9999
18	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 30 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 30, x = 22	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 30 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 30, x = 0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0003	0.0000	0.0000	0.0000	0.0000
2	0.0021	0.0003	0.0000	0.0000	0.0000
3	0.0093	0.0019	0.0003	0.0000	0.0000
4	0.0302	0.0075	0.0015	0.0002	0.0000
5	0.0766	0.0233	0.0057	0.0011	0.0002
6	0.1595	0.0586	0.0172	0.0040	0.0007
7	0.2814	0.1238	0.0435	0.0121	0.0026

Binomial Cumulative Distribution Function (continued)

n = 30 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 30, x = 8	0.4315	0.2247	0.0940	0.0312	0.0081
9	0.5888	0.3575	0.1763	0.0694	0.0214
10	0.7304	0.5078	0.2915	0.1350	0.0494
11	0.8407	0.6548	0.4311	0.2327	0.1002
12	0.9155	0.7802	0.5785	0.3592	0.1808
13	0.9599	0.8737	0.7145	0.5025	0.2923
14	0.9831	0.9348	0.8246	0.6448	0.4278

Binomial Cumulative Distribution Function (continued)

n = 30 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 30, x = 15	0.9936	0.9699	0.9029	0.7691	0.5722
16	0.9979	0.9876	0.9519	0.8644	0.7077
17	0.9994	0.9955	0.9788	0.9286	0.8192
18	0.9998	0.9986	0.9917	0.9666	0.8998
19	1.0000	0.9996	0.9971	0.9862	0.9506
20	1.0000	0.9999	0.9991	0.9950	0.9786
21	1.0000	1.0000	0.9998	0.9984	0.9919

Binomial Cumulative Distribution Function (continued)

n = 30 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 30, x = 22	1.0000	1.0000	1.0000	0.9996	0.9974
23	1.0000	1.0000	1.0000	0.9999	0.9993
24	1.0000	1.0000	1.0000	1.0000	0.9998
25	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 40, x = 0	0.1285	0.0148	0.0015	0.0001	0.0000
1	0.3991	0.0805	0.0121	0.0015	0.0001
2	0.6767	0.2228	0.0486	0.0079	0.0010
3	0.8619	0.4231	0.1302	0.0285	0.0047
4	0.9520	0.6290	0.2633	0.0759	0.0160
5	0.9861	0.7937	0.4325	0.1613	0.0433
6	0.9966	0.9005	0.6067	0.2859	0.0962
7	0.9993	0.9581	0.7559	0.4371	0.1820

Binomial Cumulative Distribution Function (continued)

	n = 40 (continued)				
p =	0.05	0.10	0.15	0.20	0.25
n = 40, x = 8	0.9999	0.9845	0.8646	0.5931	0.2998
9	1.0000	0.9949	0.9328	0.7318	0.4395
10	1.0000	0.9985	0.9701	0.8392	0.5839
11	1.0000	0.9996	0.9880	0.9125	0.7151
12	1.0000	0.9999	0.9957	0.9568	0.8209
13	1.0000	1.0000	0.9986	0.9806	0.8968
14	1.0000	1.0000	0.9996	0.9921	0.9456
15	1.0000	1.0000	0.9999	0.9971	0.9738

Binomial Cumulative Distribution Function (continued)

	n = 40 (continued)				
p =	0.05	0.10	0.15	0.20	0.25
n = 40, x = 16	1.0000	1.0000	1.0000	0.9990	0.9884
17	1.0000	1.0000	1.0000	0.9997	0.9953
18	1.0000	1.0000	1.0000	0.9999	0.9983
19	1.0000	1.0000	1.0000	1.0000	0.9994
20	1.0000	1.0000	1.0000	1.0000	0.9998
21	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 40 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 40, x = 24	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 40 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 40, x = 31	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 40 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 40, x = 0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0006	0.0001	0.0000	0.0000	0.0000
4	0.0026	0.0003	0.0000	0.0000	0.0000
5	0.0086	0.0013	0.0001	0.0000	0.0000
6	0.0238	0.0044	0.0006	0.0001	0.0000
7	0.0553	0.0124	0.0021	0.0002	0.0000

Binomial Cumulative Distribution Function (continued)

n = 40 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 40, x = 8	0.1110	0.0303	0.0061	0.0009	0.0001
9	0.1959	0.0644	0.0156	0.0027	0.0003
10	0.3087	0.1215	0.0352	0.0074	0.0011
11	0.4406	0.2053	0.0709	0.0179	0.0032
12	0.5772	0.3143	0.1285	0.0386	0.0083
13	0.7032	0.4408	0.2112	0.0751	0.0192
14	0.8074	0.5721	0.3174	0.1326	0.0403
15	0.8849	0.6946	0.4402	0.2142	0.0769

Binomial Cumulative Distribution Function (continued)

n = 40 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 40, x = 16	0.9367	0.7978	0.5681	0.3185	0.1341
17	0.9680	0.8761	0.6885	0.4391	0.2148
18	0.9852	0.9301	0.7911	0.5651	0.3179
19	0.9937	0.9637	0.8702	0.6844	0.4373
20	0.9976	0.9827	0.9256	0.7870	0.5627
21	0.9991	0.9925	0.9608	0.8669	0.6821
22	0.9997	0.9970	0.9811	0.9233	0.7852
23	0.9999	0.9989	0.9917	0.9595	0.8659

Binomial Cumulative Distribution Function (continued)

n = 40 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 40, x = 24	1.0000	0.9996	0.9966	0.9804	0.9231
25	1.0000	0.9999	0.9988	0.9914	0.9597
26	1.0000	1.0000	0.9996	0.9966	0.9808
27	1.0000	1.0000	0.9999	0.9988	0.9917
28	1.0000	1.0000	1.0000	0.9996	0.9968
29	1.0000	1.0000	1.0000	0.9999	0.9989
30	1.0000	1.0000	1.0000	1.0000	0.9997

Binomial Cumulative Distribution Function (continued)

n = 40 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 40, x = 31	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 50, x = 0	0.0769	0.0052	0.0003	0.0000	0.0000
1	0.2794	0.0338	0.0029	0.0002	0.0000
2	0.5405	0.1117	0.0142	0.0013	0.0001
3	0.7604	0.2503	0.0460	0.0057	0.0005
4	0.8964	0.4312	0.1121	0.0185	0.0021
5	0.9622	0.6161	0.2194	0.0480	0.0070
6	0.9882	0.7702	0.3613	0.1034	0.0194
7	0.9968	0.8779	0.5188	0.1904	0.0453

Binomial Cumulative Distribution Function (continued)

		n = 50 (continued)				
p =		0.05	0.10	0.15	0.20	0.25
n = 50, x =	8	0.9992	0.9421	0.6681	0.3073	0.0916
	9	0.9998	0.9755	0.7911	0.4437	0.1637
	10	1.0000	0.9906	0.8801	0.5836	0.2622
	11	1.0000	0.9968	0.9372	0.7107	0.3816
	12	1.0000	0.9990	0.9699	0.8139	0.5110
	13	1.0000	0.9997	0.9868	0.8894	0.6370
	14	1.0000	0.9999	0.9947	0.9393	0.7481
	15	1.0000	1.0000	0.9981	0.9692	0.8369

Binomial Cumulative Distribution Function (continued)

	n = 50 (continued)				
p =	0.05	0.10	0.15	0.20	0.25
n = 50, x = 16	1.0000	1.0000	0.9993	0.9856	0.9017
17	1.0000	1.0000	0.9998	0.9937	0.9449
18	1.0000	1.0000	0.9999	0.9975	0.9713
19	1.0000	1.0000	1.0000	0.9991	0.9861
20	1.0000	1.0000	1.0000	0.9997	0.9937
21	1.0000	1.0000	1.0000	0.9999	0.9974
22	1.0000	1.0000	1.0000	1.0000	0.9990
23	1.0000	1.0000	1.0000	1.0000	0.9996

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.05	0.10	0.15	0.20	0.25
n = 50, x = 24	1.0000	1.0000	1.0000	1.0000	0.9999
25	1.0000	1.0000	1.0000	1.0000	1.0000
26	1.0000	1.0000	1.0000	1.0000	1.0000
27	1.0000	1.0000	1.0000	1.0000	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)**n = 50 (continued)**

p =	0.05	0.10	0.15	0.20	0.25
n = 50, x = 32	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000
36	1.0000	1.0000	1.0000	1.0000	1.0000
37	1.0000	1.0000	1.0000	1.0000	1.0000
38	1.0000	1.0000	1.0000	1.0000	1.0000

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 50, x = 0	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.0007	0.0001	0.0000	0.0000	0.0000
6	0.0025	0.0002	0.0000	0.0000	0.0000
7	0.0073	0.0008	0.0001	0.0000	0.0000

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 50, x = 8	0.0183	0.0025	0.0002	0.0000	0.0000
9	0.0402	0.0067	0.0008	0.0001	0.0000
10	0.0789	0.0160	0.0022	0.0002	0.0000
11	0.1390	0.0342	0.0057	0.0006	0.0000
12	0.2229	0.0661	0.0133	0.0018	0.0002
13	0.3279	0.1163	0.0280	0.0045	0.0005
14	0.4468	0.1878	0.0540	0.0104	0.0013
15	0.5692	0.2801	0.0955	0.0220	0.0033

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 50, x = 16	0.6839	0.3889	0.1561	0.0427	0.0077
17	0.7822	0.5060	0.2369	0.0765	0.0164
18	0.8594	0.6216	0.3356	0.1273	0.0325
19	0.9152	0.7264	0.4465	0.1974	0.0595
20	0.9522	0.8139	0.5610	0.2862	0.1013
21	0.9749	0.8813	0.6701	0.3900	0.1611
22	0.9877	0.9290	0.7660	0.5019	0.2399
23	0.9944	0.9604	0.8438	0.6134	0.3359

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 50, x = 24	0.9976	0.9793	0.9022	0.7160	0.4439
25	0.9991	0.9900	0.9427	0.8034	0.5561
26	0.9997	0.9955	0.9686	0.8721	0.6641
27	0.9999	0.9981	0.9840	0.9220	0.7601
28	1.0000	0.9993	0.9924	0.9556	0.8389
29	1.0000	0.9997	0.9966	0.9765	0.8987
30	1.0000	0.9999	0.9986	0.9884	0.9405
31	1.0000	1.0000	0.9995	0.9947	0.9675

Binomial Cumulative Distribution Function (continued)

n = 50 (continued)

p =	0.30	0.35	0.40	0.45	0.50
n = 50, x = 32	1.0000	1.0000	0.9998	0.9978	0.9836
33	1.0000	1.0000	0.9999	0.9991	0.9923
34	1.0000	1.0000	1.0000	0.9997	0.9967
35	1.0000	1.0000	1.0000	0.9999	0.9987
36	1.0000	1.0000	1.0000	1.0000	0.9995
37	1.0000	1.0000	1.0000	1.0000	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The values z in the table are those which a random variable $Z \sim N(0, 1)$ exceeds with probability p ; that is,

$$P(Z > z) = 1 - \Phi(z) = p.$$

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION**(continued)**

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0364	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

POISSON CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a Poisson distribution with parameter λ .

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	0.5	1.0	1.5	2.0	2.5
x = 0	0.6065	0.3679	0.2231	0.1353	0.0821
1	0.9098	0.7358	0.5578	0.4060	0.2873
2	0.9856	0.9197	0.8088	0.6767	0.5438
3	0.9982	0.9810	0.9344	0.8571	0.7576
4	0.9998	0.9963	0.9814	0.9473	0.8912
5	1.0000	0.9994	0.9955	0.9834	0.9580
6	1.0000	0.9999	0.9991	0.9955	0.9858

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	0.5	1.0	1.5	2.0	2.5
$x = 7$	1.0000	1.0000	0.9998	0.9989	0.9958
8	1.0000	1.0000	1.0000	0.9998	0.9989
9	1.0000	1.0000	1.0000	1.0000	0.9997
10	1.0000	1.0000	1.0000	1.0000	0.9999
11	1.0000	1.0000	1.0000	1.0000	1.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	0.5	1.0	1.5	2.0	2.5
x = 15	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	3.0	3.5	4.0	4.5	5.0
x = 0	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.8153	0.7254	0.6288	0.5321	0.4405
5	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9665	0.9347	0.8893	0.8311	0.7622

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	3.0	3.5	4.0	4.5	5.0
$x = 7$	0.9881	0.9733	0.9489	0.9134	0.8666
8	0.9962	0.9901	0.9786	0.9597	0.9319
9	0.9989	0.9967	0.9919	0.9829	0.9682
10	0.9997	0.9990	0.9972	0.9933	0.9863
11	0.9999	0.9997	0.9991	0.9976	0.9945
12	1.0000	0.9999	0.9997	0.9992	0.9980
13	1.0000	1.0000	0.9999	0.9997	0.9993
14	1.0000	1.0000	1.0000	0.9999	0.9998

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	3.0	3.5	4.0	4.5	5.0
$x = 15$	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	5.5	6.0	6.5	7.0	7.5
x = 0	0.0041	0.0025	0.0015	0.0009	0.0006
1	0.0266	0.0174	0.0113	0.0073	0.0047
2	0.0884	0.0620	0.0430	0.0296	0.0203
3	0.2017	0.1512	0.1118	0.0818	0.0591
4	0.3575	0.2851	0.2237	0.1730	0.1321
5	0.5289	0.4457	0.3690	0.3007	0.2414
6	0.6860	0.6063	0.5265	0.4497	0.3782

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	5.5	6.0	6.5	7.0	7.5
$x = 7$	0.8095	0.7440	0.6728	0.5987	0.5246
8	0.8944	0.8472	0.7916	0.7291	0.6620
9	0.9462	0.9161	0.8774	0.8305	0.7764
10	0.9747	0.9574	0.9332	0.9015	0.8622
11	0.9890	0.9799	0.9661	0.9467	0.9208
12	0.9955	0.9912	0.9840	0.9730	0.9573
13	0.9983	0.9964	0.9929	0.9872	0.9784
14	0.9994	0.9986	0.9970	0.9943	0.9897

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	5.5	6.0	6.5	7.0	7.5
$\lambda = 15$	0.9998	0.9995	0.9988	0.9976	0.9954
16	0.9999	0.9998	0.9996	0.9990	0.9980
17	1.0000	0.9999	0.9998	0.9996	0.9992
18	1.0000	1.0000	0.9999	0.9999	0.9997
19	1.0000	1.0000	1.0000	1.0000	0.9999
20	1.0000	1.0000	1.0000	1.0000	1.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	8.0	8.5	9.0	9.5	10.0
x = 0	0.0003	0.0002	0.0001	0.0001	0.0000
1	0.0030	0.0019	0.0012	0.0008	0.0005
2	0.0138	0.0093	0.0062	0.0042	0.0028
3	0.0424	0.0301	0.0212	0.0149	0.0103
4	0.0996	0.0744	0.0550	0.0403	0.0293
5	0.1912	0.1496	0.1157	0.0885	0.0671
6	0.3134	0.2562	0.2068	0.1649	0.1301

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	8.0	8.5	9.0	9.5	10.0
$x = 7$	0.4530	0.3856	0.3239	0.2687	0.2202
8	0.5925	0.5231	0.4557	0.3918	0.3328
9	0.7166	0.6530	0.5874	0.5218	0.4579
10	0.8159	0.7634	0.7060	0.6453	0.5830
11	0.8881	0.8487	0.8030	0.7520	0.6968
12	0.9362	0.9091	0.8758	0.8364	0.7916
13	0.9658	0.9486	0.9261	0.8981	0.8645
14	0.9827	0.9726	0.9585	0.9400	0.9165

POISSON CUMULATIVE DISTRIBUTION FUNCTION**(continued)**

$\lambda =$	8.0	8.5	9.0	9.5	10.0
x = 15	0.9918	0.9862	0.9780	0.9665	0.9513
16	0.9963	0.9934	0.9889	0.9823	0.9730
17	0.9984	0.9970	0.9947	0.9911	0.9857
18	0.9993	0.9987	0.9976	0.9957	0.9928
19	0.9997	0.9995	0.9989	0.9980	0.9965
20	0.9999	0.9998	0.9996	0.9991	0.9984
21	1.0000	0.9999	0.9998	0.9996	0.9993
22	1.0000	1.0000	0.9999	0.9999	0.9997

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

The values in the table are those which a random variable with the χ^2 distribution on V degrees of freedom exceeds with the probability shown.

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.995	0.990	0.975	0.950	0.900
1	0.000	0.000	0.001	0.004	0.016
2	0.010	0.020	0.051	0.103	0.211
3	0.072	0.115	0.216	0.352	0.584
4	0.207	0.297	0.484	0.711	1.064
5	0.412	0.554	0.831	1.145	1.610
6	0.676	0.872	1.237	1.635	2.204
7	0.989	1.239	1.690	2.167	2.833

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.995	0.990	0.975	0.950	0.900
8	1.344	1.646	2.180	2.733	3.490
9	1.735	2.088	2.700	3.325	4.168
10	2.156	2.558	3.247	3.940	4.865
11	2.603	3.053	3.816	4.575	5.580
12	3.074	3.571	4.404	5.226	6.304
13	3.565	4.107	5.009	5.892	7.042
14	4.075	4.660	5.629	6.571	7.790

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.995	0.990	0.975	0.950	0.900
15	4.601	5.229	6.262	7.261	8.547
16	5.142	5.812	6.908	7.962	9.312
17	5.697	6.408	7.564	8.672	10.085
18	6.265	7.015	8.231	9.390	10.865
19	6.844	7.633	8.907	10.117	11.651
20	7.434	8.260	9.591	10.851	12.443
21	8.034	8.897	10.283	11.591	13.240

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.995	0.990	0.975	0.950	0.900
22	8.643	9.542	10.982	12.338	14.042
23	9.260	10.196	11.689	13.091	14.848
24	9.886	10.856	12.401	13.848	15.659
25	10.520	11.524	13.120	14.611	16.473
26	11.160	12.198	13.844	15.379	17.292
27	11.808	12.879	14.573	16.151	18.114
28	12.461	13.565	15.308	16.928	18.939

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION**(continued)**

v	0.995	0.990	0.975	0.950	0.900
29	13.121	14.256	16.047	17.708	19.768
30	13.787	14.953	16.791	18.493	20.599

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION**(continued)**

v	0.100	0.050	0.025	0.010	0.005
1	2.705	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.210	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.860
5	9.236	11.070	12.832	15.086	16.750
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.100	0.050	0.025	0.010	0.005
8	13.362	15.507	17.535	20.090	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.920	24.725	26.757
12	18.549	21.026	23.337	26.217	28.300
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.100	0.050	0.025	0.010	0.005
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32.000	34.267
17	24.769	27.587	30.191	33.409	35.718
18	25.989	28.869	31.526	34.805	37.156
19	27.204	30.144	32.852	36.191	38.582
20	28.412	31.410	34.170	37.566	39.997
21	29.615	32.671	35.479	38.932	41.401

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

(continued)

v	0.100	0.050	0.025	0.010	0.005
22	30.813	33.924	36.781	40.289	42.796
23	32.007	35.172	38.076	41.638	44.181
24	33.196	36.415	39.364	42.980	45.558
25	34.382	37.652	40.646	44.314	46.928
26	35.563	38.885	41.923	45.642	48.290
27	36.741	40.113	43.194	46.963	49.645
28	37.916	41.337	44.461	48.278	50.993

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION**(continued)**

v	0.100	0.050	0.025	0.010	0.005
29	39.088	42.557	45.722	49.588	52.336
30	40.256	43.773	46.979	50.892	53.672

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

These tables concern tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Sample size,	Product Moment Coefficient				
	Level				
	0.10	0.05	0.025	0.01	0.005
n = 4,	0.8000	0.9000	0.9500	0.9800	0.9900
n = 5,	0.6870	0.8054	0.8783	0.9343	0.9587
n = 6,	0.6084	0.7293	0.8114	0.8822	0.9172
n = 7,	0.5509	0.6694	0.7545	0.8329	0.8745
n = 8,	0.5067	0.6215	0.7067	0.7887	0.8343
n = 9,	0.4716	0.5822	0.6664	0.7498	0.7977

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Spearman's Coefficient****Level****Sample size,****0.05****0.025****0.01****n = 4,****1.0000****—****—****n = 5,****0.9000****1.0000****1.0000****n = 6,****0.8286****0.8857****0.9429****n = 7,****0.7143****0.7857****0.8929****n = 8,****0.6429****0.7381****0.8333****n = 9,****0.6000****0.7000****0.7833**

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Product Moment Coefficient (continued)**

Sample size,	Level				
	0.10	0.05	0.025	0.01	0.005
n = 10,	0.4428	0.5494	0.6319	0.7155	0.7646
n = 11,	0.4187	0.5214	0.6021	0.6851	0.7348
n = 12,	0.3981	0.4973	0.5760	0.6581	0.7079
n = 13	0.3802	0.4762	0.5529	0.6339	0.6835
n = 14,	0.3646	0.4575	0.5324	0.6120	0.6614
n = 15,	0.3507	0.4409	0.5140	0.5923	0.6411

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Spearman's Coefficient (continued)

Sample size,	Level		
	0.05	0.025	0.01
n = 10,	0.5636	0.6485	0.7455
n = 11,	0.5364	0.6182	0.7091
n = 12,	0.5035	0.5874	0.6783
n = 13,	0.4835	0.5604	0.6484
n = 14,	0.4637	0.5385	0.6264
n = 15,	0.4464	0.5214	0.6036

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Product Moment Coefficient (continued)

Sample size,	Level				
	0.10	0.05	0.025	0.01	0.005
n = 16,	0.3383	0.4259	0.4973	0.5742	0.6226
n = 17,	0.3271	0.4124	0.4821	0.5577	0.6055
n = 18,	0.3170	0.4000	0.4683	0.5425	0.5897
n = 19,	0.3077	0.3887	0.4555	0.5285	0.5751
n = 20,	0.2992	0.3783	0.4438	0.5155	0.5614
n = 21,	0.2914	0.3687	0.4329	0.5034	0.5487

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Spearman's Coefficient (continued)**

Sample size,	Level		
	0.05	0.025	0.01
n = 16,	0.4294	0.5029	0.5824
n = 17,	0.4142	0.4877	0.5662
n = 18,	0.4014	0.4716	0.5501
n = 19,	0.3912	0.4596	0.5351
n = 20,	0.3805	0.4466	0.5218
n = 21,	0.3701	0.4364	0.5091

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Product Moment Coefficient (continued)

Sample size,	Level				
	0.10	0.05	0.025	0.01	0.005
n = 22,	0.2841	0.3598	0.4227	0.4921	0.5368
n = 23,	0.2774	0.3515	0.4133	0.4815	0.5256
n = 24,	0.2711	0.3438	0.4044	0.4716	0.5151
n = 25,	0.2653	0.3365	0.3961	0.4622	0.5052
n = 26,	0.2598	0.3297	0.3882	0.4534	0.4958
n = 27,	0.2546	0.3233	0.3809	0.4451	0.4869

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Spearman's Coefficient (continued)**

Sample size,	Level		
	0.05	0.025	0.01
n = 22,	0.3608	0.4252	0.4975
n = 23,	0.3528	0.4160	0.4862
n = 24,	0.3443	0.4070	0.4757
n = 25,	0.3369	0.3977	0.4662
n = 26,	0.3306	0.3901	0.4571
n = 27,	0.3242	0.3828	0.4487

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Product Moment Coefficient (continued)

Sample size,	Level				
	0.10	0.05	0.025	0.01	0.005
n = 28,	0.2497	0.3172	0.3739	0.4372	0.4785
n = 29,	0.2451	0.3115	0.3673	0.4297	0.4705
n = 30,	0.2407	0.3061	0.3610	0.4226	0.4629
n = 40,	0.2070	0.2638	0.3120	0.3665	0.4026
n = 50,	0.1843	0.2353	0.2787	0.3281	0.3610
n = 60,	0.1678	0.2144	0.2542	0.2997	0.3301

CRITICAL VALUES FOR CORRELATION COEFFICIENTS

(continued)

Spearman's Coefficient (continued)

Sample size,	Level		
	0.05	0.025	0.01
n = 28,	0.3180	0.3755	0.4401
n = 29,	0.3118	0.3685	0.4325
n = 30,	0.3063	0.3624	0.4251
n = 40,	0.2640	0.3128	0.3681
n = 50,	0.2353	0.2791	0.3293
n = 60,	0.2144	0.2545	0.3005

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Product Moment Coefficient (continued)**

Sample size,	Level				
	0.10	0.05	0.025	0.01	0.005
n = 70,	0.1550	0.1982	0.2352	0.2776	0.3060
n = 80,	0.1448	0.1852	0.2199	0.2597	0.2864
n = 90,	0.1364	0.1745	0.2072	0.2449	0.2702
n = 100,	0.1292	0.1654	0.1966	0.2324	0.2565

CRITICAL VALUES FOR CORRELATION COEFFICIENTS**(continued)****Spearman's Coefficient (continued)**

Sample size,	Level		
	0.05	0.025	0.01
n = 70,	0.1982	0.2354	0.2782
n = 80,	0.1852	0.2201	0.2602
n = 90,	0.1745	0.2074	0.2453
n = 100,	0.1654	0.1967	0.2327

RANDOM NUMBERS

86 13 84 10 07 30 39 05 97 96 88 07 37 26 04 89 13 48 19 20

60 78 48 12 99 47 09 46 91 33 17 21 03 94 79 00 08 50 40 16

78 48 06 37 82 26 01 06 64 65 94 41 17 26 74 66 61 93 24 97

80 56 90 79 66 94 18 40 97 79 93 20 41 51 25 04 20 71 76 04

99 09 39 25 66 31 70 56 30 15 52 17 87 55 31 11 10 68 98 23

56 32 32 72 91 65 97 36 56 61 12 79 95 17 57 16 53 58 96 36

66 02 49 93 97 44 99 15 56 86 80 57 11 78 40 23 58 40 86 14

31 77 53 94 05 93 56 14 71 23 60 46 05 33 23 72 93 10 81 23

98 79 72 43 14 76 54 77 66 29 84 09 88 56 75 86 41 67 04 42

50 97 92 15 10 01 57 01 87 33 73 17 70 18 40 21 24 20 66 62

RANDOM NUMBERS (continued)

90 51 94 50 12 48 88 95 09 34 09 30 22 27 25 56 40 76 01 59
31 99 52 24 13 43 27 88 11 39 41 65 00 84 13 06 31 79 74 97
22 96 23 34 46 12 67 11 48 06 99 24 14 83 78 37 65 73 39 47
06 84 55 41 27 06 74 59 14 29 20 14 45 75 31 16 05 41 22 96
08 64 89 30 25 25 71 35 33 31 04 56 12 67 03 74 07 16 49 32
86 87 62 43 15 11 76 49 79 13 78 80 93 89 09 57 07 14 40 74
94 44 97 13 77 04 35 02 12 76 60 91 93 40 81 06 85 85 72 84
63 25 55 14 66 47 99 90 02 90 83 43 16 01 19 69 11 78 87 16
11 22 83 98 15 21 18 57 53 42 91 91 26 52 89 13 86 00 47 61
01 70 10 83 94 71 13 67 11 12 36 54 53 32 90 43 79 01 95 15

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION

The values in the table are those which a random variable with Student's t distribution on v degrees of freedom exceeds with the probability shown.

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
32	1.309	1.694	2.037	2.449	2.738

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
34	1.307	1.691	2.032	2.441	2.728
36	1.306	1.688	2.028	2.435	2.719
38	1.304	1.686	2.024	2.429	2.712
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660

PERCENTAGE POINTS OF STUDENT'S T DISTRIBUTION**(continued)**

v	0.10	0.05	0.025	0.01	0.005
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.369	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617

PERCENTAGE POINTS OF THE F DISTRIBUTION

The values in the table are those which a random variable with the **F** distribution on v_1 and v_2 degrees of freedom exceeds with probability 0.05 or 0.01

If an upper percentage point of the **F** distribution on v_1 and v_2 degrees of freedom is f , then the corresponding lower percentage point of the **F** distribution on v_2 and v_1 degrees of freedom is $1/f$.

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

Probability	v_1	1	2	3	4
F = 0.05,					
	$v_2 = 1$	161.4	199.5	215.7	224.6
	2	18.51	19.00	19.16	19.25
	3	10.13	9.55	9.28	9.12
	4	7.71	6.94	6.59	6.39
	5	6.61	5.79	5.41	5.19
	6	5.99	5.14	4.76	4.53

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	1	2	3	4
F = 0.05,					
	$v_2 = 7$	5.59	4.74	4.35	4.12
	8	5.32	4.46	4.07	3.84
	9	5.12	4.26	3.86	3.63
	10	4.96	4.10	3.71	3.48
	11	4.84	3.98	3.59	3.36
	12	4.75	3.89	3.49	3.26

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	1	2	3	4
F = 0.05,					
	$v_2 = 14$	4.60	3.74	3.34	3.11
	16	4.49	3.63	3.24	3.01
	18	4.41	3.55	3.16	2.93
	20	4.35	3.49	3.10	2.87
	25	4.24	3.39	2.99	2.76
	30	4.17	3.32	2.92	2.69

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	1	2	3	4
F = 0.05,					
	$v_2 =$				
	40	4.08	3.23	2.84	2.61
	60	4.00	3.15	2.76	2.53
	120	3.92	3.07	2.68	2.45
	∞	3.84	3.00	2.60	2.37

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	5	6	8	10
F = 0.05,					
	$v_2 = 1$	230.2	234.0	238.9	241.9
	2	19.30	19.33	19.37	19.40
	3	9.01	8.94	8.85	8.79
	4	6.26	6.16	6.04	5.96
	5	5.05	4.95	4.82	4.74
	6	4.39	4.28	4.15	4.06

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	5	6	8	10
F = 0.05,					
	$v_2 = 7$	3.97	3.87	3.73	3.64
	8	3.69	3.58	3.44	3.35
	9	3.48	3.37	3.23	3.14
	10	3.33	3.22	3.07	2.98
	11	3.20	3.09	2.95	2.85
	12	3.11	3.00	2.85	2.75

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	5	6	8	10
F = 0.05,					
	$v_2 = 14$	2.96	2.85	2.70	2.60
	16	2.85	2.74	2.59	2.49
	18	2.77	2.66	2.51	2.41
	20	2.71	2.60	2.45	2.35
	25	2.60	2.49	2.34	2.24
	30	2.53	2.42	2.27	2.16

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	5	6	8	10
F = 0.05,					
	$v_2 = 40$	2.45	2.34	2.18	2.08
	60	2.37	2.25	2.10	1.99
	120	2.29	2.18	2.02	1.91
	∞	2.21	2.10	1.94	1.83

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	12	24	∞
F = 0.05,				
	$v_2 = 1$	243.9	249.1	254.3
	2	19.41	19.46	19.50
	3	8.74	8.64	8.53
	4	5.91	5.77	5.63
	5	4.68	4.53	4.37
	6	4.00	3.84	3.67

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	12	24	∞
F = 0.05,				
	$v_2 = 7$	3.57	3.41	3.23
	8	3.28	3.12	2.93
	9	3.07	2.90	2.71
	10	2.91	2.74	2.54
	11	2.79	2.61	2.40
	12	2.69	2.51	2.30

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	12	24	∞
F = 0.05,				
	$v_2 = 14$	2.53	2.35	2.13
	16	2.42	2.24	2.01
	18	2.34	2.15	1.92
	20	2.28	2.08	1.84
	25	2.16	1.96	1.71
	30	2.09	1.89	1.62

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.05, continued)

Probability	v_1	12	24	∞
F = 0.05,				
	$v_2 = 40$	2.00	1.79	1.51
	60	1.92	1.70	1.39
	120	1.83	1.61	1.25
	∞	1.75	1.52	1.00

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

Probability	v_1	1	2	3	4
F = 0.01,					
	$v_2 =$				
	1	4052.	5000.	5403.	5625.
	2	98.50	99.00	99.17	99.25
	3	34.12	30.82	29.46	28.71
	4	21.20	18.00	16.69	15.98
	5	16.26	13.27	12.06	11.39
	6	13.70	10.90	9.78	9.15

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.01, continued)

Probability	v_1	1	2	3	4
F = 0.01,					
	$v_2 = 7$	12.20	9.55	8.45	7.85
	8	11.30	8.65	7.59	7.01
	9	10.60	8.02	6.99	6.42
	10	10.00	7.56	6.55	5.99
	11	9.65	7.21	6.22	5.67
	12	9.33	6.93	5.95	5.41

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.01, continued)

Probability	v_1	1	2	3	4
F = 0.01,					
	$v_2 = 14$	8.86	6.51	5.56	5.04
	16	8.53	6.23	5.29	4.77
	18	8.29	6.01	5.09	4.58
	20	8.10	5.85	4.94	4.43
	25	7.77	5.57	4.68	4.18
	30	7.56	5.39	4.51	4.02

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.01, continued)

Probability	v_1	1	2	3	4
F = 0.01,					
	$v_2 = 40$	7.31	5.18	4.31	3.83
	60	7.08	4.98	4.13	3.65
	120	6.85	4.79	3.95	3.48
	∞	6.63	4.61	3.78	3.32

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.01, continued)

Probability	v_1	5	6	8	10
F = 0.01,					
	$v_2 =$				
	1	5764.	5859.	5982.	6056.
	2	99.30	99.33	99.37	99.40
	3	28.24	27.91	27.49	27.23
	4	15.52	15.21	14.80	14.55
	5	10.97	10.67	10.29	10.05
	6	8.75	8.47	8.10	7.87

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	5	6	8	10
F = 0.01,					
	$v_2 = 7$	7.46	7.19	6.84	6.62
	8	6.63	6.37	6.03	5.81
	9	6.06	5.80	5.47	5.26
	10	5.64	5.39	5.06	4.85
	11	5.32	5.07	4.74	4.54
	12	5.06	4.82	4.50	4.30

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	5	6	8	10
F = 0.01,					
	$v_2 = 14$	4.70	4.46	4.14	3.94
	16	4.44	4.20	3.89	3.69
	18	4.25	4.01	3.71	3.51
	20	4.10	3.87	3.56	3.37
	25	3.86	3.63	3.32	3.13
	30	3.70	3.47	3.17	2.98

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	5	6	8	10
F = 0.01,					
	$v_2 = 40$	3.51	3.29	2.99	2.80
	60	3.34	3.12	2.82	2.63
	120	3.17	2.96	2.66	2.47
	∞	3.02	2.80	2.51	2.32

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)

(F = 0.01, continued)

Probability	v_1	12	24	∞
F = 0.01,				
	$v_2 = 1$	6106.	6235.	6366.
	2	99.42	99.46	99.50
	3	27.05	26.60	26.13
	4	14.37	13.93	13.45
	5	9.89	9.47	9.02
	6	7.72	7.31	6.88

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	12	24	∞
<hr/>				
F = 0.01,				
	$v_2 = 7$	6.47	6.07	5.65
	8	5.67	5.28	4.86
	9	5.11	4.73	4.31
	10	4.17	4.33	3.91
	11	4.40	4.02	3.60
	12	4.16	3.78	3.36

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	12	24	∞
<hr/>				
F = 0.01,				
	$v_2 = 14$	3.80	3.43	3.00
	16	3.55	3.18	2.75
	18	3.37	3.00	2.57
	20	3.23	2.86	2.42
	25	2.99	2.62	2.17
	30	2.84	2.47	2.01

PERCENTAGE POINTS OF THE F DISTRIBUTION (continued)
(F = 0.01, continued)

Probability	v_1	12	24	∞
F = 0.01,				
	$v_2 = 40$	2.66	2.29	1.80
	60	2.50	2.12	1.60
	120	2.34	1.95	1.38
	∞	2.18	1.79	1.00