Normal Approximation to Binomial Distribution (Edexcel and OCR MEI) MS

Q1.

Question	Scheme		Marks	AOs
(a)(i)	$X \sim B(15, 0.48)$		M1	3.3
	$P(X = 3) = 0.019668...$ awrt 0.0197		A1	3.4
(ii)	$[P(X \geqslant 5) = 1 - P(X \leqslant 4)] = 0.92013...$ awrt 0.920		A1	1.1b
			(3)	
(b)	Y is the number of hits	M is the number of misses		
	$Y \sim N(120, 62.4)$	$M \sim N(130, 62.4)$	B1	3.3
	$P(X > 110) \approx P(Y > 110.5)$ $\left[=P\left(Z > \frac{110.5 - "120"}{\sqrt{"62.4"}}\right)\right]$	$P(X > 110) \approx P(M < 139.5)$ $\left[=P\left(Z < \frac{139.5 - "130"}{\sqrt{"62.4"}}\right)\right]$	M1	3.4
	$= 0.88544...$		A1	1.1b
			(3)	
				(6 marks)

Notes:		
(a)	M1	Writing or using the binomial distribution in (i) or (ii) Allow for sight of B(15, 0.48) or in words: binomial with $n = 15$ and $p = 0.48$ may be implied in (i) or (ii) by one correct answer to 3sf or sight of $P(X \leqslant 4) = 0.07986...$ i.e. awrt 0.0799. Allow for ${}^{15}C_3 \times 0.48^3 \times 0.52^{12}$ as this is "correct use" Condone B(0.48, 15)
(i)	A1	awrt 0.0197
(ii)	A1	awrt 0.920 (Allow 0.92)
(b)	B1	Setting up a correct Normal model. Allow sight of $N(120, 62.4)$ or $N(130, 62.4)$ or $N\left(120, \frac{312}{5}\right)$ or $N\left(130, \frac{312}{5}\right)$ or may be awarded if used correctly in standardisation or in words: Normal with mean = 120/130 and variance = 62.4 or sd = $\sqrt{62.4}$ condone $N(120, \sqrt{62.4})$ or $N(130, \sqrt{62.4})$ or sd = 62.4 Look out for $\sigma = \frac{\sqrt{1560}}{5}$ or $\frac{2\sqrt{390}}{5}$ or awrt 7.90 (condone 7.9) This may be implied by sight of 0.897 or 0.8854...
	M1	Sight of the continuity correction with a normal distribution
		110.5 or 111.5 or 109.5 / **139.5** or 140.5 or 138.5
		NB we will also allow **129.5** or 130.5 or 128.5 / NB we will also allow **120.5** or 119.5 or 121.5
		Continuity correction may be seen in standardisation NB No continuity correction(CC) gives awrt 0.897 which is M0 unless CC seen
	A1	awrt 0.8854 or awrt 0.885 dependent on sight of >110.5 or <129.5 or < 139.5 or >120.5 Allow $\leqslant$ or $\geqslant$ instead of < or > NB 0.885548... from B(250, 0.48) scores M0A0

Q2.

Question Number	Scheme	Marks
(a)	$H_0 : p = 0.15 \quad H_1 : p \neq 0.15$	B1 B1
	$X \sim B(30, 0.15)$	M1
	$P(X \leq 1) = 0.0480$ or CR: $X = 0$	A1
	$(0.0480 > 0.025)$	
	not a significant result or do not reject H_0 or not in CR	M1
	there is no evidence of a change in the proportion of customers buying an item from the display.	A1ft
		(6)
(b)	$H_0 : p = 0.2 \quad H_1 : p > 0.2$	B1
	Let S = the number who buy sandwiches, $S \sim B(120, 0.2)$,	
	$S \approx W \sim N\left(24, \sqrt{19.2}^2\right)$	M1 A1
	$P(S \geq 31) = P(W \geq 30.5)$	M1
	$= P\left(Z > \frac{30.5 - 24}{\sqrt{19.2}}\right)$ or $\frac{x - 0.5 - 24}{\sqrt{19.2}} = 1.2816$	M1
	$[= P(Z > 1.48..)]$	
	$= 1 - 0.9306$	M1
	$= 0.0694$ $\quad x = 30.1$	A1
	< 0.10 so a significant result, there is evidence that more customers are purchasing sandwiches or the shopkeepers claim is correct.	B1ft
		(8)
		14

Notes:

(a)

1st B1 for H_0 must use p 2nd B1 for H_1 must use p

1st M1 for writing or using B(30,0.15) – may be implied by correct CR

1st A1 0.0480 or $X = 0$. Allow $X \leq 0$. Ignore upper CR. NB Allow CR $X \leq 1$ if using one tail test.

2nd M1 A correct statement (see table below) Do not allow non-contextual conflicting statements eg"significant" and "accept H_0". **Ignore comparisons**

2nd A1 for a correct statement in context. For context we need idea of change/decrease in number of customers buying from display – may use different words. NB A correct contextual statement on its own scores M1A1

	Two tail $0.025 < p < 0.975$ **or** **One tail** $0.05 < p < 0.95$	**Two tail** $p < 0.025$ or $p > 0.975$ **or** **One tail** $p < 0.05$ or $p > 0.95$
2nd M1	not significant/ accept H_0/ Not in CR or contextual	significant/ reject H_0/ In CR or contextual
2nd A1	There is no evidence of a change/decrease in the proportion of customers buying an item from the display	There is evidence of a change/decrease in the proportion of customers buying an item from the display.

(b)

1st B1 both hypotheses correct – must use p.

1st M1 for a normal approx

1st A1 for correct mean and sd

2nd M1 for use of continuity correction, either 30.5 or 31.5 or $(x \pm 0.5)$ seen

3rd M1 standardising with their mean and their sd and 30.5, 31 or 31.5 or x or $(x \pm 0.5)$)

4th M1 for 1 - tables value or 1.2816

2nd A1 for awrt 0.069 or $x = 30.1$

2nd B1ft For a correct conclusion in context using their probability and 0.1 For context we need idea of more customers buying sandwiches – may use different words

	One tail $0.1 < p < 0.9$ **or Two tail** $0.05 < p < 0.95$	**One tail** $p < 0.1$ or $p > 0.9$ **or Two tail** $p < 0.05$ or $p > 0.95$
2nd M1	not significant/ accept H_0/ Not in CR or contextual	significant/ reject H_0/ In CR or contextual
2nd A1	There is no evidence of an increase in the proportion of customers buying sandwiches	There is evidence of a change/increase in the proportion of customers buying sandwiches.

Q3.

Question Number	Scheme		Marks
	N($0.2n$, $0.16n$)	B1: Mean = $0.2n$ and Var = $0.16n$ oe this may be awarded if they appear in the standardisation as $0.2n$ and either $0.16n$ or $\sqrt{0.16n}$	B1
	$P\left(Z > \frac{55.5-0.2n}{\sqrt{0.16n}}\right) = 0.0401$	M1: Using a continuity correction either 55.5 or 54.5	M1
	$\frac{55.5-0.2n}{\sqrt{0.16n}} = 1.75$	B1: Using a z = awrt $\pm$ 1.75 M1: Standardising using either 55.5, 54.5 or 55 and equal to a z value. Follow through their mean and variance. If they have not given the mean and Var earlier then they must be correct A1: A correct equation. May be awarded for $\frac{55.5-0.2n}{\sqrt{0.16n}} = 1.75$ Condone use of an inequality sign rather than an equals sign	B1M1A1
	$0.2n + 0.7\sqrt{n} - 55.5 = 0$	M1d: This is dependent on the previous method mark being awarded. Using either the quadratic formula or completing the square or factorising or any correct method to solve **their 3 term** equation. If they write the formula down then allow a slip. If no formula written down then it must be correct for their equation. May be implied by correct answer or $\sqrt{n} = 15$ or 342.25 NB you may award this mark if they use 54.5 for awrt 14.9, -18.4, 221 or 337 55 for awrt -18.4, 14.9,223 or -117 If the answer is not one of these then the method for solving their 3 term equation must be seen.	M1d
	$\sqrt{n} = 15$	A1: Allow 15 or -18.5 do not need to see n or $\sqrt{n}$. Condone $n = 15$ or n = -18.5	A1
	$n = 225$	A1 : cao 225 do not need to see n or $\sqrt{n}$	A1 (8)

	Alternative method for last 3 marks $(0.2n - 55.5)^2 = (-0.7\sqrt{n})^2$ $0.04n^2 - 22.69n + 3080.25 = 0$ n = 225 or 1369/4 n = 225	M1 solving 3 term quadratic in n as above A1 either 225 or 1369/4 or 342.25 A1must select 225	**Total 8**

Q4.

Question Number	Scheme		Marks
(a)	$P(M<10)=P\left(Z<\frac{12-14}{\sigma}\right)=0.1$		
	$\Rightarrow \frac{12-14}{\sigma}=,\ -1.2816$	**M1** standardising ($\pm$) with 12, 14 and σ and setting equal to a z value where $\lvert z\rvert>1$ **B1** ± 1.2816 or better	**M1** **B1**
	$\sigma=1.5605\ldots\ldots$ =awrt 1.56 minutes	**A1** awrt 1.56 Do **not** allow answer written as an exact fraction.	**A1** **(3)**
(b)	T represents number less than 12 minutes. $T\sim B(15, 0.1)$	**B1** Writing or using B(15, 0.1) .	**B1**
	$P(T\leq 1)$	**M1** writing $P(T\leq 1)$ or $P(T<2)$ any letter may be used.	**M1**
	$=0.549$	**A1** awrt 0.549	**A1**
		NB 0.549 gets B1 M1 A1	**(3)**
(c)	[$T\sim$ number of people who take less than 12 mins to complete the test] $T\sim B(n, 0.1)$		
	T can be approximated by $N(0.1n, 0.09n)$	**B1** mean = $0.1n$ and Var = $0.09n$ **oe** may be seen in an attempt at standardisation	**B1**
	$P\left(Z<\frac{8.5-0.1n}{\sqrt{0.09n}}\right)=0.3085$	**M1** using a continuity correction either 8.5 or 7.5 in an attempt at standardised form. Allow 0.09 for sd.	**M1**
	$\frac{8.5-0.1n}{\sqrt{0.09n}}=-0.5$ or $\frac{8.5-0.1x^2}{0.3x}=-0.5$	**B1** a z value of awrt $\pm$ 0.5	**B1**
		M1 standardising using their mean and sd. (If these have not been given then they must be correct here) **and** one of 7.5, 8, 8.5, 9 or 9.5 **and** equal to a z value where $\lvert z\rvert>0.4$. Allow any form	**M1**
		A1 A correct equation in **any form**. ISW. Do **not** allow if they have $0.3n$ rather than $0.3\sqrt{n}$	**A1**
	$0.1n-0.15\sqrt{n}-8.5=0$ $\sqrt{n}=10$	**M1** using either the quadratic formula or completing the square or factorising or any correct method to solve **their 3 term quadratic**. If they write the quadratic formula down then allow one slip. If no formula written down then it must be correct for their equation. May be implied by seeing 10 or 8.5. They must show working if the equation used is not correct. **2nd A1** awrt 10.0 – do not need to see n or $\sqrt{n}$. Allow $n=10$ May be implied by 100	**M1A1**
	$n=100$	**3rd A1 cso** 100 If they have a second answer of 72.25 they must reject it to get this final mark.	**A1cso** **(8)**
			(Total 14)

Q5.

Question	Scheme	Marks	AOs
(a)	The seeds would be destroyed in the process so they would have none to sell	B1	2.4
		(1)	
(b)	[S = no. of seeds out of 24 that germinate, $S \sim B(24, 0.55)$]		
	T = no. of trays with at least 15 germinating. $T \sim B(10, p)$	M1	3.3
	$p = P(S \geqslant 15) = 0.299126...$	A1	1.1b
	So $P(T \geqslant 5) = 0.1487...$ awrt **0.149**	A1	1.1b
		(3)	
(c)	n is large and p close to 0.5	B1	1.2
		(1)	
(d)	$X \sim N(132, 59.4)$	B1	3.4
	$P(X \geqslant 149.5) = P\left(Z \geqslant \frac{149.5-132}{\sqrt{59.4}}\right)$	M1	1.1b
	$= 0.01158...$ awrt **0.0116**	A1cso	1.1b
		(3)	
(e)	e.g The probability is very small therefore there is evidence that the company's claim is incorrect.	B1	2.2b
		(1)	
			(9 marks)

Notes:

(a)
B1: cao

(b)
M1: for selection of an appropriate model for T
1st A1: for a correct value of the parameter p (accept 0.3 or better)
2nd A1: for awrt 0.149

(c)
B1: both correct conditions

(d)
B1: for correct normal distribution
M1: for correct use of continuity correction
A1: cso

(e)
B1: correct statement