Hypothesis Testing for the Sample Mean of a Normal Distribution (From OCR 4767)

Q1, (OCR 4767, Jun 2006, Q2)

The head circumference of 3-year-old boys is known to be Normally distributed with mean 49.7 cm and standard deviation 1.6 cm.

(i) Find the probability that the head circumference of a randomly selected 3-year-old boy will be

(*A*) over 51.5 cm,

(*B*) between 48.0 and 51.5 cm. [5]

(ii) Four 3-year-old boys are selected at random. Find the probability that exactly one of them has head circumference between 48.0 and 51.5 cm. [3]

(iii) The head circumference of 3-year-old girls is known to be Normally distributed with mean μ and standard deviation σ. Given that 60% of 3-year-old girls have head circumference below 49.0 cm and 30% have head circumference below 47.5 cm, find the values of μ and σ. [4]

A nutritionist claims that boys who have been fed on a special organic diet will have a larger mean head circumference than other boys. A random sample of ten 3-year-old boys who have been fed on this organic diet is selected. It is found that their mean head circumference is 50.45 cm.

(iv) Using the null and alternative hypotheses H_0: $\mu = 49.7$ cm, H_1: $\mu > 49.7$ cm, carry out a test at the 10% significance level to examine the nutritionist's claim. Explain the meaning of μ in these hypotheses. You may assume that the standard deviation of the head circumference of organically fed 3-year-old boys is 1.6 cm. [6]

Q2, (Jun 2007, Q1i,ii,iv,v)

The random variable X represents the time taken in minutes for a haircut at a barber's shop. X is Normally distributed with mean 11 and standard deviation 3.

(i) Find $P(X < 10)$. [4]

(ii) Find the probability that exactly 3 out of 8 randomly selected haircuts take less than 10 minutes. [3]

A new hairdresser joins the shop. The shop manager suspects that she takes longer on average than the other staff to do a haircut. In order to test this, the manager records the time taken for 25 randomly selected cuts by the new hairdresser. The mean time for these cuts is 12.34 minutes. You should assume that the time taken by the new hairdresser is Normally distributed with standard deviation 3 minutes.

(iv) Write down suitable null and alternative hypotheses for the test. [3]

(v) Carry out the test at the 5% level. [5]

Q3, (Jan 2007, Q2)

(a) A farmer grows Brussels sprouts. The diameter of sprouts in a particular batch, measured in mm, is Normally distributed with mean 28 and variance 16. Sprouts that are between 24 mm and 33 mm in diameter are sold to a supermarket.

(i) Find the probability that the diameter of a randomly selected sprout will be within this range. [4]

(ii) The farmer sells the sprouts in this range to the supermarket for 10 pence per kilogram. The farmer sells sprouts under 24 mm in diameter to a frozen food factory for 5 pence per kilogram. Sprouts over 33 mm in diameter are thrown away. Estimate the total income received by the farmer for the batch, which weighs 25 000 kg. [3]

(iii) By harvesting sprouts earlier, the mean diameter for another batch can be reduced to k mm. Find the value of k for which only 5% of the sprouts will be above 33 mm in diameter. You may assume that the variance is still 16. [3]

(b) The farmer also grows onions. The weight in kilograms of the onions is Normally distributed with mean 0.155 and variance 0.005. He is trying out a new variety, which he hopes will yield a higher mean weight. In order to test this, he takes a random sample of 25 onions of the new variety and finds that their total weight is 4.77 kg. You should assume that the weight in kilograms of the new variety is Normally distributed with variance 0.005.

(i) Write down suitable null and alternative hypotheses for the test in terms of μ. State the meaning of μ in this case. [2]

(ii) Carry out the test at the 1% level. [6]

Q4, (Jun 2009, Q3i,ii,iv,v)

Intensity of light is measured in lumens. The random variable X represents the intensity of the light from a standard 100 watt light bulb. X is Normally distributed with mean 1720 and standard deviation 90. You may assume that the intensities for different bulbs are independent.

(i) Show that $P(X < 1700) = 0.4121$. **[4]**

(ii) These bulbs are sold in packs of 4. Find the probability that the intensities of exactly 2 of the 4 bulbs in a randomly chosen pack are below 1700 lumens. **[3]**

A manufacturer claims that the average intensity of its 25 watt low energy light bulbs is 1720 lumens. A consumer organisation suspects that the true figure may be lower than this. The intensities of a random sample of 20 of these bulbs are measured. A hypothesis test is then carried out to check the claim.

(iv) Write down a suitable null hypothesis and explain briefly why the alternative hypothesis should be $H_1 : \mu < 1720$. State the meaning of μ. **[3]**

(v) Given that the standard deviation of the intensity of such bulbs is 90 lumens and that the mean intensity of the sample of 20 bulbs is 1703 lumens, carry out the test at the 5% significance level. **[5]**

Q5, (Jun 2010, Q3)

In a men's cycling time trial, the times are modelled by the random variable X minutes which is Normally distributed with mean 63 and standard deviation 5.2.

(i) Find

(*A*) $P(X < 65)$,

(*B*) $P(60 < X < 65)$. **[6]**

(ii) Find the probability that 5 riders selected at random all record times between 60 and 65 minutes. **[2]**

(iii) A competitor aims to be in the fastest 5% of entrants (i.e. those with the lowest times). Find the maximum time that he can take. **[3]**

It is suggested that holding the time trial on a new course may result in lower times. To investigate this, a random sample of 15 competitors is selected. These 15 competitors do the time trial on the new course. The mean time taken by these riders is 61.7 minutes. You may assume that times are Normally distributed and the standard deviation is still 5.2 minutes. A hypothesis test is carried out to investigate whether times on the new course are lower.

(iv) Write down suitable null and alternative hypotheses for the test. Carry out the test at the 5% significance level. **[8]**

Q6, (Jun 2012, Q4b)

Mary buys flour in bags which are labelled as containing 5 kg. She suspects that the average contents of these bags may be less than 5 kg. In order to test this, she selects a random sample of 8 bags and weighs their contents. Assuming that weights are Normally distributed with standard deviation 0.0072 kg, carry out a test at the 5% level, given that the weights of the 8 bags in kg are as follows.

4.992	4.981	5.006	4.982	4.996	5.009	4.991	5.003	**[9]**

Q7, (Jan 2013, Q4b)

At a canning factory, cans are filled with tomato purée. The machine which fills the cans is set so that the volume of tomato purée in a can, measured in millilitres, is Normally distributed with mean 420 and standard deviation 3.5. After the machine is recalibrated, a quality control officer wishes to check whether the mean is still 420 millilitres. A random sample of 10 cans of tomato purée is selected and the volumes, measured in millilitres, are as follows.

417.2	422.6	414.3	419.6	420.4	410.0	418.3	416.9	418.9	419.7

Carry out a test at the 1% significance level to investigate whether the mean is still 420 millilitres. You should assume that the volumes are Normally distributed with unchanged standard deviation. **[9]**

Q8, (Jun 2014, Q3)

The wing lengths of native English male blackbirds, measured in mm, are Normally distributed with mean 130.5 and variance 11.84.

(i) Find the probability that a randomly selected native English male blackbird has a wing length greater than 135 mm. **[3]**

(ii) Given that 1% of native English male blackbirds have wing length more than k mm, find the value of k. **[3]**

(iii) Find the probability that a randomly selected native English male blackbird has a wing length which is 131 mm correct to the nearest millimetre. **[3]**

It is suspected that Scandinavian male blackbirds have, on average, longer wings than native English male blackbirds. A random sample of 20 Scandinavian male blackbirds has mean wing length 132.4 mm. You may assume that wing lengths in this population are Normally distributed with variance 11.84 mm^2.

(iv) Carry out an appropriate hypothesis test, at the 5% significance level. **[8]**

(v) Discuss briefly one advantage and one disadvantage of using a 10% significance level rather than a 5% significance level in hypothesis testing in general. **[2]**