Hypothesis Testing for the Sample Mean of a Normal Distribution (From OCR 4767)

Q1, (OCR 4767, Jun 2006, Q2)

(i)	$X \sim N(49.7, 1.6^2)$ (*A*) $P(X > 51.5) = P\left(Z > \frac{51.5 - 49.7}{1.6}\right)$ $= P(Z > 1.125)$ $= 1 - \Phi(1.125) = 1 - 0.8696 = 0.1304$ (*B*) $P(X < 48.0) = P\left(Z < \frac{48.0 - 49.7}{1.6}\right)$ $= P(Z < -1.0625) = 1 - \Phi(1.0625)$ $= 1 - 0.8560 = 0.1440$ $P(48.0 < X < 51.5) = 1 - 0.1304 - 0.1440 = 0.7256$	M1 for standardizing M1 for prob. calc. A1 (at least 2 s.f.) M1 for appropriate prob' calc. A1 (0.725 – 0.726)	**5**
(ii)	P(one over 51.5, three between 48.0 and 51.5) $= \binom{4}{1} \times 0.7256 \times 0.2744^3 = 0.0600$	M1 for coefficient M1 for 0.7256×0.2744^3 A1 FT (at least 2 sf)	**3**
(iii)	From tables, $\Phi^{-1}(0.60) = 0.2533$, $\Phi^{-1}(0.30) = -0.5244$ $49.0 = \mu + 0.2533\sigma$ $47.5 = \mu - 0.5244\sigma$ $1.5 = 0.7777\sigma$ $\sigma = 1.929$, $\mu = 48.51$	B1 for 0.2533 or 0.5244 seen M1 for at least one correct equation μ & σ M1 for attempt to solve two correct equations A1 CAO for both	**4**
(iv)	Where μ denotes the mean circumference of the entire population of organically fed 3-year-old boys. $n = 10$, Test statistic $Z = \frac{50.45 - 49.7}{1.6/\sqrt{10}} = \frac{0.75}{0.5060} = 1.482$ 10% level 1 tailed critical value of z is 1.282 1.482 > 1.282 so significant. There is sufficient evidence to reject H_0 and conclude that organically fed 3-year-old boys have a higher mean head circumference.	E1 M1 A1(at least 3sf) B1 for 1.282 M1 for comparison leading to a conclusion A1 for conclusion in context	**6**
			18

Q2, (Jun 2007, Q1i,ii,iv,v)

(i)	$X \sim \mathrm{N}(11, 3^2)$ $\mathrm{P}(X < 10) = \mathrm{P}\left(Z < \frac{10-11}{3}\right)$ $= \mathrm{P}(Z < -0.333)$ $= \Phi(-0.333) = 1 - \Phi(0.333)$ $= 1 - 0.6304 = 0.3696$	M1 for standardizing M1 for use of tables with their z-value M1 *dep* for correct tail A1CAO (must include use of differences)	**4**
(ii)	P(3 of 8 less than ten) $= \binom{8}{3} \times 0.3696^3 \times 0.6304^5 = 0.2815$	M1 for coefficient M1 for $0.3696^3 \times 0.6304^5$ A1 FT (min 2sf)	**3**
(iv)	H_0: $\mu = 11$; H_1: $\mu > 11$ Where μ denotes the mean time taken by the new hairdresser	B1 for H_0, as seen. B1 for H_1, as seen. B1 for definition of μ	**3**
(v)	Test statistic $= \frac{12.34-11}{3/\sqrt{25}} = \frac{1.34}{0.6}$ $= 2.23$ 5% level 1 tailed critical value of z = 1.645 2.23 > 1.645, so significant. There is sufficient evidence to reject H_0 It is reasonable to conclude that the new hairdresser does take longer on average than other staff.	M1 must include √25 A1 (FT their μ) B1 for 1.645 M1 for sensible comparison leading to a conclusion A1 for conclusion in words in context (FT their μ)	**5**
			19

Q3, (Jan 2007, Q2)

(a) (i)	$X \sim N(28,16)$ $P(24 < X < 33) = P\left(\frac{24-28}{4} < Z < \frac{33-28}{4}\right)$ $= P(-1 < Z < 1.25)$ $= \Phi(1.25) - (1 - \Phi(1))$ $= 0.8944 - (1 - 0.8413)$ $= 0.8944 - 0.1587$ $= 0.7357$ (4 s.f.) *or* 0.736 (to 3 s.f.)	M1 for standardizing A1 for 1. 25 and -1 M1 for prob. with tables and correct structure A1 CAO (min 3 s.f., to include use of difference column)	**4**
(ii)	$25000 \times 0.7357 \times 0.1 = £1839$ $25000 \times 0.1587 \times 0.05 = £198$ Total = £1839 + £198 = £2037	M1 for either product, (with or without price) M1 for sum of both products with price A1 CAO awrt £2040	**3**
(iii)	$X \sim N(k, 16)$ From tables $\Phi^{-1}(0.95) = 1.645$ $\frac{33-k}{4} = 1.645$ $33 - k = 1.645 \times 4$ $k = 33 - 6.58$ $k = 26.42$ (4 s.f.) *or* 26.4 (to 3 s.f.)	B1 for ±1.645 seen M1 for correct equation in *k* with positive z-value A1 *CAO*	**3**
(b) (i)	H_0: $\mu = 0.155$; H_1: $\mu > 0.155$ Where μ denotes the mean weight in kilograms of the population of onions of the new variety	B1 for both correct & ito μ B1 for definition of μ	**2**
(ii)	Mean weight = 4.77/25 = 0.1908 Test statistic = $\frac{0.1908-0.155}{\sqrt{0.005}/\sqrt{25}} = \frac{0.0358}{0.01414}$ $= 2.531$ 1% level 1-tailed critical value of z = 2.326 2.531 > 2.236 so significant. There is sufficient evidence to reject H_0 It is reasonable to conclude that the new variety has a higher mean weight.	B1 M1 must include √25 A1FT B1 for 2.326 M1 For sensible comparison leading to a conclusion A1 for correct, consistent conclusion in words and in context	**6**
			18

Q4, (Jun 2009, Q3i,ii,iv,v)

(i)	$X \sim N(1720, 90^2)$ $P(X < 1700) = P\left(Z < \frac{1700-1720}{90}\right)$ $= P(Z < -0.2222)$ $= \Phi(-0.2222) = 1 - \Phi(0.2222)$ $= 1 - 0.5879$ $= 0.4121$	M1 for standardising A1 M1 use of tables (correct tail) A1CAO **NB ANSWER GIVEN**	**4**
(ii)	P(2 of 4 below 1700) $= \binom{4}{2} \times 0.4121^2 \times 0.5879^2 = 0.3522$	M1 for coefficient M1 for $0.4121^2 \times 0.5879^2$ A1 FT (min 2sf)	**3**
(iv)	H_0: $\mu = 1720$; H_1 is of this form since the consumer organisation suspects that the mean is below 1720 μ denotes the mean intensity of 25 Watt low energy bulbs made by this manufacturer.	B1 E1 B1 for definition of μ	**3**
(v)	Test statistic $= \frac{1703-1720}{90/\sqrt{20}} = \frac{-17}{20.12}$ $= -0.8447$ Lower 5% level 1 tailed critical value of $z = -1.645$ $-0.8447 > -1.645$ so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the mean intensity of bulbs made by this manufacturer is less than 1720	M1 must include $\sqrt{20}$ A1FT B1 for –1.645 No FT from here if wrong. Must be –1.645 unless it is clear that absolute values are being used. M1 for sensible comparison leading to a conclusion. FT only candidate's test statistic A1 for conclusion in words in context	**5**
		TOTAL	**20**

Q5, (Jun 2010, Q3)

(i)	(*A*) $P(X < 65) =$ $P\left(Z < \frac{65-63}{5.2}\right)$ $= P(Z < 0.3846)$ $= \Phi(0.3846) = 0.6497$	M1 for standardizing M1 for structure A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only	**3**
	(*B*) $P(60 < X < 65) = P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ $= P(-0.5769 < Z < 0.3846)$ $= \Phi(0.3846) - (1 - \Phi(0.5769))$ $= 0.6497 - (1 - 0.7181)$ $= 0.3678$	M1 for standardizing both M1 for correct structure A1 CAO 3s.f.	**3**
(ii)	P(All 5 between 60 and 65) $= 0.3678^5 = 0.00673$	M1 A1 FT (min 2sf)	**2**
(iii)	From tables $\Phi^{-1}(0.95) = 1.645$ $\frac{k-63}{5.2} = -1.645$ $x = 63 - 5.2 \times 1.645 = 54.45$ mins	B1 for ±1.645 seen M1 for correct equation in *k* A1 CAO	**3**
(iv)	H_0: $\mu = 63$ minutes; H_1: $\mu < 63$ minutes. Where μ denotes the population mean time on the new course.	B1 for use of 63 B1 for both correct B1 for definition of μ	**3**
	Test statistic $= \frac{61.7-63}{5.2/\sqrt{15}} = \frac{-1.3}{1.3426}$ $= -0.968$ 5% level 1 tailed critical value of $z = 1.645$ $-0.968 > -1.645$ so not significant. There is not sufficient evidence to reject H_0 There is insufficient evidence to conclude that the new course results in lower times.	M1 must include √15 A1 B1 for ±1.645 M1 for sensible comparison leading to a conclusion A1 FT for correct conclusion in words in context	**5**
			19

Q6, (Jun 2012, Q4b)

$\bar{x} = 4.995$	B1	For 4.995 seen
H_0: $\mu = 5$	B1	For use of 5 in hypotheses.
H_1 : $\mu < 5$	B1	For both correct. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population mean.
Where μ denotes the **mean** content of the bags of flour (in the population)	B1	For definition of μ in context. Condone omission of "population" if correct notation μ is used, but if μ is defined as the **sample** mean then award **B0**.
Test statistic $= \dfrac{4.995 - 5.0}{0.0072/\sqrt{8}} = \dfrac{-0.005}{0.002546} = -1.964$	M1* A1	must include √8 FT their $\bar{x}$. Allow +1.964 only if later compared with +1.645
Lower 5% level 1 tailed critical value of $z = -1.645$ $P(X \leq 4.995) = 0.02475 < 0.05$	B1*	For −1.645 No FT from here if wrong. Must be −1.645 unless it is clear that absolute values are being used.
$-1.964 < -1.645$ so significant.	M1 dep*	For sensible comparison with correct c.v. leading to a conclusion.
There is sufficient evidence to reject H_0 There is sufficient evidence to suggest that the average contents of bags is less than 5kg.	A1 **[9]**	For non-assertive conclusion in words and in context. No FT here. See additional notes.

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Q7, (Jan 2013, Q4b)

$\bar{x} = 417.79$	B1	For $\bar{x}$	
H_0: $\mu = 420$;	B1	For use of 420 in hypotheses. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population mean.	
H_1 : $\mu \neq 420$	B1	For both correct	
Where μ denotes the mean volume of the cans of tomato purée (in the population)	B1	For definition of μ. Condone omission of "population" if correct notation μ is used, but if μ is defined as the **sample** mean then award **B0**.	
Test statistic $= \dfrac{417.79-420}{3.5/\sqrt{10}} = \dfrac{-2.21}{1.107} = -1.997$	M1* A1	must include √10 FT their $\bar{x}$	
Lower 1% level 2 tailed critical value of z = -2.576 $P(X \leq 417.79) = 0.0229 > 0.005$ (2 tail)	B1*	For –2.576 Must be –2.576 unless it is clear that absolute values are being used.	
$-1.997 > -2.576$	M1 dep*	For sensible comparison leading to a conclusion.	
So not significant. There is insufficient evidence to reject H0			
There is insufficient evidence to suggest that the average volumes of the cans of tomato purée is not 420ml	A1 **[9]**	For conclusion in words in context provided that correct cv used. FT only candidate's test statistic.	

Q8, (Jun 2014, Q3)

(i)		$P(X > 135) = P\left(Z > \frac{135-130.5}{\sqrt{11.84}}\right)$	M1	For standardising. Penalise use of "continuity corrections"
		$= P(Z > 1.308) = 1 - \Phi(1.308) = 1 - 0.9045$	M1	For correct structure i.e. finding the area to the right of their z
		=0.0955 (3s.f.)	A1	CAO inc use of diff tables Allow 0.0954 and 0.0956 If numerator reversed, give BOD only if $P(Z < -1.308)$ is used
			[3]	
(ii)		From tables $\Phi^{-1}(0.99) = 2.326$	B1	±2.326 or (better) seen, not ±2.33
		$\frac{k-130.5}{\sqrt{11.84}} = 2.326$ oe	M1	For sensible equation in k with their z **value**. Note that use of $z = 0.8389$ from $\Phi(0.99)$ gets B0M0A0, as 0.8389 is clearly a **probability**. Allow use of -2.326 (or their negative z) with numerator reversed. Condone use of σ = 11.84 if also used in part (i). Condone use of "$k \pm 0.5$" for k in equation. 0/3 for trial and improvement
		$k = 130.5 + 2.326 \times \sqrt{11.84} = 138.50$	A1	CAO Allow 138.504. Accept 138.5 Do not accept final answers of 139 or 138.
			[3]	

(iii)		P(Wing length = 131) = $P\left(\frac{130.5-130.5}{\sqrt{11.84}} \le Z \le \frac{131.5-130.5}{\sqrt{11.84}}\right)$	B1	For both limits correct, soi. e.g. use of 0.5 in probability calculation implies correct lower limit.
		$= P(0 < Z < 0.2906)$ $= \Phi(0.2906) - \Phi(0)$ $= 0.6143 - 0.5$	M1	For **correct structure** using their standardised values. i.e. **Finding the area between their z values found using μ = 130.5** Condone use of σ = 11.84 if also used in part (i) or part (ii).
		$= 0.1143$	A1	CAO inc use of diff tables Allow 0.1145 Allow 0.114 www
			[3]	

Question			Answer	Marks	Guidance
3	(iv)		H_0: $\mu = 130.5$ H_1 : $\mu > 130.5$	B1	For both correct Hypotheses in words must refer to population. Do not allow other symbols unless clearly defined as population mean
			Where μ denotes the **mean wing length** (in the population) (of Scandinavian male blackbirds).	B1	For definition of μ in context. Do not allow "sample mean wing length" or "mean wing length of English blackbirds"
			Test statistic $= \dfrac{132.4 - 130.5}{\sqrt{11.84} / \sqrt{20}} = \dfrac{1.90}{0.7694}$	M1*	must include √20 Condone use of σ = 11.84 if also used in part (i), part (ii) or part (iii). Condone numerator reversed for max M1*A0B1M0depA0A0 (max 4/8)
			= 2.469	A1	Allow 2.47
			Upper 5% level 1 tailed critical value of $z =$ 1.645	B1	For 1.645. Must be positive. B0 if -1.645 seen. No further A marks from here if wrong.
			2.469 > 1.645	M1dep*	For sensible comparison leading to a conclusion.
			The result is significant. There is sufficient evidence to reject H_0	A1	For correct conclusion. e.g. for "significant" oe FT only candidate's test statistic if cv = 1.645
			There is sufficient evidence to **suggest** that the <u>mean wing length</u> (of this population of birds) <u>is greater</u> (than 130.5mm).	A1	For **non-assertive** conclusion in <u>context</u>, consistent with their result Condone use of "average" for "mean" FT only candidate's test statistic if cv = 1.645
				[8]	

Question			Answer	Marks	Guidance
	(v)		With a 10% significance level rather than a 5% significance level, Advantage: One is less likely to accept the null hypothesis when it is false. Disadvantage: One is more likely to reject the null hypothesis when it is true.	 E1 E1 **[2]**	 Accept equivalent wording. Note – Unless stated otherwise, assume the first comment relates to an advantage and the second comment relates to a disadvantage.