Hypothesis Testing for the Sample Mean of a Normal Distribution (Sheet 2)

Q1.

A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

(c) Test Hannah's belief at the 5% level of significance.

You should state your hypotheses clearly.

(5)

(Total for question = 13 marks)

Q2.

A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

(Total for question = 12 marks)

Q3.

A manufacturing company produces solar panels. The output of each solar panel is normally distributed with standard deviation 6 watts. It is thought that the mean output, μ, is 160 watts.

A researcher believes that the mean output of the solar panels is greater than 160 watts. He writes down the output values of 5 randomly selected solar panels. He uses the data to carry out a hypothesis test at the 5% level of significance.

He tests H_0: $\mu = 160$ against H_1: $\mu > 160$

On reporting to his manager, the researcher can only find 4 of the output values. These are shown below

168.2 157.4 173.3 161.1

Given that the result of the hypothesis test is that there is significant evidence to reject H_0 at the 5% level of significance, calculate the minimum possible missing output value, α. Give your answer correct to 1 decimal place.

(6)

(Total 6 marks)

Q4.

A random sample of size n is to be taken from a population that is normally distributed with mean 40 and standard deviation 3. Find the minimum sample size such that the probability of the sample mean being greater than 42 is less than 5%.

(5)

(Total 5 marks)

Q5.

The time, in minutes, it takes Robert to complete the puzzle in his morning newspaper each day is normally distributed with mean 18 and standard deviation 3. After taking a holiday, Robert records the times taken to complete a random sample of 15 puzzles and he finds that the mean time is 16.5 minutes. You may assume that the holiday has not changed the standard deviation of times taken to complete the puzzle.

Stating your hypotheses clearly test, at the 5% level of significance, whether or not there has been a reduction in the mean time Robert takes to complete the puzzle.

(7)

(Total 7 marks)

Q6.

A report states that employees spend, on average, 80 minutes every working day on personal use of the Internet. A company takes a random sample of 100 employees and finds their mean personal Internet use is 83 minutes with a standard deviation of 15 minutes. The company's managing director claims that his employees spend more time on average on personal use of the Internet than the report states.

Test, at the 5% level of significance, the managing director's claim. State your hypotheses clearly.

(7)

(Total 7 marks)

Q7.

The lifetime, L hours, of a battery has a normal distribution with mean 18 hours and standard deviation 4 hours.

Alice's calculator requires 4 batteries and will stop working when any one battery reaches the end of its lifetime.

(a) Find the probability that a randomly selected battery will last for longer than 16 hours.

(1)

At the start of her exams Alice put 4 new batteries in her calculator.
She has used her calculator for 16 hours, but has another 4 hours of exams to sit.

(b) Find the probability that her calculator will not stop working for Alice's remaining exams.

(5)

Alice only has 2 new batteries so, after the first 16 hours of her exams, although her calculator is still working, she randomly selects 2 of the batteries from her calculator and replaces these with the 2 new batteries.

(c) Show that the probability that her calculator will not stop working for the remainder of her exams is 0.199 to 3 significant figures.

(3)

After her exams, Alice believed that the lifetime of the batteries was more than 18 hours. She took a random sample of 20 of these batteries and found that their mean lifetime was 19.2 hours.

(d) Stating your hypotheses clearly and using a 5% level of significance, test Alice's belief.

(5)

(Total for question = 14 marks)

Q8.

A machine fills packets with X grams of powder where X is normally distributed with mean μ. Each packet is supposed to contain 1 kg of powder.

To comply with regulations, the weight of powder in a randomly selected packet should be such that $P(X < \mu - 30) = 0.0005$

(a) Show that this requires the standard deviation to be 9.117 g to 3 decimal places.

(3)

A random sample of 10 packets is selected from the machine. The weight, in grams, of powder in each packet is as follows

999.8 991.6 1000.3 1006.1 1008.2 997.0 993.2 1000.0 997.1 1002.1

(b) Assuming that the standard deviation of the population is 9.117 g, test, at the 1% significance level, whether or not the machine is delivering packets with mean weight of less than 1 kg. State your hypotheses clearly.

(7)

(Total 10 marks)