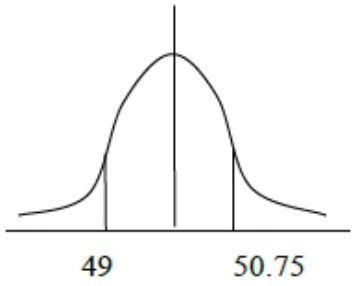
Hypothesis Testing for the Sample Mean of a Normal Distribution (Sheet 2) Mark Scheme

Q1.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\frac{24.63-25}{\sigma}=-1.0364$	M1	This mark is given for standardising as part of a method to find σ
	$\sigma=0.357$	A1	This mark is given for a correct value of σ
	$P(D>K)=0.4$ or $P(D<K)=0.6$	B1	This mark is given for
	$\frac{k-25}{\sigma}=\frac{k-25}{0.357}=0.2533$	M1	This mark is given for using a normal model to find the probability
	$k=25.09$	A1	This mark is given for a correct value for k
(b)	$Y\sim B(200, 0.45)$ so $W\sim N(90, 49.5)$	B1	This mark is given for setting up the normal distribution approximation of the binomial
	$P(Y<100)\approx P(W<99.5)=P\left(Z<\frac{99.5-90}{\sqrt{49.5}}\right)$	M1	This mark is given for using the normal model with a continuity correction
	$=0.912$	A1	This mark is given for finding a correct value of the probability
(c)	$H_0: \mu=25$ $H_1: \mu<25$	B1	This mark is given for both hypotheses in terms of μ found correctly
	$\overline{D}\sim N\left(25, \frac{0.16^2}{20}\right)$	M1	This mark is given for a method to set up the normal distribution
	$P(\overline{D}<24.94)=0.0468$	A1	This mark is given for using the model to find a correct p-value
	$p=0.0468<0.05$, so reject H_0	M1	This mark is given for a correct comparison and non-contextual conclusion
	There is sufficient evidence to support Hannah's belief	A1	This mark is given for a correct conclusion in context stated
			(Total 13 marks)

Q2.

Question	Scheme	Marks	AOs
(a)			
	P $(L > 50.98) = 0.025$	B1cao	3.4
	$\therefore \frac{50.98-\mu}{0.5} = 1.96$	M1	1.1b
	$\therefore \quad \mu = 50$	A1cao	1.1b
	$P(49 < L < 50.75)$	M1	3.4
	$= 0.9104\ldots$ awrt **0.910**	A1ft	1.1b
		(5)	
(b)	S = number of strips that cannot be used so $S \sim B(10, 0.090)$	M1	3.3
	$= P(S \leqslant 3) = 0.991166\ldots$ awrt 0.991	A1	1.1b
		(2)	
(c)	$H_0 : \mu = 50.1$ $H_1 : \mu > 50.1$	B1	2.5
	$\bar{X} \sim N\left(50.1, \frac{0.6^2}{15}\right)$ and $\bar{X} > 50.4$	M1	3.3
	$P(\bar{X} > 50.4) = 0.0264$	A1	3.4
	$p = 0.0264 > 0.01$ or $z = 1.936\ldots < 2.3263$ and not significant	A1	1.1b
	There is insufficient evidence that the **mean length** of strips is **greater than 50.1**	A1	2.2b
		(5)	
			(12 marks)

Q3.

Question Number	Scheme	Marks
	Test statistic, $z = \frac{132+\frac{\alpha}{5}-160}{\frac{6}{\sqrt{5}}}$	M1A1ft
	Critical z values is 1.6449	B1
	Therefore the test statistic is significant if $\frac{132+\frac{\alpha}{5}-160}{\frac{6}{\sqrt{5}}} > 1.6449$	M1
	Therefore $132+\frac{\alpha}{5}-160 > 1.6449\times\frac{6}{\sqrt{5}}$ $\alpha > 5\left(1.6449\times\frac{6}{\sqrt{5}}+28\right)$ $\alpha > 162.0686493\ldots$ Accept awrt 162.1	A1
		(6) (6 marks)
	Notes	
	1st A1 ft on their $\bar{x}$ 1st B1 given for 1.6449 seen (condone sign) 3rd M1 inequality using their test statistic, accept incorrect signs for M1	

Q4.

Question Number	Scheme	Marks
	$X \sim N(40, 3^2)$ $\quad \bar{X} \sim N\left(40, \frac{9}{n}\right)$ $\quad$ (Condone $Y \sim N\left(40, \frac{9}{n}\right)$)	B1
	$P(\bar{X} > 42) = P\left(Z > \frac{42-40}{\sqrt{\frac{9}{n}}}\right)$	M1
	$\frac{42-40}{\sqrt{\frac{9}{n}}} \geq 1.6449$	B1 dM1
	$n \geq 6.087$ $n = 7$	A1 **[Total 5]**
	1st B1 for stating the correct distribution for $\bar{X}$. May be implied if correctly used in line 2 and no incorrect version seen elsewhere. 1st M1 for an attempt to standardise with 42, 40 and their $\sqrt{\frac{9}{n}}$, must have n. Allow $\pm$ 2nd B1 for using $z = \pm 1.6449$ (or better) 2nd dM1 Dep on 1st M1 for forming an equation in n or $\sqrt{n}$. Allow "=" or "<" i.e. setting their standardised expression = their z value ($\lvert z \rvert > 1.5$) A1 for $n = 7$ only The A1 must follow from correct working so e.g. $n < 6.087$ leading to $n = 7$ is A0	

Q5.

Question number	Scheme	Marks
	$H_0 : \mu = 18, \quad H_1 : \mu < 18$	B1, B1
	$z = \frac{16.5-18}{3/\sqrt{15}} =, -1.9364...$ AWRT – 1.94	M1, A1
	5% one tail c.v. is z = (-) 1.6449 or probability (AWRT 0.026) (±) 1.6449	B1
	- 1.94 < -1.6449 or significant or reject H_0 or in critical region	M1
	There is evidence that the (mean) time to complete the puzzles has reduced Or Robert is getting faster (at doing the puzzles)	A1f.t. 7
	1st & 2nd B1 must see μ and 18 1st M1 for attempting test statistic, allow ±. Or attempt at critical value for $\bar{X}$: $\mu - z \times \frac{3}{\sqrt{15}}$ 1st A1 for AWRT – 1.94. Allow use of $\lvert z \rvert = +1.94$ to score M1A1. Or critical value = AWRT 16.7. 3rd B1 for AWRT 0.026 (i.e. correct probability only) or ± 1.6449. (May be seen in cv formula) 2nd M1 for correct comparison or statement relating their test statistic and 1.6449 or their probability and 0.05. Ignore their hypotheses if any or assume they were correct. 2nd A1f.t. for conclusion in context which refers to "speed" or "time". Depends only on previous M	

Q6.

Question Number	Scheme	Marks
	$H_0: \mu = 80$, $H_1: \mu > 80$	B1,B1
	$z = \dfrac{83-80}{\frac{15}{\sqrt{100}}} = 2$	M1A1
	$2 > 1.6449$ (accept 1.645 or better)	B1
	Reject H_0 or significant result or in the critical region	M1
	Managing director's claim is supported.	A1
		7

	1st B1 for H_0. They must use μ not x, p, λ or $\bar{x}$ etc
	2nd B1 for H_1 (must be > 80). Same rules about μ.
	1st M1 for attempt at standardising using 83, 80 and $\frac{15}{\sqrt{100}}$. Can accept $\pm$.
	May be implied by $z = \pm 2$
	1st A1 for + 2 only
	3rd B1 for ± 1.6449 seen (or probability of 0.0228 or better)
	2nd M1 for a correct statement about "significance" or rejecting H_0 (or H_1) based on their z value and their 1.6449 (provided it is a recognizable critical value from normal tables) or their probability (< 0.5) and significance level of 0.05. Condone their probability > 0.5 compared with 0.95 for the 2nd M1
	2nd A1 for a correct contextualised comment. Must mention "director" and "claim" or "time" and "use of Internet". No follow through.
2nd M1A1	If no comparison or statement is made but a correct contextualised comment is given the M1 can be implied. If a comparison is made it must be compatible with statement otherwise M0 e.g. comparing 0.0228 with 1.6449 is M0 or comparing probability 0.9772 with 0.05 is M0 comparing -2 with - 1.6449 is OK provided a correct statement accompanies it condone -2 >-1.6449 provided their statement correctly rejects H_0.
Critical Region	They may find a critical region for $\bar{X}$: $\bar{X} > 80 + \frac{15}{\sqrt{100}} \times 1.6449 =$ awrt 82.5
	1st M1 for $80 + \frac{15}{\sqrt{100}} \times (z \text{ value})$
	3rd B1 for 1.645 or better
	1st A1 for awrt 82.5
	The rest of the marks are as per the scheme.

Q7.

Qu	Scheme	Marks	AO
(a)	$P(L > 16) = 0.69146\ldots$ awrt 0.691	B1 (1)	1.1b
(b)	$P(L > 20 \mid L > 16) = \dfrac{P(L > 20)}{P(L > 16)}$	M1	3.1b
	$= \dfrac{0.308537\ldots}{(a)}$ or $\dfrac{1-(a)}{(a)}, = 0.44621\ldots$	A1ft, A1	1.1b 1.1b
	For calc to work require $(0.44621\ldots)^4 = 0.03964\ldots$ awrt 0.0396	dM1 A1 (5)	2.1 1.1b
(c)	Require: $[P(L > 4)]^2 \times [P(L > 20 \mid L > 16)]^2$	M1	1.1a
	$= (0.99976\ldots)^2 \times (\text{"}0.44621\ldots\text{"})^2$	A1ft	1.1b
	$= 0.19901\ldots$ awrt 0.199 (*)	A1cso* (3)	1.1b
(d)	$H_0 : \mu = 18 \quad H_1 : \mu > 18$	B1	2.5
	$\bar{L} \sim N\left(18, \left(\dfrac{4}{\sqrt{20}}\right)^2\right)$	M1	3.3
	$P(\bar{L} > 19.2) = P(Z > 1.3416\ldots) = 0.089856\ldots$	A1	3.4
	(0.0899 > 5%) or (19.2 < 19.5) or 1.34 < 1.6449 so not significant	A1	1.1b
	Insufficient evidence to support Alice's claim (or belief)	A1 (5)	3.5a
		(14 marks)	

	Notes
(a)	B1 for evaluating probability using their calculator (awrt 0.691) Accept 0.6915
(b)	1st M1 for a first step of identifying a suitable conditional probability (either form)
	1st A1ft for a ratio of probabilities with numerator = awrt 0.309 or 1 – (a) and denom = their (a)
	2nd A1 for awrt 0.446 (o.e.) Accept 0.4465 (from $\frac{0.3085}{0.691} = 0.44645\ldots$)
	NB $\frac{P(16<L<20)}{P(L>16)} = 0.5538\ldots$ scores M1A1A1 when they do 1 – 0.5538 = 0.4462…
	2nd M1 (dep on 1st M1) for 2nd correct step i.e. (their 0.446…)^4 or X~B(4, "0.446") and $P(X = 4)$
	3rd A1 for awrt 0.0396
(c)	1st M1 for a correct approach to solving the problem (May be implied by A1ft)
	1st A1ft for $P(L > 4)$ = awrt 0.9998 used and ft their 0.44621 in correct expression
	If use $P(L > 20) = 0.3085..$ as 0.446.. in (b) then M1 for $(0.3085..)^2 \times [P(L > 4)]^2$; A1ft as above
*	2nd A1cso for 0.199 or better with clear evidence of M1 [NB $(0.4662..)^2 = 0.199\ldots$ is M0A0A0]
	Must see M1 scored by correct expression in symbols or values (M1A1ft)
(d)	B1 for both hypotheses in terms of μ.
	M1 for selecting a suitable model. Sight of normal, mean 18, sd $\frac{4}{\sqrt{20}}$ (o.e.) or variance = 0.8
	1st A1 for using the model correctly. Allow awrt 0.0899 or 0.09 from correct prob. statement
ALT	CR $(\bar{L}) > 19.471\ldots$ (accept awrt 19.5) or CV of 1.6449 (or better: calc 1.6448536..)
	2nd A1 for correct non-contextual conclusion. Wrong comparison or contradictions A0
	Error giving 2nd A0 implies 3rd A0 but just a correct contextual conclusion can score A1A1
	3rd A1 dep on M1 and 1st A1 for a correct contextual conclusion mentioning Alice's claim /belief or there is insufficient evidence that the mean lifetime is more than 18 hours

Q8.

Question Number	Scheme	Marks
(a)	$z = \pm 3.2905$	B1
	$\sigma = \frac{30}{3.2905}$	M1
	$\sigma = 9.117$ **	A1cso
		(3)
(b)	$H_0: \mu = 1000$ $H_1: \mu < 1000$	B1
	mean weight = 999.54	B1
	$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{(999.54-1000)}{\frac{9.117}{\sqrt{10}}} = -0.160$ or $\frac{c-1000}{\sqrt{83.12/10}} = -2.3263 \therefore$ CR $c < 993.29$	M1A1
	1% critical value = -2.3263	B1
	$-2.3263 < -0.160$	
	Accept H_0 / not in critical region	dM1
	There is no evidence that that the machine is delivering packets of mean weight less than 1 kg	A1ft
		(7)
		Total 10

Notes

(a)	M1 for 30/'their $\lvert z \rvert$ ' >1
	A1 cso as given answer
(b)	1st B1 both hypotheses correct.
	Accept 1kg in hypotheses if consistent units used in working usually either kg or g.
	2nd B1 999.54 (g) or 0.99954 (kg)
	1st M1 for standardising using their mean allow ± , 1000 and $\frac{9.117}{\sqrt{10}}$ o.e. in kg
	1st A1 awrt -0.160 unless clearly using $\lvert z \rvert$ (stated) then accept 0.160 or CR awrt 993
	Condone -0.16 if fully correct expression seen.
	3rd B1 ± 2.3263 sign consistent with test statistic or $p = 0.4364 > 0.01$ NB $p = 0.5636 < 0.99$
	2nd dM1 dependent upon 1st M for a correct statement linking their test statistic and their cv
	Contradictory statements score M0 e.g. "significant, do not reject H_0"
	2nd A1 for correct conclusion in context. Must mention 'machine' and 'packets'.