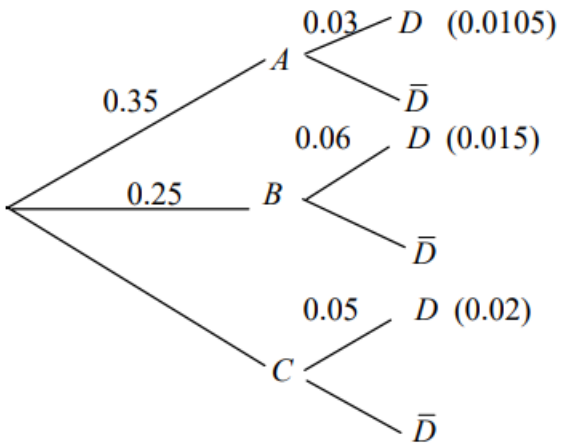
Conditional Probability with Tree Diagrams and Two-Way Tables (From Edexcel 6683)

Q1, (Jun 2005, Q7)

	Glasses / No Glasses / Totals Science 18 **12** 30 Arts **27** **23** **50** — 50 may be seen in (a) Humanities 44 **24** 68 — 23 may be seen in (b) Totals 89 **59** 148		B1 B1
(a)	$P(\text{Arts}) = \frac{50}{148} = \frac{25}{74} = 0.338$	a number/148	M1 A1 (4)
(b)	$P(\text{No glasses / Arts}) = \frac{23/148}{50/148} = \frac{23}{50} = 0.46$	$\frac{\text{prob}}{\text{their(a)prob}}$ or $\frac{\text{number}}{\text{their } 50}$	M1 A1 (2)
(c)	$P(\text{Right Handed}) = (\frac{30}{148} \times 0.8) + (\frac{50}{148} \times 0.7) + (\frac{68}{148} \times 0.75)$	attempt add three prob A1 √ on their (a)	M1 A1 √
	$= \frac{55}{74} = 0.743$	awrt 0.743	A1 (3)
(d)	$P(\text{Science /Right handed}) = \frac{\frac{30}{148} \times 0.8}{(c)} = \frac{12}{55} = 0.218$	√ on their (c)	M1 A1√ A1 (3)

Q2, (Jan 2007, Q2)

(a)



Correct tree shape — M1

A, *B* and *C* and 0.35 and 0.25 — A1

D (x3) and 0.03, 0.06, 0.05 — A1 (3)

(May be implied by seeing $P(A \cap D)$ etc at the ends)

(b)(i) $P(A \cap D) = 0.35 \times 0.03$, $= \underline{\mathbf{0.0105}}$ or $\frac{21}{2000}$ — M1, A1

$P(C) = 0.4$ (anywhere) — B1

(ii) $P(D) =$ (i) + 0.25x 0.06 + (0.4x0.05) — M1

$= \underline{\mathbf{0.0455}}$ or $\frac{91}{2000}$ — A1 (5)

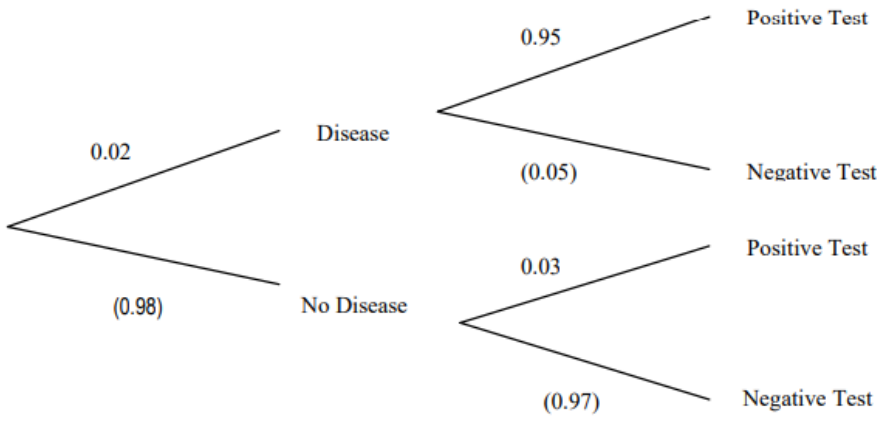
(c) $P(C|D) = \frac{P(C \cap D)}{P(D)}$, $= \frac{0.4 \times 0.05}{\text{(ii)}}$ — M1, A1ft

$= 0.43956\ldots$ or $\frac{40}{91}$ — $\underline{\mathbf{0.44}}$ or awrt $\underline{\mathbf{0.440}}$ — A1 (3)

[Correct answers only score full marks in each part] — **11 marks**

Q3, (Jun 2008, Q1)

(a)



Tree without probabilities or labels **M1**

0.02(Disease), 0.95(Positive) on correct branches **A1**

0.03(Positive) on correct branch. **A1**

[3]

(b) P(Positive Test) $= 0.02 \times 0.95 + 0.98 \times 0.03$ **M1A1ft**

$= 0.0484$ **A1**

[3]

(c) $P(\text{Do not have disease} \mid \text{Postive test}) = \dfrac{0.98 \times 0.03}{0.0484}$ **M1**

$= 0.607438..$ awrt 0.607 **A1**

[2]

(d) Test not very useful OR
High probability of not having the disease for a person with a positive test **B1**

[1]

Total 9

Q4, (Jan 2009, Q2)

E = take regular exercise B = always eat breakfast

(a) $P(E \cap B) = P(E \mid B) \times P(B)$ M1

$= \dfrac{9}{25} \times \dfrac{2}{3} = 0.24$ or $\dfrac{6}{25}$ or $\dfrac{18}{75}$ A1 (2)

(b) $P(E \cup B) = \dfrac{2}{3} + \dfrac{2}{5} - \dfrac{6}{25}$ or $P(E' \mid B')$ or $P(B' \cap E)$ or $P(B \cap E')$ M1

$= \dfrac{62}{75}$ $= \dfrac{13}{25}$ $= \dfrac{12}{75}$ $= \dfrac{32}{75}$ A1

$P(E' \cap B') = 1 - P(E \cup B) = \dfrac{13}{75}$ or $0.17\dot{3}$ M1 A1 (4)

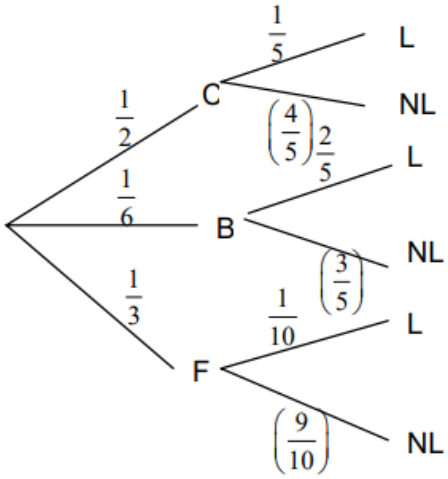
(c) $P(E \mid B) = 0.36 \neq 0.40 = P(E)$ or $P(E \cap B) = \dfrac{6}{25} \neq \dfrac{2}{5} \times \dfrac{2}{3} = P(E) \times P(B)$ M1 A1 (2)

So E and B are <u>not</u> statistically independent

[8]

Q5, (Jun 2009, Q2)

(a)



Correct tree B1
All labels B1
Probabilities on correct branches B1

(3)

(b)(i) $\frac{1}{3}\times\frac{1}{10}=\frac{1}{30}$ or equivalent M1 A1

(2)

(ii) $CNL + BNL + FNL = \frac{1}{2}\times\frac{4}{5}+\frac{1}{6}\times\frac{3}{5}+\frac{1}{3}\times\frac{9}{10}$ M1

$= \frac{4}{5}$ or equivalent A1

(2)

(c) $P(F'/L)=\frac{P(F'\cap L)}{P(L)}$ Attempt correct conditional probability **but see notes** M1

$=\frac{\frac{1}{6}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{5}}{1-(ii)}$ $\frac{\text{numerator}}{\text{denominator}}$ $\frac{A1}{A1ft}$

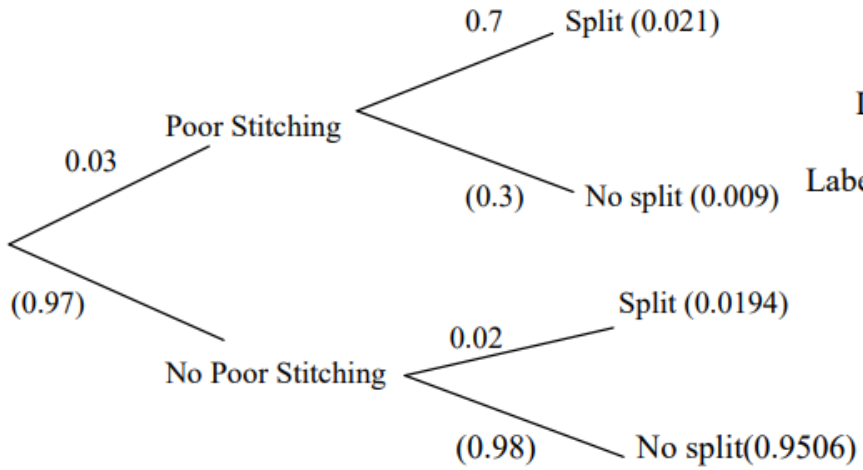
$=\frac{\frac{5}{30}}{\frac{1}{5}}=\frac{5}{6}$ or equivalent cao A1

(4)

[11]

Q6, (Jun 2012, Q7)

(a)



Shape B1

Labels & 0.03 B1

Labels & 0.7,0.02 B1

(3)

(b) P(Exactly one defect) = $0.03\times0.3+0.97\times0.02$ or $P(PS\cup Split)-2P(PS\cap Split)$ M1A1ft

$=[0.009 + 0.0194 =]$ **0.0284** A1 cao (3)

(c) P(No defects) $=(1-0.03)\times(1-0.02)\times(1-0.05)$ (or better) M1

$=0.90307$ **awrt 0.903** A1 cao (2)

(d) P(Exactly one defect) = (b)$\times(1-0.05)$ + $(1-0.03)\times(1-0.02)\times0.05$ M1 M1

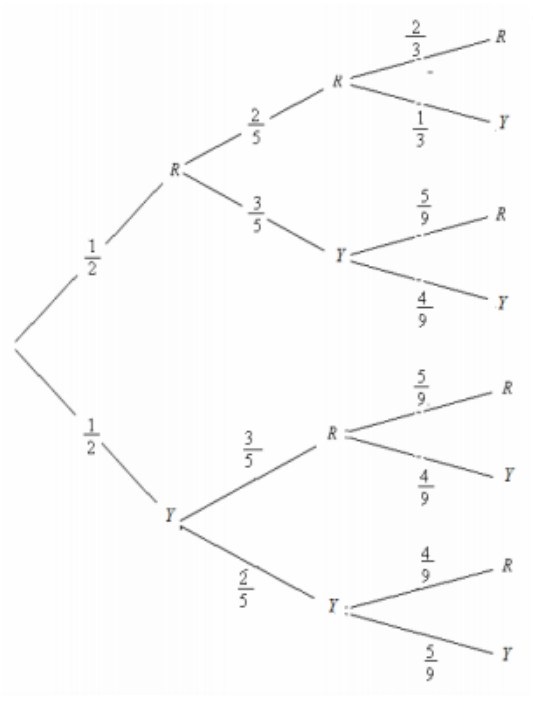
= "0.0284"$\times0.95+0.97\times0.98\times0.05$ A1ft

$=[0.02698 + 0.04753] = 0.07451$ **awrt 0.0745** A1 cao (4)

[12]

Q7, (Jan 2011, Q7)

(a)



$\frac{2}{15}$

$\frac{1}{6}$

$\frac{1}{6}$

$\frac{4}{45}$

$\left(\frac{1}{9}\right)$

both $\frac{2}{3},\frac{1}{3}$ B1

$\frac{4}{9}$ B1

both $\frac{3}{5},\frac{2}{5}$ B1

all three of $\frac{4}{9},\frac{4}{9},\frac{5}{9}$ B1

(4)

(b) $P(A)=P(RR)+P(YY)=\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times"\frac{2}{5}"=\frac{2}{5}$

B1 for $\frac{1}{2}\times\frac{2}{5}$ (oe) seen at least once

B1 M1 A1

(3)

(c) $P(B)=P(RRR)+P(RYR)+P(YRR)+P(YYR)$

M1 for at least 1 case of 3 balls identified. (Implied by 2nd M1)

M1

$\left(\frac{1}{2}\times\frac{2}{5}\times"\frac{2}{3}"\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{3}{5}"\times\frac{5}{9}\right)+\left(\frac{1}{2}\times"\frac{2}{5}"\times"\frac{4}{9}"\right)$ $\underline{=\frac{5}{9}}$ (*)

M1,A1cso

(3)

(d) $P(A\cap B)=P(RRR)+P(YYR)$

M1 for identifying both cases and + probs.
may be implied by correct expressions

M1

$=\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{4}{9}\right)$ $\underline{=\frac{2}{9}}$ (*)

A1cso

(2)

(e) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$="\frac{2}{5}"+\frac{5}{9}-\frac{2}{9}$ $\underline{=\frac{11}{15}}$

Must have some attempt to use

M1

A1cao

(2)

(f) $\frac{P(RRR)}{P(RRR)+P(YYY)}=\frac{\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}}{\left(\frac{1}{2}\times\frac{2}{5}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{5}{9}\right)}=\frac{6}{11}$

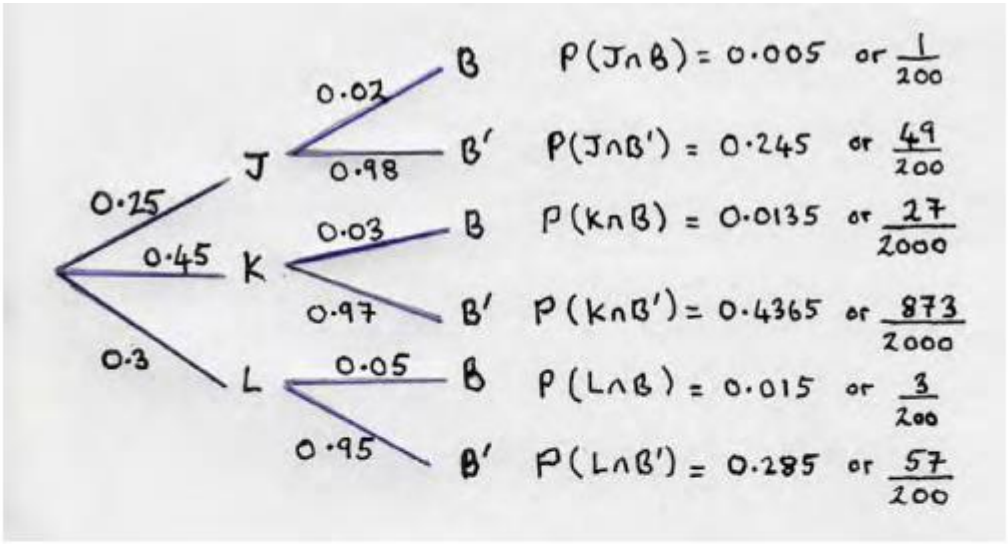
Probabilities must come from the product of 3 probs. from their tree diagram.

M1
A1ft
A1 cao

(3)

[17]

Q8, (Jun 2014, Q4)

(a)		M1 A1 **(2)**
(b)	0.25×0.98,	$= \mathbf{0.245}$ (or exact equiv. e.g. $\frac{49}{200}$)	M1A1 **(2)**
(c)	$0.25\times0.02+0.45\times0.03+0.3\times0.05$,	$= \mathbf{0.0335}$ (or exact equiv. e.g. $\frac{67}{2000}$)	M1A1 **(2)**
(d)	$[\mathrm{P}(J\cup L\mid B)] = \frac{0.25\times0.02+0.3\times0.05}{0.0335}$ or $\frac{0.0335-0.45\times0.03}{0.0335}$ $= 0.5970...$	awrt **0.597** (or $\frac{40}{67}$ or exact equiv.)	M1A1ft A1 **(3)**
			Total 9