Conditional Probability with the Normal Distribution (Sheet 2)

Q1.

Yuto works in the quality control department of a large company. The time, T minutes, it takes Yuto to analyse a sample is normally distributed with mean 18 minutes and standard deviation 5 minutes.

(a) Find the probability that Yuto takes longer than 20 minutes to analyse the next sample.

(3)

The company has a large store of samples analysed by Yuto with the time taken for each analysis recorded. Serena is investigating the samples that took Yuto longer than 15 minutes to analyse.

She selects, at random, one of the samples that took Yuto longer than 15 minutes to analyse.

(b) Find the probability that this sample took Yuto more than 20 minutes to analyse.

(4)

Serena can identify, in advance, the samples that Yuto can analyse in under 15 minutes and in future she will assign these to someone else.

(c) Estimate the median time taken by Yuto to analyse samples in future.

(5)

(Total for question = 12 marks)

Q2.

The time, in minutes, taken by men to run a marathon is modelled by a normal distribution with mean 240 minutes and standard deviation 40 minutes.

(a) Find the proportion of men that take longer than 300 minutes to run a marathon.

(3)

Nathaniel is preparing to run a marathon. He aims to finish in the first 20% of male runners.

(b) Using the above model estimate the longest time that Nathaniel can take to run the marathon and achieve his aim.

(3)

The time, W minutes, taken by women to run a marathon is modelled by a normal distribution with mean μ minutes.

Given that $P(W < \mu + 30) = 0.82$

(c) find $P(W < \mu - 30 \mid W < \mu)$

(3)

(Total for question = 9 marks)

Q3.
The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

(a) Find the probability that a randomly selected adult female has a height greater than 170 cm.

(3)

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

(b) find the probability that she has a height greater than 180 cm.

(4)

Half of tall adult females have a height greater than h cm.

(c) Find the value of h.

(5)

(Total 12 marks)

Q4.
The random variable $Z \sim N(0, 1)$

A is the event $Z > 1.1$
B is the event $Z > -1.9$
C is the event $-1.5 < Z < 1.5$

(a) Find

(i) P(A)
(ii) P(B)
(iii) P(C)
(iv) P($A \cup C$)

(6)

The random variable X has a normal distribution with mean 21 and standard deviation 5

(b) Find the value of w such that $P(X > w \mid X > 28) = 0.625$

(6)

(Total for question = 12 marks)

Q5.
The length of time, L hours, that a phone will work before it needs charging is normally distributed with a mean of 100 hours and a standard deviation of 15 hours.

(a) Find $P(L > 127)$.

(3)

(b) Find the value of d such that $P(L < d) = 0.10$

(3)

Alice is about to go on a 6 hour journey.
Given that it is 127 hours since Alice last charged her phone,

(c) find the probability that her phone will not need charging before her journey is completed.

(4)

(Total 10 marks)

Q6.

The time, in minutes, taken to fly from London to Malaga has a normal distribution with mean 150 minutes and standard deviation 10 minutes.

(a) Find the probability that the next flight from London to Malaga takes less than 145 minutes.

(3)

The time taken to fly from London to Berlin has a normal distribution with mean 100 minutes and standard deviation d minutes.

Given that 15% of the flights from London to Berlin take longer than 115 minutes,

(b) find the value of the standard deviation d.

(4)

The time, X minutes, taken to fly from London to another city has a normal distribution with mean μ minutes.

Given that $P(X < \mu - 15) = 0.35$

(c) find $P(X > \mu + 15 \mid X > \mu - 15)$.

(3)

(Total 10 marks)