Conditional Probability with the Normal Distribution (Sheet 2) Mark Scheme

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$[P(T>20)=]\ P\left(Z>\frac{20-18}{5}\right)$	M1
	$P(Z>0.4)=1-0.6554$	M1
	$=$ **0.3446** or awrt **0.345**	A1
		(3)
(b)	Require $P(T>20 \mid T>15)$ or $\frac{P(T>20)}{P(T>15)}$	M1
	$\frac{"(a)"}{P(Z>\frac{15-18}{5})}=\frac{"(a)"}{P(Z>-0.6)}$, $=\frac{"0.3446"}{0.7257}$ or $\frac{"0.345"}{0.726}$	M1, A1ft
	$=0.47485\ldots=$ awrt **0.475**	A1
		(4)
(c)	$P(T>d \mid T>15)=0.5$ or $P(T<d \mid T>15)=0.5$	M1
	$P(T>d)$ or $P(15<T<d)=0.5\times$"0.7257" =[0.36285]	A1ft
	$P(T<d)=$"0.63715"	M1
	So $\frac{d-18}{5}=0.35$ (calculator gives 0.35085…)	A1
	$d=19.754\ldots$ = awrt **19.8**	A1cso
	(Accept 19 mins 45(secs) or 19:45 but 19.45 is A0)	(5)
		[12]

	Notes
(a)	1st M1 for standardising with 20, 18 and 5. Accept ±
	2nd M1 for attempting $1-p$ [where $0.5<p<0.7$]. Beware $1-0.4$ (or their z value) is M0
	A1 for awrt 0.345 (Correct ans only 3/3)
(b)	1st M1 for either correct conditional probability statement (allow "in words" or any letter except Z)
	1st M1 can be implied by 2nd M1 so a mark of M0M1 should not be given.
	2nd M1 for using their (a) on num. and attempting to standardise $P(T>15)$ (no ±)on denom.
	Num.>Deno. is M0
	Allow one digit transcription errors from (a) e.g. 0.3464 or 0.3466 etc for 2nd M1 and 1st A1ft
	1st A1ft for their 0.3446 on numerator and denominator of 0.7257 (or better: 0.7257469…)
	provided Num < Denom. Allow 0.726 on the denominator
	Sight of $\frac{"0.3446"}{0.7257 \text{ or } 0.726}$ will score M1M1A1ft
	2nd A1 for awrt 0.475
(c)	1st M1 for a correct conditional probability statement that includes the 0.5
	1st A1ft for $P(T>d)$ or $P(15<T<d)=0.5\times$ their $P(T>15)$ [provided $P(T>15)>0.5$]
	Follow through (3sf) their $P(T>15)=0.7257$ or better from part (b). (Allow 0.726)
	Sight of $0.5\times$their $0.7257=$"0.36285" or better scores 1st M1 and 1st A1ft (Allow 0.363)
	2nd M1 (dep on 1st M1) for $P(T<d)=1-$"0.36285" or "0.36285" $+1-$"0.7257"
	$=$ [0.6371~0.6372]
	Sight of their 0.63715 or better (calc: 0.637126…) scores first 3 marks (Allow 0.637)
	2nd A1 for $\frac{d-18}{5}=0.35$ (or better) (Calc could give 0.350788…)
	3rd A1cso for ($d=$) awrt 19.8 (accept 19.7 not awrt 19.7) **Must** come from correct work.
Beware!	$0.5\times0.7257=0.36285$ and using this (instead of 0.35) as z value leads to 19.8 but is A0A0

Q2.

Question Number	Scheme	Marks
(a)	[$T \sim N(240, 40^2)$…require $P(T > 300)$]	
	$P\left(Z > \frac{300-240}{40}\right)$	M1
	$= 1 - P(Z < 1.5)$ or $1 - 0.9332$	M1
	= awrt **0.0668** or 6.68%	A1
		(3)
(b)	$[P(T < n) = 0.20 \Rightarrow]$ $\frac{n-240}{40} = -0.8416$	M1 B1
	n = awrt **206** minutes	A1
		(3)
(c)	$[P(W < \mu - 30 \mid W < \mu) =]$ $\frac{P(W < \mu - 30)}{P(W < \mu)}$	M1
	$= \frac{1-0.82}{0.50}$	A1
	$= \mathbf{0.36}$	A1cao
		(3)
		[9 marks]

	Notes
(a)	1st M1 for standardising with 300, 240 and 40. May be implied by use of 1.5 Allow $\pm$ 2nd M1 for $1 - P(Z <$ "1.5") i.e. a correct method for finding $P(Z >$ "1.5") e.g. $1 - p$ where $0.5 < p < 0.99$ A1 for awrt 0.0668 (Answer only 3/3)
(b)	M1 for an attempt to standardise with 240, 40 and n and set = $\pm z$ $(0.8 < \lvert z \rvert < 0.9)$ B1 for $z = \pm 0.8416$ (or better) used as a z value. Do not allow for $1 - 0.8416$ Calc gives 0.8416212…[**May be implied by awrt 206.34**, give B1 as well as A1 if seen] A1 for awrt 206 (can be scored for using a z value of 0.84 or even 0.85) Must follow from correct working but a range of possible z values are OK
Ans only	If answer is awrt 206 score M1B0A1 (unless of course z = 0.8416 seen) but awrt 206.34 scores 3/3
(c)	M1 for the correct ratio expression (Not $P([W < 30 - \mu] \cap [W < \mu])$ on numerator) Condone use of Z instead of W only if they later get a correct numerical ratio otherwise M0 However they may write $P\left(Z < \frac{-30}{\sigma}\right)$ etc which is of course fine 1st A1 for a correct numerical ratio
Use tables	May see use of $z = 0.92$ or better (calc: 0.9153650…) or $\sigma = 32.6$~32.8 allow:
ALT	1st M1 for $\frac{P(Z < -0.92)}{P(Z < 0)}$ and 1st A1 for $\frac{1-0.8212}{0.5}$ or $\frac{0.1788}{0.5}$
	2nd A1 for 0.36 or an exact equivalent e.g. $\frac{9}{25}$ (Answer only M1A1A0) The final answer of 0.36 must come from exact values; 0.36 rounded from 0.3576 etc is A0

Q3.

Question Number	Scheme	Marks
(a)	The random variable $H \sim$ height of females	
	$P(H>170) = P\left(Z > \frac{170-160}{8}\right)$ $[= P(Z>1.25)]$	M1
	$= 1-0.8944$	M1
	$= 0.1056$ (calc 0.1056498…) **awrt 0.106** (accept 10.6%)	A1 **(3)**
(b)	$P(H>180) = P\left(Z > \frac{180-160}{8}\right)$ $[=1-0.9938]$	M1
	$= 0.0062$ (calc 0.006209…) awrt 0.0062 or $\frac{31}{5000}$	A1
	$[P(H>180 \mid H>170)] = \frac{0.0062}{0.1056}$	M1
	$= 0.0587$ (calc 0.0587760…) **awrt 0.0587 or 0.0588**	A1 **(4)**
(c)	$P(H>h \mid H>170)\ (=0.5)$ or $\frac{P(H>h)}{P(H>170)}\ (=0.5)$	M1
	$[P(H>h)] = 0.5\times$"0.1056" $= 0.0528$ (calc 0.0528249…) or $[P(H<h)] = 0.9472$	A1ft
	$\frac{h-160}{8} = 1.62$ (calc 1.6180592…)	M1 B1
	$h =$ awrt 173 cm **awrt 173**	A1 **(5)**
		Total 12

	Notes
(a)	1st M1 for attempt at standardising with 170, 160 and 8. Allow ± i.e. for $\pm\frac{170-160}{8}$
	2nd M1 for attempting $1-p$ where $0.8<p<1$. Correct answer only 3/3
(b)	1st M1 for standardising with 180, 160 and 8
	1st A1 for 0.0062 seen, maybe seen as part of another expression/calculation.
	2nd M1 using conditional probability with denom = their (a) and num < their denom. Values needed.
	2nd A1 for awrt 0.0587 or 0.0588. Condone 5.87% or 5.88% or $\frac{31}{528}$
	Correct answer only 4/4
(c)	1st M1 for a correct conditional probability statement. Either line and don't insist on 0.5, ft (a)
	1st A1ft for $[P(H>h)] = 0.5\times$ their(a)
	Award M1A1ft for correct evaluation of $0.5\times$ their(a) or sight of 0.0528 or better
	2nd M1 for attempt to standardise (±) with 160 and 8 and set equal to $\pm z$ value $(1.56<\lvert z\rvert<1.68)$
	B1 for $(z=)$ awrt ± 1.62 (seen)
	2nd A1 for awrt 173 but dependent on both M marks.

Q4.

Question	Scheme	Marks
(a)(i)	$P(A) = P(Z > 1.1) = 1 - 0.8643 = \underline{0.1357}$ (accept awrt 0.136)	B1
(ii)	$P(B) = P(Z > -1.9) = \underline{0.9713}$ (accept awrt 0.971)	B1
(iii)	$P(C) = [P(-1.5 < Z < 1.5)] = 0.9332 - (1 - 0.9332)$ or $(0.9332 - 0.5) \times 2$ $= \underline{0.8664}$ (accept awrt 0.866)	M1 A1
(iv)	$P(A \cup C) = P(Z > -1.5)$ or $P(Z < 1.5)$ or $= P(A) + P(C) - P(A \cap C) = "0.1357" + "0.8664" - (0.9332 - 0.8643)$ $= \underline{0.9332}$ (accept awrt 0.933)	M1 A1 (6)
(b)	$[P(X > w \mid X > 28) =] \dfrac{P(X > w)}{P(X > 28)} = [0.625]$	M1
	$P(X > 28) = P\left(Z > \dfrac{28-21}{5}\right) = P(Z > 1.4) = [0.0808$ calc: 0.80756..$]$	M1
	$P(X > w) = 0.0808 \times 0.625$ (= 0.0505) or ($P(X < w) = 0.9495$)	A1
	$\dfrac{w-21}{5} = 1.64$	M1 B1
	$w =$ awrt $\underline{29.2}$	A1 (6) (12 marks)

	Notes
	Mark final answer here so in (ii) 0.9713 followed by 1 – 0.9713 is B0 but for rounding errors e.g. 29.245 followed by 29.3 apply ISW and award for 29.245
(a)(iii)	M1 for correct expression with probability values . Correct ans implies M1A1
(iv)	M1 for a correct addition formula with some correct substitution (or correct ft) or $P(Z > -1.5)$ (o.e) or for a fully correct expression with correct probabilities A1 for 0.9332 (accept 0.933) Correct answer only is M1A1
(b)	M1 for correct expression for conditional probability- must have $P(X > w)$ as num' May be implied by $P(X > w) = 0.625 \times$ (any probability) M1 for standardising 28 with 21 and 5 Allow $\pm$ (May be implied by 0.0808 [or awrt 0.081] seen in correct position) A1 for $P(X > w) = 0.0808 \times 0.625$ or $P(X > w) = 0.0505$ or $P(X < w) = 0.9495$) This A1 depends on both Ms but seeing $P(X > w) = 0.0808 \times 0.625$ scores M1M1A1
1st 3 marks	Allow $P\left(Z > \dfrac{w-21}{5}\right)$ instead of $P(X > w)$ for these first 3 marks
	M1 for standardising w with 21 and 5 (allow $\pm$) and setting equal to a z-value $\lvert z \rvert > 1$ Allow any letter instead of w B1 for 1.64 (or better) used correctly. [Calculator gives: 1.6402851...] A1 allow awrt 29.2

Q5.

Question Number	Scheme	Marks
(a)	$\frac{127-100}{15}$	M1
	So $P(L>127) = P(Z>1.8)$ or $1-P(Z<1.8)$ o.e.	A1
	$= 1-0.9641 = \underline{0.0359}$ (awrt $\underline{0.0359}$)	A1
		(3)
(b)	$\frac{d-100}{15} = -1.2816$ (Calculator gives $-1.2815515\ldots$)	M1, B1
	$d = 80.776$ (awrt $\underline{80.8}$)	A1
		(3)
(c)	Require $P(L>133 \mid L>127)$	M1
	$= \left[\frac{P(L>133)}{P(L>127)}\right] = \frac{P(Z>2.2)}{P(L>127)}$	dM1
	$= \left[\frac{1-0.9861}{1-0.9641}\right] = \frac{0.0139}{[0.0359]}$	A1
	$= 0.3871\ldots$ $=$ awrt $\underline{0.39}$	A1
		(4)
S.C.	An attempt at $P(L<133 \mid L>127)$ that leads to awrt 0.61 (M0M1A0A0)	
		10

Notes

(a)

M1 for attempting to standardise with 127, 100 and 15 . Allow $\pm$

1st A1 for $Z>1.8$. Allow a diagram but must have 1.8 and correct area indicated.
Must have the Z so $P(L>127)$ with or without a diagram is insufficient. May be implied by 0.0359

2nd A1 for awrt 0.0359 (calc. gives 0.035930266…). Correct ans only 3/3. M1A0A1 not poss.

(b)

M1 for an attempt to standardise with 100 and 15 and set = $\pm$ any z value ($|z|>1$)

B1 for $z = \pm 1.2816$ (or better) seen anywhere [May be implied by 80.776(72…) or better seen]

A1 for awrt 80.8 (can be scored for using 1.28 but then they get M1B0A1)
The 80.8 must follow from correct working.

Calc

If answer is awrt 80.8 **and** awrt 80.777 or 80.776… or better seen then award M1B1A1
If answer is awrt 80.8 or 80.77 then award M1B0A1 (unless of course $z = 1.2816$ is seen)

(c)

1st M1 for clear indication of correct conditional probability or attempt at correct ratio
So clear attempt at $\frac{P(L>133)}{P(L>127)}$ is sufficient for the 1st M1

2nd dM1 dependent on 1st M1 for $P(L>133)$ leading to $P(Z>2.2)$.

1st A1 for 0.0139 or better seen coming from $P(Z>2.20)$. Dependent on both Ms

2nd A1 for awrt 0.39. Both Ms required

ALT

If they assume Alice did not check that the phone was working you may see:
$[P(L<127).0] + P(L>127).\underline{P(L>133 \mid L>127)}$ Provided the conditional probability is seen as part of this calculation the 1st M1 can be scored and their final answer will be 0.0139(4/4)
An answer of 0.0139 without sight of the conditional probability is 0/4.

Q6.

Question	Scheme	Marks
(a)	$[P(M<145)=]\ P\left(Z<\frac{145-150}{10}\right)$	M1
	$=P(Z<-0.5)$ or $P(Z>0.5)$	A1
	= awrt **0.309**	A1 (3)
(b)	$[P(B>115)=0.15\Rightarrow]\ \frac{115-100}{d}=1.0364$ (Calc gives 1.036433...)	M1B1A1
	$d=14.5$ (Calc gives 14.4727...)	A1 (4)
(c)	$[P(X>\mu+15\mid X>\mu-15)=]\ \frac{P(X>\mu+15)}{P(X>\mu-15)}$	M1
	$=\frac{0.35}{1-0.35}$	A1
	$=\frac{7}{13}$ or **awrt 0.538**	A1 (3)
		[10]

	Notes
	Condone poor use of notation if a correct line appears later.
(a)	M1 for standardising with 145, 150 and 10. Allow $\pm$ and use of symmetry so 155 instead of 145
	1st A1 for $P(Z<-0.5)$ or $P(Z>0.5)$ i.e. a z value of ± 0.5 and a correct region indicated
	2nd A1 for awrt 0.309 Answer only is 3/3
(b)	M1 for $\pm\frac{115-100}{d}=z$ where $\lvert z\rvert>1$ Condone MR of $\mu=150$ instead of 100 for M1B1only
	B1 for a standardised expression $=\pm 1.0364$ (do not allow for use of $1-1.0364$)
	1st A1 for $z=$ awrt 1.04 and compatible signs i.e. a correct equation with $z=$ awrt 1.04
	2nd A1 for awrt 14.5 (allow awrt 14.4 if $z=$ awrt 1.04 is seen)
Calc	Answer only of awrt 14.473 scores M1B1A1A1
	Answer only of awrt 14.48 scores M1B0A1A1
(c)	M1 for a correct ratio expression need $P(X>\mu+15)$ on numerator. Allow use of a value for μ May be implied by next line.
	NB $\frac{0.35\times0.65}{0.65}=\frac{0.2275}{0.65}$ is M0
	1st A1 for a correct ratio of probabilities
	2nd A1 for awrt 0.538 or $\frac{7}{13}$ (o.e.). Allow 0.5385 provided 2nd A1 is scored.