Conditional Probability and Venn Diagrams (Sheet 2)

Q1.

Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

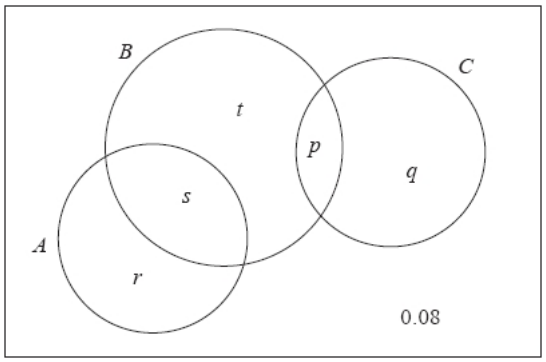
(d) Find $P([B \cup C]')$

(2)

(Total for question = 10 marks)

Q2.

The Venn diagram shows three events A, B and C, where p, q, r, s and t are probabilities.



$P(A) = 0.5$, $P(B) = 0.6$ and $P(C) = 0.25$ and the events B and C are independent.

(a) Find the value of p and the value of q.

(2)

(b) Find the value of r.

(2)

(c) Hence write down the value of s and the value of t.

(2)

(d) State, giving a reason, whether or not the events A and B are independent.

(2)

(e) Find $P(B \mid A \cup C)$.

(3)

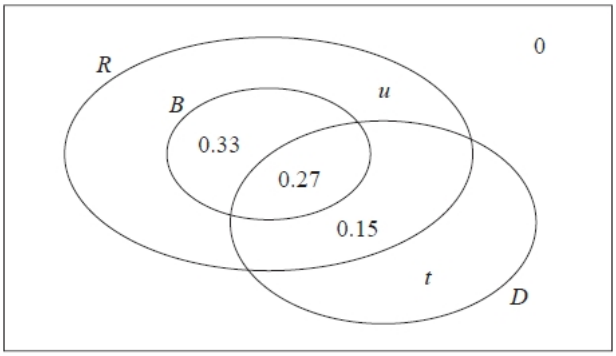
(Total for question = 11 marks)

Q3.

The Venn diagram shows the probabilities of customer bookings at Harry's hotel.

R is the event that a customer books a room
B is the event that a customer books breakfast
D is the event that a customer books dinner

u and *t* are probabilities.



(a) Write down the probability that a customer books breakfast but does not book a room.

(1)

Given that the events *B* and *D* are independent

(b) find the value of *t*

(4)

(c) hence find the value of *u*

(2)

(d) Find

(i) $P(D|R \cap B)$
(ii) $P(D|R \cap B')$

(4)

A coach load of 77 customers arrive at Harry's hotel.

Of these 77 customers

40 have booked a room and breakfast
37 have booked a room without breakfast

(e) Estimate how many of these 77 customers will book dinner.

(2)

(Total for question = 13 marks)

Q4.

For the events A and B,

$$P(A' \cap B) = 0.22 \text{ and } P(A' \cap B') = 0.18$$

(a) Find $P(A)$.

(1)

(b) Find $P(A \cup B)$.

(1)

Given that $P(A \mid B) = 0.6$

(c) find $P(A \cap B)$.

(3)

(d) Determine whether or not A and B are independent.

(2)

(Total 7 marks)

Q5.

A and B are two events such that

$$P(B) = \tfrac{1}{2} \quad P(A \mid B) = \tfrac{2}{5} \quad P(A \cup B) = \tfrac{13}{20}$$

(a) Find $P(A \cap B)$.

(2)

(b) Draw a Venn diagram to show the events A, B and all the associated probabilities.

(3)

Find

(c) $P(A)$

(1)

(d) $P(B \mid A)$

(2)

(e) $P(A' \cap B)$

(1)

(Total 9 marks)

Q6.

A college has 80 students in Year 12.

20 students study Biology
28 students study Chemistry
30 students study Physics
7 students study both Biology and Chemistry
11 students study both Chemistry and Physics
5 students study both Physics and Biology
3 students study all 3 of these subjects

(a) Draw a Venn diagram to represent this information.

(5)

A Year 12 student at the college is selected at random.

(b) Find the probability that the student studies Chemistry but not Biology or Physics.

(1)

(c) Find the probability that the student studies Chemistry or Physics or both.

(2)

Given that the student studies Chemistry or Physics or both,

(d) find the probability that the student does not study Biology.

(2)

(e) Determine whether studying Biology and studying Chemistry are statistically independent.

(3)

(Total for question = 13 marks)

Q7.

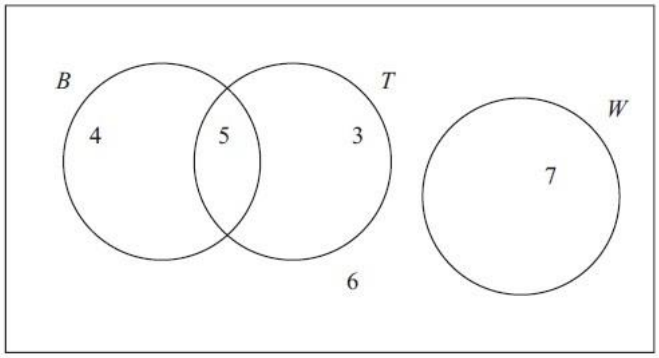


Figure 1

Figure 1 shows how 25 people travelled to work.

Their travel to work is represented by the events

B bicycle
T train
W walk

(a) Write down 2 of these events that are mutually exclusive. Give a reason for your answer.

(2)

(b) Determine whether or not *B* and *T* are independent events.

(3)

One person is chosen at random.

Find the probability that this person

(c) walks to work,

(1)

(d) travels to work by bicycle and train.

(1)

(e) Given that this person travels to work by bicycle, find the probability that they will also take the train.

(2)

(Total 9 marks)

Q8.

(a) State in words the relationship between two events R and S when $P(R \cap S) = 0$

(1)

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$

Find

(b) $P(B)$

(4)

(c) $P(A' \cap B)$

(2)

(d) $P(B' | A)$

(2)

(Total 9 marks)

Q9.

Jake and Kamil are sometimes late for school.
The events J and K are defined as follows

J = the event that Jake is late for school
K = the event that Jake is late for school

$P(J) = 0.25$, $P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

(1)

(b) Kamil is late for school.

(2)

Given that Jake is late for school,

(c) find the probability that Kamil is late.

(3)

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

(2)

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

(1)

(Total 9 marks)