



Conditional Probability and Venn Diagrams (Sheet 2) Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$P(A' B') = \frac{P(A' \cap B')}{P(B')} \text{ or } \frac{0.33}{0.55}$	M1	3.1a
	$= \frac{3}{5} \text{ or } 0.6$	A1	1.1b
		(2)	
(b)	e.g. $P(A) \times P(B) = \frac{7}{20} \times \frac{9}{20} = \frac{63}{400} \neq P(A \cap B) = 0.13 = \frac{52}{400}$ or $P(A' B') = 0.6 \neq P(A') = 0.65$	B1	2.4
		(1)	
(c)		B1	2.5
		M1	3.1a
		A1	1.1b
		M1	1.1b
		A1	1.1b
	(5)		
(d)	$P(B \cup C)' = 0.22 + 0.22 \text{ or } 1 - [0.56]$ or $1 - [0.13 + 0.23 + 0.09 + 0.11]$ o.e.	M1	1.1b
	$= 0.44$	A1	1.1b
		(2)	

(10 marks)

Notes:	
(a)	M1: for a correct ratio of probabilities formula and at least one correct value. A1: a correct answer
(b)	for a fully correct explanation: correct probabilities and correct comparisons.
(c)	B1: for box with B intersecting A and C but C not intersecting A. (Or accept three intersecting circles, but with zeros entered for $A \cap C$ and $A \cap B \cap C$) No box is B0 M1: for method for finding $P(B \cap C)$ A1: for 0.09 M1: for 0.13 and their 0.09 in correct places and method for their 0.23 A1: fully correct
(d)	M1: for a correct expression – ft their probabilities from their Venn diagram. A1: cao

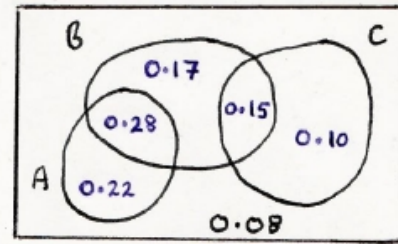


Q2.

Question Number	Scheme	Marks
(a)	$p = P(B \cap C) = P(B) \times P(C) = 0.6 \times 0.25 = \underline{0.15}$ $q = [P(C) - p] = \underline{0.10}$	M1 A1 (2)
(b)	$r = 1 - 0.08 - [P(B) + q] = 1 - 0.08 - 0.6 - 0.1$ (o.e.) <u>or</u> $1 - 0.08 - (0.6 + 0.25 - p)$ $= \underline{0.22}$	M1 A1cao (2)
(c)	$s = [P(A) - r] = \underline{0.28}$ $t = [P(B) - p - s$ <u>or</u> use $P(B \cap C') - s = 0.6 \times 0.75 - "0.28"] = \underline{0.17}$	B1ft B1ft (2)
(d)	$P(A) \times P(B) = 0.5 \times 0.6 = 0.3$ which is <u>not</u> equal to $s (= 0.28)$ So A and B are <u>not</u> independent	M1 A1 (2)
(e)	$\frac{(s+p) \text{ or } (0.6-t)}{P(A \cup C) \text{ or } [P(A) + P(C)] \text{ or } (r+s+p+q)} = \frac{("0.28"+"0.15") \text{ or } (0.6-"0.17")}{0.5+0.25}$ $= \underline{\frac{43}{75}}$	M1, A1ft A1 (3)

[11]

Notes	
(a)	M1 for a correct expression (using independence) for p <u>or</u> 0.15 A1 for $q = 0.10$ (both correct 2/2)
Mark (b) & (c) together	
(b)	M1 for a correct expression for r using $P(B \cup C)$. Can ft their $q \in [0, 0.32]$ A1cao for $r = 0.22$ (correct ans only 2/2)
(c)	1 st B1ft for $s = 0.28$ <u>or</u> $0.5 -$ their "0.22" 2 nd B1ft for $t = 0.17$ <u>or</u> $0.6 -$ their "0.15" - their "0.28"
ALT	Find t then s then r
(c)	2 nd B1 for $t = 0.17$ [from $1 - 0.08 - P(A) - P(C)$] 1 st B1ft for $s = 0.28$ <u>or</u> $P(B) - "0.17" - "0.15"$
(b)	M1 for $r = P(A) - s$ and the A1 for 0.22
$s = 0.3$	They assume A and B are independent and get $s = 0.3$ [from $P(A) \times P(B)$]
(c)	1 st B0 for $s = 0.3$ BUT can get 2 nd B1ft for either case in the scheme
(b)	M1 for $r = P(A) - s$ BUT then A0cao for $r = 0.2$
(d)	M1 for a correct $P(A) \times P(B) = 0.5 \times 0.6$ <u>or</u> 0.3 and a clear comparison with their $s (\neq 0.3)$ <u>Or</u> calculation of $P(A B) = \frac{7}{15}$ <u>or</u> 0.467 <u>or</u> $\frac{\text{their } s}{0.6}$ and comparison with $P(A) = 0.5$ (o.e.) A1 dep. on M1 being earned and clear statement that A and B are <u>not</u> independent
SC $s = 0.3$	dep on 1 st B1ft for $s = 0.5 - 0.2$ in (c); for correct calc. and conclusion seen (B1). On open M0A1
(e)	M1 for a correct ratio expression of probs: num. < den. Allow $1 - (0.08 + \text{their "t"})$ on den. Any sight of multiplication on the numerator e.g. 0.6×0.75 is M0 1 st A1ft for correct ratio or ft using their values in numerator but correct denominator. 2 nd A1 for $\frac{43}{75}$ or accept awrt 0.573



Fully correct Venn diagram will score the first 6 marks

If text and VD disagree use text values



Q3.

Question Number	Scheme	Marks
(a)	$[P(B \cap R') =] \underline{0}$	B1 (1)
(b)	$P(B) = 0.27 + 0.33 = 0.6$, $P(D) = 0.27 + 0.15 + t$, $P(B \cap D) = 0.27$ $[P(B) \times P(D) = P(B \cap D) \text{ gives}] \quad 0.6 \times (0.42 + t) = 0.27$ $0.42 + t = \frac{0.27}{0.6} \quad \text{or} \quad 0.6t = 0.018$ $t = \underline{0.03}$	M1 M1 A1 A1 (4)
(c)	$[u =] \quad 1 - (0.6 + 0.15 + t)$ $u = \underline{0.22}$	M1 A1ft (2)
(d)(i)	$\left[\frac{P(D \cap R \cap B)}{P(R \cap B)} = \right] = \frac{0.27}{0.27 + 0.33} \quad \text{or} \quad P(D R \cap B) = P(D B) = P(D)$ $= \underline{0.45}$	M1 A1
(ii)	$\left[\frac{P(D \cap [R \cap B'])}{P(R \cap B')} = \right] = \frac{0.15}{0.15 + u}$ $= \frac{15}{37}$	M1 A1 (4)
(e)	$40 \times "0.45" \quad \text{and} \quad 37 \times \frac{15}{37}$ $= \underline{33}$	M1 A1 (2)
[13 marks]		

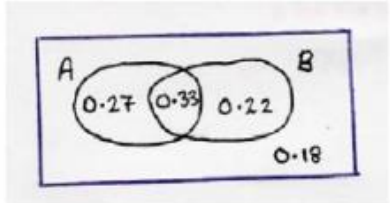
Notes	
(b)	<p>1st M1 for attempting 3 suitable probabilities, one involving t (at least 2 correct) e.g. sight of 0.6, 0.27, $0.42 + t$ correctly labelled in terms of B, D, R <u>or</u> in a correct equation. May see e.g. $P(B D) = \frac{0.27}{0.42 + t}$</p> <p>2nd M1 for using the independence to form a linear equation in t. ft their probs if stated. 1st A1 for solving leading to a correct equation as far as $p + t = q$ <u>or</u> $pt = q$ 2nd A1 for 0.03 or exact equivalent</p>
(c)	<p>M1 for a correct expression for u. Allow their t or just letter t in a correct expression A1ft for 0.22 (or exact equivalent) <u>or</u> ft their t, i.e. $u = 0.25 - t$ provided u & t are probs Can score M1A1ft provided their $u + t = 0.25$ where u and t are both in $[0, 1]$</p>
(d)(i)	<p>M1 for a correct numerical ratio of probabilities A1 for 0.45 or exact equivalent (Answer only 2/2)</p>
(ii)	<p>M1 for a correct numerical ratio of probabilities, ft their u, provided u is a probability A1 for $\frac{15}{37}$ or 0.405 <u>or</u> allow awrt 0.41 following a correct expression (Ans only 2/2)</p>
(e)	<p>M1 for a correct method for <u>both</u> 18 and 15 ft their 0.45 and their $\frac{15}{37}$ provided both in $[0, 1]$ NB $P(D) \times 77$ is M0 A1 for 33 only NB $\frac{27}{33} \times 40 = 32.7\dots$ which rounds to 33 but scores M0A0. (Ans only send to review)</p>

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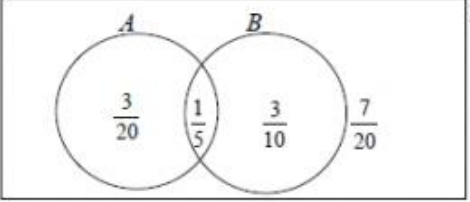
Q4.

Question Number	Scheme	Marks			
(a)	$[P(A) = 1 - 0.18 - 0.22] = 0.6$ (or exact equivalent)	B1 (1)			
(b)	$P(A \cup B) = "0.6" + 0.22 = 0.82$ (or exact equivalent)	B1ft (1)			
(c)	<table border="0" style="width: 100%;"> <tr> <td style="width: 30%; vertical-align: top;"> $x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$ </td> <td style="width: 30%; vertical-align: top;"> Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent) </td> <td style="width: 30%; vertical-align: top;"> Establish independence before or after 1st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$ </td> </tr> </table>	$x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$	Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent)	Establish independence before or after 1 st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$	M1 dM1 A1cso (3)
$x = P(A \cap B)$ $\frac{x}{x+0.22} = 0.6$ $x = 0.6x + 0.132$ $0.4x = 0.132$	Use $P(B)P(A' B) = P(A' \cap B)$ $P(B) \times [1 - 0.6] = 0.22$ Use $P(A \cap B) = P(A B)P(B)$ $P(A \cap B) = 0.6 \times 0.55$ $x = 0.33$ (or exact equivalent)	Establish independence before or after 1 st M1 and score marks for (d) (RH ver) Find $P(B)$ Use $P(B)P(A) = P(A \cap B)$ $P(A \cap B) = 0.6 \times 0.55$			
(d)	<table border="0" style="width: 100%;"> <tr> <td style="width: 40%; vertical-align: top;"> $P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6$ $= 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent </td> <td style="width: 60%; vertical-align: top;"> or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent </td> </tr> </table>	$P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6$ $= 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent	or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent	M1 A1cso (2)	
$P(B) = 0.55$ $P(B) \times P(A) = 0.55 \times 0.6$ $= 0.33$ $P(B) \times P(A) = P(A \cap B)$ therefore (statistically) independent	or stating $P(A) = P(A B) [= 0.6]$ or $P(A) = P(A B)$ therefore (statistically) independent				
Total 7					
Notes					
(b)	B1ft for their (a) + 0.22 or $1 - P(A' \cap B')$ Do not fit their (a) if it is > 0.78 NB 3 versions for (c). Check carefully that Ms are genuinely scored. Look out for <u>assuming independence</u> and if you see $P(B) = 0.55$ check it is <u>derived</u> properly				
(c)	1 st M1 for a correct equation for x e.g. $\frac{x}{x+0.22} = 0.6$ or a correctly derived equation for $P(B)$ 2 nd dM1 for solving to get in form $kx = L$ or <u>correct</u> use of $P(B)$ to find $P(A \cap B)$ [2 nd or 3 rd ver] or $P(A \cap B) = P(B) - 0.22$ A1cso for 0.33 Dep. on <u>both</u> Ms and no incorrect working seen.				
(d)	M1 for finding $P(B) \times P(A) = 0.33$ (values needed) or stating $P(A) = P(A B)$ (= 0.6 not needed) A1cso for a correct statement: $P(B) \times P(A) = P(A \cap B)$ or $P(A) = P(A B)$ <u>and</u> stating independent NB The M1 in (d) using $P(A \cap B)$ requires $P(B) = 0.55$ There is no fit of an incorrect $P(B)$ Full marks in (d) is OK even if 0/3 in (c)				
	{This Venn diagram may be helpful.}				





Q5.

Question Number	Scheme	Marks
(a)	$P(A \cap B) = P(A B) \times P(B)$ $P(A \cap B) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$	M1 A1 (2)
(b)	 <p>2 intersecting circles and 'P(A ∩ B)'</p>	B1ft B1 Box and $\frac{7}{20}$ B1 (3)
(c)	$\left[P(A) = \frac{3}{20} + \frac{1}{5} \right] = \frac{7}{20} \text{ or } 0.35$	B1ft (1)
(d)	$P(B A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}}$ $= \frac{4}{7}$	M1 A1 cao (2)
(e)	0.3	B1ft (1)
Notes		
(a)	M1 for $\frac{2}{5} \times \frac{1}{2}$ <u>or</u> a correct probability product expression and one correct prob. Ans only 2/2	
(b)	1 st B1 for 2 intersecting circles labelled A and B and ft their prob. for intersection Condone missing labels for 2 nd and 3 rd B marks	
(c)	B1ft for 0.35 (o.e.) if no Venn diagram <u>or</u> correct follow through from their diagram <u>or</u> allow 0.35 (or correct ft) from correct working e.g. $0.65 - 0.5 + (a)$ B0 for 0.35 if their diagram does not give 0.35 unless it comes from correct work Don't insist on $P(A) = \dots$ but do not award for $P(A' \cap B') = \frac{7}{20}$	
(d)	M1 for $\frac{\text{their (a)}}{\text{their (c)}}$ <u>or</u> a correct ratio of probabilities from their diagram NB incorrect use of $P(A' \cap B') = \frac{7}{20}$ scores M0 and num \geq denom scores M0 A1 for $\frac{4}{7}$ only	
(e)	B1ft for 0.3 <u>or</u> correct ft from their Venn diagram <u>or</u> ft from $\frac{11}{20} - \text{their (c)}$	
[Total 9]		



Q6.

Question	Scheme	Marks
(a)		B1 M1 A1 A1 B1 (5)
(b)	$\frac{13}{80}$ <u>or</u> 0.1625	B1ft (1)
(c)	$\frac{28+30-11}{80}$ <u>or</u> $\frac{2+3+4+8+13+17}{80}$ <u>or</u> $1 - \frac{(11+22)}{80} = \frac{47}{80}$ <u>or</u> 0.5875	M1 A1 (2)
(d)	$\frac{"17+8+13"}{"47"}$ <u>or</u> $\frac{"38"}{"47"}$ <u>or</u> $1 - \frac{"2+3+4"}{"47"} = \frac{38}{47}$ (condone awrt 0.809)	M1 A1cao (2)
(e)	$P(B C) = \frac{7}{28}$, $P(B) = \frac{20}{80}$ $P(C B) = \frac{7}{20}$, $P(C) = \frac{28}{80}$ $P(B \cap C) = \frac{7}{80}$, $P(B) = \frac{20}{80}$, $P(C) = \frac{28}{80}$ $P(B C) = P(B)$, $P(C B) = P(C)$ these may be implied by correct conclusion $P(B \cap C) = P(B) \times P(C)$ this approach requires the product to be seen So, they are independent.	M1 M1 A1 (3) (13 marks)

	Notes
(a)	B1 for 3 intersecting circles with 3 in the centre. Allow probs. or integers in diagram. M1 for some correct subtraction e.g. at least one of 2, 4, 8 <u>or</u> for B: 20 – their(2+3+4) etc A1 for 2, 4 and 8 (ignore labels) A1 for 11, 13 and 17 (must be in compatible regions with 2, 4, 8 if no labels) B1 for correct labels and 22 and box (Do not treat "blank" as 0 so can't use 0 for ft in (c))
(c)	M1 for a correct expression seen in (c) (<u>or</u> ft their diagram). Correct ans M1A1
(d)	M1 for denominator of 47 or ft their numerator from part (c) <u>and</u> numerator of 38 or their (17 + 8 + 13) or (their 47) – their (2 + 3 + 4). Correct ans M1A1
(e)	M1 for stating at least the required probs. & labelled for a correct test (can ft their diagram) M1 for <u>use</u> of a correct test with B and C Must see product attempted for $P(B \cap C)$ test. A1 for a correct test with all probabilities correct <u>and</u> a correct concluding statement. NB M0M1A0 should be possible but A1 requires both Ms



Q7.

Question	Scheme	Marks
(a)	B, W <u>or</u> T, W [accept $B \cup T, W$ <u>or</u> $B \cap T, W$] [Condone $P(B), P(W)$ etc] Since there is no <u>overlap</u> between the events <u>or</u> cannot happen together (o.e.) (Accept comment in context e.g. "no one walks and takes the train")	B1 B1 (2)
(b)	e.g. $P(B) = \frac{9}{25}, P(T) = \frac{8}{25}, P(B \cap T) = \frac{5}{25}$ $P(B \cap T) \neq P(B) \times P(T)$ [0.2 \neq 0.36 \times 0.32 = 0.1152 o.e.] So B and T are <u>not</u> independent	M1 M1 A1cso (3)
(c)	$[P(W) =] \frac{7}{25}$ <u>or</u> 0.28	B1 (1)
(d)	$[P(B \cap T) =] \frac{5}{25}$ <u>or</u> $\frac{1}{5}$ <u>or</u> 0.2	B1 (1)
(e)	$[P(T B) =] \frac{P(T \cap B)}{P(B)} = \frac{\text{"(d)"}}{(5+4)/25}$ $= \frac{5}{9}$ <u>or</u> 0.55	M1 A1 (2)
[9]		
Notes		
(a)	1 st B1 for a suitable pair. Do not accept universally exclusive pairs such as B and B' etc 2 nd B1 for any <u>correct</u> statement. Accept use of symbols e.g.: $B \cap W = \emptyset$ <u>or</u> $P(T \cap W) = 0$ etc But $T \cap W = 0$ is B0 (since it is not a correct statement)	
(b)	1 st M1 for an attempt at all required probabilities with labels for a suitable test (allow one error). Accept use of A and B as long as they can be identified as B and T by correct probabilities Must be probabilities not integers such as 5, 9, 8 etc for both these M marks 2 nd M1 for $P(B) \times P(T)$ evaluated (correct for <u>their</u> probabilities) <u>or</u> $P(B \cap T) \neq P(B) \times P(T)$ stated or implied in symbols or using their probabilities. <u>or</u> $P(B T) \neq P(B)$ <u>or</u> $P(T B) \neq P(T)$ stated or implied in symbols or using their probabilities. A1 for a conclusion of <u>not</u> independent. Requires all probabilities used to be correct and seen. This A mark is dependent on both Ms	
	NB $P(B T) = \frac{5}{8}$ & $P(B) = \frac{9}{25}$ <u>or</u> $P(T B) = \frac{5}{9}$ & $P(T) = \frac{8}{25}$ seen, followed by a correct conclusion scores 3/3	
(e)	M1 for a correct ratio of probabilities e.g. $\frac{5/25}{(5+4)/25}$ <u>or</u> $\frac{5}{5+4}$ <u>or</u> A correct ratio expression and at least one correct (or correct f.t.) probability substituted. A1 for $\frac{5}{9}$ with no incorrect working seen but $\frac{5}{9}$ following from $P(B T)$ is 0/2. $\frac{5}{9}$ alone is 2/2	

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Q8.

Question Number	Scheme	Marks
(a)	(R and S are mutually) exclusive.	B1 (1)
(b)	$\frac{2}{3} = \frac{1}{4} + P(B) - P(A \cap B)$ <p style="text-align: right;">use of Addition Rule</p> $\frac{2}{3} = \frac{1}{4} + P(B) - \frac{1}{4} \times P(B)$ <p style="text-align: right;">use of independence</p> $\frac{5}{12} = \frac{3}{4} P(B)$ $P(B) = \frac{5}{9}$	M1 M1 A1 A1 (4)
(c)	$P(A' \cap B) = \frac{3}{4} \times \frac{5}{9} = \frac{15}{36} = \frac{5}{12}$	M1A1ft (2)
(d)	$P(B A) = \frac{(1 - (b)) \times 0.25}{0.25} \quad \text{or } P(B') \text{ or } \frac{1}{4}$ $= \frac{4}{9}$	M1 A1 (2)
		Total 9



Q9.

Question Number	Scheme	Marks
(a)	$P(J \cup K) = 1 - 0.7$ or $0.1 + 0.15 + 0.05 = \underline{0.3}$	B1 (1)
(b)	$P(K) = 0.05 + 0.15$ or “0.3” $- 0.25 + 0.15$ or “0.3” $= 0.25 + P(K) - 0.15$ May be seen on Venn diagram $= \underline{0.2}$	M1 A1 (2)
(c)	$[P(K J)] = \frac{P(K \cap J)}{P(J)}$ $= \frac{0.15}{0.25}$ $= \underline{\frac{3}{5}}$ or $\underline{0.6}$	M1 A1 A1 (3)
(d)	$P(J) \times P(K) = 0.25 \times 0.2 (= 0.05)$, $P(J \cap K) = 0.15$ or $P(K J) = 0.6$, $P(K) = 0.2$ or may see $P(J K) = 0.75$ and $P(J) = 0.25$ not equal therefore not independent	M1 A1ft (2)
(e)	Not independent so confirms the teacher’s suspicion or they are linked (This requires a statement about independence in (d) or in (e))	B1ft (1)
(9 marks)		
Notes		
(b)	M1 for a complete method, follow through their 0.3, leading to a linear equation for $P(K)$ NB You may see this Venn diagram. A correct diagram (Venn or table) implies M1 in (b) Need not include box or 0.7 Correct answer only is 2/2	
(c)	In parts (c) and (d) they must have defined A and B M1 for a correct expression (including ratio) in symbols. 1 st A1 for a correct ratio of probabilities (if this is seen the M1 is awarded by implication) Must be in (c). Condone no LHS but wrong LHS (e.g. $P(K)$ or $P(J K)$) is M0A0 2 nd A1 for correct answer as printed only. Correct answer only 3/3	
(d)	Mark (d) and (e) together M1 for a correct comparison of known probabilities for an independence test - ft their values. E.g. $P(J) \times P(K)$ with $P(J \cap K)$ or $P(K J)$ with $P(K)$ [Must have expressions] The values of these probabilities should be given unless they are in the question or stated elsewhere. A1ft for correct calculations and correct comment for their probabilities	
(e)	B1ft ft their conclusion on independence so not independent confirms teacher...independent contradicts teacher. Methods leading to negative probabilities should score M0	

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