Vectors in 3d Mark Scheme (Sheet 2)

Q1.

Question	Scheme	Marks	AOs
(a)	Attempts $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB}=5\mathbf{i}+10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB\rvert=\sqrt{(5)^2+(10)^2}$	M1	1.1b
	$\lvert AB\rvert=5\sqrt{5}$	A1ft	1.1b
		(2)	
			(4 marks)

Notes

(a) M1: Attempts subtraction but may omit brackets
A1: cao (allow column vector notation)

(b) M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)
A1ft: $\lvert AB\rvert=5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Q2.

Question	Scheme	Marks	AOs
	$\overrightarrow{OA}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $\overrightarrow{OB}=4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $\overrightarrow{OC}=a\mathbf{i}+5\mathbf{j}-2\mathbf{k}$, $a<0$ $\overrightarrow{AB}=\overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert=4$		
(a)	E.g. $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}$ or $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA}=2\overrightarrow{OB}-\overrightarrow{OA}$ or $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AB}$		
	$=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}$ $\left\{=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ or $=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)$ $\left\{=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1	3.1a
	$=\begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1	1.1b
	$\{\lvert\overrightarrow{AC}\rvert=4\Rightarrow\}(a-2)^2+(5-3)^2+(-2--4)^2=(4)^2$ $\Rightarrow(a-2)^2=8\Rightarrow a=\ldots$ or $\Rightarrow a^2-4a-4=0\Rightarrow a=\ldots$	dM1	2.1
	(as $a<0\Rightarrow$) $a=2-2\sqrt{2}$ (or $a=2-\sqrt{8}$)	A1	1.1b
		(3)	
			(5 marks)

Q3.

Question	Scheme	Marks	AOs
	Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=2\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mathbf{i}-9\mathbf{j}+3\mathbf{k}=3\mathbf{i}-6\mathbf{j}+4\mathbf{k}$	M1	3.1a
	Attempts to find any one length using 3-d Pythagoras	M1	2.1
	Finds all of $\lvert AB\rvert=\sqrt{14},\quad \lvert AC\rvert=\sqrt{61},\quad \lvert BC\rvert=\sqrt{91}$	A1ft	1.1b
	$\cos BAC=\dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$	M1	2.1
	angle $BAC=105.9°$ *	A1*	1.1b
		(5)	
			(5 marks)

Notes:

M1: Attempts to find $\overrightarrow{AC}$ by using $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

M1: Attempts to find any one length by use of Pythagoras' Theorem

A1ft: Finds all three lengths in the triangle. Follow through on their $\lvert AC\rvert$

M1: Attempts to find BAC using $\cos BAC=\dfrac{\lvert AB\rvert^2+\lvert AC\rvert^2-\lvert BC\rvert^2}{2\lvert AB\rvert\lvert AC\rvert}$

Allow this to be scored for other methods such as $\cos BAC=\dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert AB\rvert\lvert AC\rvert}$

A1*: This is a show that and all aspects must be correct. Angle $BAC=105.9°$

Q4.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AB}=(3\mathbf{i}-3\mathbf{j}-4\mathbf{k})-(2\mathbf{i}+5\mathbf{j}-6\mathbf{k})$	M1	1.1b
	$=\mathbf{i}-8\mathbf{j}+2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC}=2\times\overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB, so $OABC$ is a trapezium.	A1	2.4
		(2)	
			(4 marks)
Notes:			

Q5.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k} + \mathbf{i} + \mathbf{j} + 4\mathbf{k} = \ldots$	M1	1.1b
	$= -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $(AC^2) = "2^2 + 3^2 + 1^2", (AB^2) = 3^2 + 4^2 + 5^2, (BC^2) = 1^2 + 1^2 + 4^2$	M1	1.1b
	$2^2 + 3^2 + 1^2 = 3^2 + 4^2 + 5^2 + 1^2 + 1^2 + 4^2 - 2\sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2}\cos ABC$	M1	3.1a
	$14 = 50 + 18 - 2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC = \frac{50 + 18 - 14}{2\sqrt{50}\sqrt{18}} = \frac{9}{10}*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2 = 3^2 + 4^2 + 5^2,\ BC^2 = 1^2 + 1^2 + 4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC} = (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \bullet (\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = 27 = \sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2}\cos ABC$	M1	3.1a
	$27 = \sqrt{50}\sqrt{18}\cos ABC \Rightarrow \cos ABC = \frac{27}{\sqrt{50}\sqrt{18}} = \frac{9}{10}*$	A1*	2.1
			(5 marks)
	Notes		