Trapezium Rule Mark Scheme (Edexcel)

Q1.

Question Number	Scheme		Marks
(a)	$\sqrt{7}$ and $\sqrt{15}$	Both $\sqrt{7}$ and $\sqrt{15}$. Allow awrt 2.65 and 3.87	B1
			[1]
(b)	$\text{Area}(R) \approx \frac{1}{2}\times 2;\times\left\{\sqrt{3}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)+\sqrt{19}\right\}$	Outside brackets $\frac{1}{2}\times 2$ or 1 (may be implied)	B1;
		For structure of {..............}	M1
	Note decimal values are $\frac{1}{2}\times 2;\times\left\{\sqrt{3}+\sqrt{19}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)\right\}=\frac{1}{2}\times 2;\times\left\{6.0909..+19.6707...\right\}$		
	M1 requires the correct structure for the *y* values. It needs to contain first *y* value **plus** last *y* value and the second bracket to be multiplied by 2 and to be the summation of the remaining *y* values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2(…..) bracket this may be regarded as a slip and the M mark can be allowed (nb: an extra repeated term, however, forfeits the M mark). M0 if any values used are *x* values instead of *y* values. Bracketing mistakes: e.g. $\left(\frac{1}{2}\times 2\right)\times\left(\sqrt{3}+\sqrt{19}\right)+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)$ $\left(\frac{1}{2}\times 2\right)\times\sqrt{3}+\sqrt{19}+2\left(\sqrt{7}+\sqrt{11}+\sqrt{15}\right)$ Both score B1 M1 **Alternative:** Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{2}\times 2(\sqrt{3}+\sqrt{7})+\frac{1}{2}\times 2(\sqrt{7}+\sqrt{11})+\frac{1}{2}\times 2(\sqrt{11}+\sqrt{15})+\frac{1}{2}\times 2(\sqrt{15}+\sqrt{19})\right]$ B1 for $\frac{1}{2}\times 2$, M1 for correct structure		
	$=1\times 25.76166865... = 25.76166... = \underline{25.76}$ (2dp)	25.76	A1 cao
			[3]
(c)	underestimate	Accept 'under', 'less than' etc.	B1
			[1]
			Total 5

Q2.

Question	Scheme	Marks	AOs
(a)	States or uses $h = 1.5$	B1	1.1a
	Full attempt at the trapezium rule $= \frac{...}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$	M1	1.1b
	$=$ awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt}133$ or e.g. $\frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x\,dx = \int_3^9 \log_3(9\times 2x)\,dx = \int_3^9 2 + \log_3 2x\,dx$ $= [2x]_3^9 + \int_3^9 \log_3 2x\,dx = 18 - 6 + \int_3^9 \log_3 2x\,dx = ...$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

B1: States or uses $h = 1.5$

M1: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2}\{1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)\}$ but condone copying slips

Note that $\frac{\text{their } h}{2}1.63 + 2.63 + 2\times(2 + 2.26 + 2.46)$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$$\frac{\text{their } h}{2}\{1.63+2\} + \frac{\text{their } h}{2}\{2+2.26\} + \frac{\text{their } h}{2}\{2.26+2.46\} + \frac{\text{their } h}{2}\{2.46+2.63\}$$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3

Use of $h = -1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

B1ft: Deduces that $\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. 10 × "13.3" = "133"

Note that $\int_3^9 \log_3(2x)^{10}\,dx = 133.2414316...$ so a correct method must be seen to award marks.

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the <u>answer</u> to part (a).

(b)(ii)

M1: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $\left[2x\right]_3^9 + \int_3^9 \log_3 2x \, dx = \ldots$ with correct use of limits on $\left[2x\right]_3^9$ which may be implied or equivalent work e.g. finds the area of the rectangle as 2×6

A1ft: Correct working followed by awrt 25.3 but ft on their 13.3 so allow for 12 + their answer to part (a) following correct work as shown.

Note that $\int_3^9 \log_3 18x \, dx = 25.32414\ldots$ so a correct method must be seen to award marks.

Some examples of an acceptable method are:

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times 2x) \, dx = \int_3^9 2+\log_3 2x \, dx = 6\times 2 + \text{"}13.3\text{"} = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times 2x) \, dx = \int_3^9 2+\log_3 2x \, dx = 12 + \text{"}13.3\text{"} = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times 2x) \, dx = \int_3^9 2+\log_3 2x \, dx = \left[2x\right]_3^9 + \int_3^9 \log_3 2x \, dx = 25.3$$

BUT just $12 + \text{"}13.3\text{"} = 25.3$ scores M0

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).

Q3.

Question Number	Scheme	Marks
Q (a)	$x = 2$ gives 2.236 (allow AWRT) Accept $\sqrt{5}$	B1
	$x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 (2)
(b)	$\left(\frac{1}{2} \times \frac{1}{2}\right)$, $[(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)]$	B1,[M1A1ft]
	$= 6.133$ (AWRT 6.13, even following minor slips)	A1 (4)
(c)	Overestimate	B1
	'Since the trapezia lie above the curve', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	dB1 (2) [8]

(b)	B1 for $\frac{1}{2} \times \frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to omit one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\left(\frac{1}{2} \times \frac{1}{2}\right)(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). Alternative: Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.414 + 1.554) + \frac{1}{4}(1.554 + 1.732) + \ldots\ldots\ldots\ldots + \frac{1}{4}(2.580 + 3)\right]$ 1st A1ft for correct expression, ft their 2.236 and their 2.580
(c)	1st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2nd B1 is dependent upon the 1st B1 (overestimate).

Q4.

Question Number	Scheme	Marks
	x: 0, 0.5, 1, 1.5, 2; y: 1, 2.821, **6**, 12.502, 26.585	
(a)	$\{\text{At } x=1,\}\ y=6$ (allow 6.000 or even 6.00)	B1 cao
		(1)
(b)	$\frac{1}{2}\times 0.5$;	B1 oe
	$\underline{\{1+26.585+2(2.821+\text{their } 6+12.502)\}}$ — <u>For structure of {...............}</u>;	M1<u>A1ft</u>
	$\frac{1}{2}\times 0.5\,\underline{\{1+26.585+2(2.821+6+12.502)\}}\ \{=\frac{1}{4}(70.231)=17.557..\}=$ awrt 17.56	A1
		(4)
(c)	$10+$"17.56" $=$"27.56"	B1ft
		(1)
		[6]

x	y	
0	1	

	Notes
(a)	B1: 6
(b)	B1: for using $\frac{1}{2}\times 0.5$ or $\frac{1}{4}$ or equivalent.
	M1: requires the correct {......} bracket structure. It needs the first bracket to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values
	A1ft: for the correct bracket {......} following through candidate's y value found in part (a).
	A1: for answer which rounds to 17.56
	NB: Separate trapezia may be used: B1 for 0.25, M1 for $1/2\,h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) Then A1 as before.
	Special case: Bracketing mistake $0.25\times(1+26.585)+2(2.821+\text{their } 6+12.502)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 49.542 usually indicates this error.
(c)	B1ft: 10 + their answer to part (b)
	(May be obtained by using the trapezium rule again with all values for y increased by 5)

Q5.

Question	Scheme	Marks	AOs
(a)	$h = 0.2$	B1	1.1b
	$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$	M1	1.1b
	e.g. $\Rightarrow a+13.5+2b+111.4=175.9 \Rightarrow a+2b=51$*	A1*	2.1
		(3)	
(b)	$a+16.8+b+20.2+18.7+13.5=97.2 \Rightarrow a+b=28 \Rightarrow a=...$ (or $b=...$)	M1	3.1a
	$a=5$ or $b=23$	A1	1.1b
	$a=5$ and $b=23$	A1	1.1b
		(3)	
		(6 marks)	

Notes

(a)

B1: States or uses $h = 0.2$ o.e.

M1: Forms the equation $\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)\}=17.59$ o.e. but condone copying slips. They may have added some of the y values together so as a minimum accept e.g. $"0.1"\times\{a+13.5+2(55.7+b)\}=17.59$
Condone invisible brackets as long as they are recovered or implied in further work before achieving the given answer. Condone the use of $\approx$ for this mark.
Allow this mark if they add the areas of individual trapezia e.g.
$\frac{\text{their } h}{2}\{a+16.8\}+\frac{\text{their } h}{2}\{16.8+b\}+\frac{\text{their } h}{2}\{b+20.2\}+\frac{\text{their } h}{2}\{20.2+18.7\}+\frac{\text{their } h}{2}\{18.7+13.5\}$
Condone copying slips but it must be a complete method using all the trapezia. h must be numerical but condone $h = 1$

A1*: A rigorous argument leading to $a+2b=51$ from correct working and no errors seen including brackets, although do not penalise a missing trailing bracket at the end e.g.
$\frac{1}{2}\times"0.2"\times\{a+13.5+2(16.8+b+20.2+18.7)=17.59 \Rightarrow ... \Rightarrow a+2b=51$ could score B1M1A1but
$\frac{1}{2}\times"0.2"\times a+13.5+2(16.8+b+20.2+18.7)=17.59 \Rightarrow ... \Rightarrow a+2b=51$ could score max B1M1A0
provided later work implied correct brackets.
Both sets of brackets must be dealt with correctly before proceeding to the final answer such that
e.g. $... \Rightarrow a+2b+124.9=175.9 \Rightarrow a+2b=51$ is M1A1*
$... \Rightarrow a+13.5+33.6+2b+40.4+37.4=175.9 \Rightarrow a+2b=51$ is M1A1*
$... \Rightarrow 0.1a+1.35+3.36+0.2b+4.04+3.74=17.59 \Rightarrow a+2b=51$ is M1A1*
Note that $a+2b\approx 51$ as the **final** answer is A0*

(b)

M1: Attempts to form the equation $a+16.8+b+20.2+18.7+13.5=97.2$, condoning copying errors, (may just be stated as e.g. $a+b=28$ o.e.) and attempts to solve their equation simultaneously with the given equation (or condone their equation from part (a)). Do not be too concerned with the process here as calculators may be used. Score if values for a or b are reached from a pair of simultaneous equations.

A1: for $a=5$ or $b=23$

A1: for both $a=5$ and $b=23$

Q6.

Question	Scheme	Marks	AOs
(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	3 × their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3 × 1.5
	
	If (a) is incorrect allow 3 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	

Question	Scheme	Marks	AOs
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as $4.535 \approx 4.50$ • Very accurate as 4.535 to 2 sf is 4.5 • $4.51425 < 4.535$ so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • $4.535 - 4.5 = 0.035$ so not far out But not just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert \times 100 = 0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert \times 100 = 0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert \times 100 = 0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert \times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
			(5 marks)

Notes:

(a)

B1: States or uses $h = 0.5$. May be implied by $\frac{1}{4} \times \{...$ below.

M1: Correct attempt at the trapezium rule.

Look for $\frac{1}{2}h \times \{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$ condoning slips on the terms but must use all y values with no repeats.

There must be a clear attempt at $\frac{1}{2}h \times$(first y + last y + $2\times$"sum of the rest")

Give M0 for $\frac{1}{2} \times \frac{1}{2} 0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)$ unless the missing brackets are implied.

NB this incorrect method gives 5.85…

May be awarded for separate trapezia e.g.

$$\frac{1}{4}(0.5774 + 0.7071) + \frac{1}{4}(0.7071 + 0.7746) + \frac{1}{4}(0.7746 + 0.8165) + \frac{1}{4}(0.8165 + 0.8452)$$

May be awarded for using the function e.g. $\frac{1}{2}h \times \left\{\sqrt{\frac{0.5}{1+0.5}} + \sqrt{\frac{2.5}{1+2.5}} + 2\left(\sqrt{\frac{1}{1+1}} + \sqrt{\frac{1.5}{1+1.5}} + \sqrt{\frac{2}{1+2}}\right)\right\}$

A1: Awrt 1.50 (Apply isw if necessary)

Correct answers with no working – send to review

(b)

B1ft: See main scheme. Must be considering $3 \times$ (a) and not e.g. attempting trapezium rule again.

(c)

B1: See scheme

Q7.

Question	Scheme	Marks	AOs
(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\}=4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$=\frac{x^3}{3}\ln x - \int \frac{x^2}{3} dx$	A1	1.1b
	$\int -2x+5 \, dx = -x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S = \int_1^3 \frac{x^2 \ln x}{3} - 2x+5 \, dx = \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2+5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S = \frac{28}{27} + \ln 27 \quad (a=28, b=27, c=27)$	A1	1.1b
		(6)	
			(10 marks)

Notes:	
(a)	
B1:	States or uses the strip width $h = 0.5$. This can be implied by the sight of $\frac{0.5}{2}\{...\}$ in the trapezium rule
M1:	For the correct form of the bracket in the trapezium rule. Must be y values rather than x values $\{\text{first } y \text{ value} + \text{last } y \text{ value} + 2\times(\text{sum of other } y \text{ values})\}$
A1:	4.393
(b)	
B1:	See scheme
(c)	
M1:	Uses integration by parts the right way around. Look for $\int x^2 \ln x \, dx = Ax^3 \ln x - \int Bx^2 \, dx$
A1:	$\frac{x^3}{3}\ln x - \int \frac{x^2}{3} \, dx$
B1:	Integrates the $-2x+5$ term correctly $= -x^2 + 5x$
M1:	All integration completed and limits used
M1:	Simplifies using ln law(s) to a form $\frac{a}{b} + \ln c$
A1:	Correct answer only $\frac{28}{27} + \ln 27$

Q8.

Question Number	Scheme		Marks
(a)	$6.248046798... = 6.248$ (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2} \times 2 \,; \times \left[3 + 2(7.107 + 7.218 + \text{their } 6.248) + 5.223\right]$		B1; M1
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int (4te^{-\frac{1}{3}t} + 3)\, dt\right\} = -12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}\, \{dt\}$	$\pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0, B \neq 0$	M1
		See notes.	A1
	$+ 3t$	$3 \to 3t$	B1
	$= -12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} \{+ 3t\}$	$-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$	A1
	$\left[-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)e^{-\frac{1}{3}(8)} - 36e^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)e^{-\frac{1}{3}(0)} - 36e^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96e^{-\frac{8}{3}} - 36e^{-\frac{8}{3}} + 24\right) - (0 - 36 + 0)$		
	$= 60 - 132e^{-\frac{8}{3}}$	$60 - 132e^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $= \left\|60 - 132e^{-\frac{8}{3}} - 49.37\right\| = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1
			[1]
			11

Notes for Question

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2} \times 2$ or 1
	M1: For structure of trapezium rule[.............]. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37)
	Note: Working must be seen to demonstrate the use of the trapezium rule. Note: actual area is 50.828…
	Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $1 + 3 + 2(7.107 + 7.218 + \text{their } 6.248) + 5.223$ (nb: answer of 50.369).

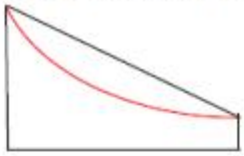
Notes for Question Continued	
(b) ctd	*Alternative method for part (b): Adding individual trapezia*
	Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]=49.369$
	B1: 2 and a divisor of 2 on all terms inside brackets.
	M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1: anything that rounds to 49.37
(c)	**M1:** For $4te^{-\frac{1}{3}t} \to \pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0, B \neq 0$
	A1: For $te^{-\frac{1}{3}t} \to \left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark).
	or $4te^{-\frac{1}{3}t} \to 4\left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ or $-12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}$ or $12\left(-te^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t}\right)$
	These results can be implied. They can be simplified or un-simplified.
	B1: $3 \to 3t$ or $3 \to 3x$ (bod).
	Note: Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4.
	Be careful some candidates will factorise out 4 and have $4\left(\ldots + \frac{3}{4}\right) \to 4\left(\ldots + \frac{3}{4}t\right)$
	which would then be fine for B1.
	Note: Allow B1 for $\int_0^8 3\,dt = 24$
	A1: For correct integration of $4te^{-\frac{1}{3}t}$ to give $-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$ or $4\left(-3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t}\right)$ or equivalent.
	This can be simplified or un-simplified.
	dM1: Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.
	Note: Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0.
	A1: An exact answer of $60 - 132e^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0.
	Note: A decimal answer of 50.82818444... without a correct exact answer is A0.
	Note: If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0.
	IMPORTANT: that is fine for candidates to work in terms of x rather than t in part (c).
	Note: The "$3t$" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46.
	Candidates may give correct decimal answers of 1.458184439... or 1.459184439...
	Note: You can award this mark whether or not the candidate has answered part (c) correctly.

Q9.

Question Number	Scheme		Marks
	x: 1, 2, 3, 4; y: 1.42857, 0.90326, *0.682116...*, 0.55556	$y = \frac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp)	0.68212	B1 cao
			[1]
(b)	$\frac{1}{2} \times 1 \times \left[1.42857 + 0.55556 + 2(0.90326 + \text{their } 0.68212)\right]$	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$	B1 aef
		For structure of [..............]	M1
	$\left\{= \frac{1}{2}(5.15489)\right\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	• Overestimate and a reason such as • {top of} trapezia lie above the curve • a diagram which gives reference to the extra area • concave or convex • $\frac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards		B1
			[1]
(d)	$\{u = \sqrt{x} \Rightarrow\}\ \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du} = 2u$		B1
	$\int \frac{10}{2u^2 + 5u} \cdot 2u\, du$	Either $\left\{\int\right\} \frac{\pm ku}{\alpha u^2 \pm \beta u} \{du\}$ or $\left\{\int\right\} \frac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\}$	M1
	$\left\{= \int \frac{20}{2u + 5}\, du\right\} = \frac{20}{2}\ln(2u + 5)$	$\pm\lambda\ln(2u + 5)$ or $\pm\lambda\ln\left(u + \frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**	M1
		$\frac{20}{2u + 5} \to \frac{20}{2}\ln(2u + 5)$ or $10\ln\left(u + \frac{5}{2}\right)$	A1 cso
	$\left\{\left[\frac{20}{2}\ln(2u + 5)\right]_1^2\right\} = 10\ln(2(2) + 5) - 10\ln(2(1) + 5)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$		A1 oe cso
			[6]
			11

Question Notes

(a)	B1	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	B1	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent.
	M1	For structure of trapezium rule[............]
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

(b) contd	Note	Award B1M1A1 for $\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445$
		<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly
		award B1M0A0 for $\frac{1}{2}\times 1 + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556$ (nb: answer of 5.65489).
		award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)$ (nb: answer of 4.162825).
		<u>Alternative method: Adding individual trapezia</u>
		Area $\approx 1\times\left[\frac{1.42857+0.90326}{2} + \frac{0.90326+\text{"}0.68212\text{"}}{2} + \frac{\text{"}0.68212\text{"}+0.55556}{2}\right] = 2.577445$
	B1	**B1:** 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1:** anything that rounds to 2.5774
(c)	B1	Overestimate **and** either trapezia lie above curve or a diagram that gives reference to the extra area
		eg. This diagram is sufficient. It must show the top of a trapezium lying above the curve. 
		or concave or convex or $\frac{d^2y}{dx^2} > 0$ (can be implied) or bends inwards or curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.
(d)	B1	$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $du = \frac{1}{2\sqrt{x}}\,dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2u\,du$ or $\frac{dx}{du} = 2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$,
		$k, \alpha, \beta \neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**
	A1	cso. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent.
		So $\int\frac{10}{2u+5}\,du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$
		or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283… (without a correct **exact** answer) is A0.

Q10.

Question Number	Scheme	Notes	Marks
	x: 0, 0.2, 0.4, 0.6, 0.8, 1; y: 2, **1.8625426...**, 1.71830, 1.56981, 1.41994, 1.27165	$y = \dfrac{6}{(2+e^x)}$	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	1.86254	B1 cao
	Note: Look for this value on the given table or in their working.		[1]
(b)	$\frac{1}{2}(0.2)\left[2+1.27165+2(\text{their } 1.86254+1.71830+1.56981+1.41994)\right]$	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$ or $\frac{1}{2}\times\frac{1}{5}$	B1 o.e.
		For structure of $[\ldots\ldots]$	M1
	$\left\{=\frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	anything that rounds to 1.6413	A1
			[3]
(c)	$\{u = e^x \text{ or } x = \ln u \Rightarrow\}$		
	$\frac{du}{dx} = e^x$ or $\frac{du}{dx} = u$ or $\frac{dx}{du} = \frac{1}{u}$ or $du = u\,dx$ etc., and $\int \frac{6}{(e^x+2)}dx = \int \frac{6}{(u+2)u}du$	See notes	B1 *
	$\{x=0\} \Rightarrow a = e^0 \Rightarrow \underline{a=1}$; $\{x=1\} \Rightarrow b = e^1 \Rightarrow \underline{b=e}$	$a=1$ and $b=e$ or $b=e^1$ or evidence of $0\to1$ and $1\to e$	B1
	NOTE: 1st B1 mark CANNOT be recovered for work in part (d) **NOTE: 2nd B1 mark CAN be recovered for work in part (d)**		[2]

Question Number	Scheme	Notes	Marks
(d) **Way 1**	$\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$ $\Rightarrow 6 \equiv A(u+2) + Bu$	Writing $\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$, o.e. or $\frac{1}{u(u+2)} \equiv \frac{P}{u} + \frac{Q}{(u+2)}$, o.e., and a complete method for finding the value of at least one of **their** A or **their** B (or **their** P or **their** Q)	M1
	$u = 0 \Rightarrow A = 3$; $u = -2 \Rightarrow B = -3$	Both **their** $A = 3$ **and their** $B = -3$. (Or **their** $P = \frac{1}{2}$ **and their** $Q = -\frac{1}{2}$ with the factor of 6 in front of the integral sign)	A1
	$\int \frac{6}{u(u+2)}du = \int \left(\frac{3}{u} - \frac{3}{(u+2)}\right)du$	Integrates $\frac{M}{u} \pm \frac{N}{u \pm k}$, $M, N, k \neq 0$; (i.e. ***a two term partial fraction***) to obtain either $\pm\lambda\ln(\alpha u)$ or $\pm\mu\ln(\beta(u\pm k))$; $\lambda, \mu, \alpha, \beta \neq 0$	M1
	$= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	Integration of both terms is **correctly followed through** from their M and from their N.	A1 ft
	$\left\{\text{So } \left[3\ln u - 3\ln(u+2)\right]_1^e\right\}$ $= \left(3\ln(e) - 3\ln(e+2)\right) - \left(3\ln 1 - 3\ln 3\right)$ [Note: A proper consideration of the limit of $u=1$ is required for this mark]	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq1, a>0$) in u or applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3 - 3\ln(e+2) + 3\ln 3$ or $3(1-\ln(e+2)+\ln 3)$ or $3 + 3\ln\left(\frac{3}{e+2}\right)$ or $3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$ or $3 - 3\ln\left(\frac{e+2}{3}\right)$ or $3\ln\left(\frac{3e}{e+2}\right)$ or $\ln\left(\frac{27e^3}{(e+2)^3}\right)$	see notes	A1 cso
	Note: Allow e^1 in place of e for the final A1 mark.		[6]
	Note: Give final A0 for $3 - 3\ln e + 2 + 3\ln 3$ (i.e. bracketing error) unless recovered.		12
	Note: Give final A0 for $3 - 3\ln(e+2) + 3\ln 3 - 3\ln 1$, where $3\ln 1$ has not been simplified to 0		
	Note: Give final A0 for $3\ln e - 3\ln(e+2) + 3\ln 3$, where $3\ln e$ has not been simplified to 3		

	Question Notes	
(b)	**Note**	**M1:** Do not allow an extra y-value ***or*** a repeated y value in their [...] Do not allow an omission of a y-ordinate in their [...] for M1 **unless** they give the correct answer of awrt 1.6413, in which case both M1 and A1 can be scored.
	Note	**A1:** Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.64150274...)
	Note	Full marks can be gained in part (b) for awrt 1.6413 even if B0 is given in part (a)
	Note	Award B1M1A1 for $\frac{1}{10}(2+1.27165)+\frac{1}{5}(\text{their } 1.86254+1.71830+1.56981+1.41994)=\text{awrt } 1.6413$
	<u>Bracketing mistakes:</u> Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2)+2+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165$ $(=16.51283)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2+1.27165)+2(\text{their } 1.86254+1.71830+1.56981+1.41994)$ $(=13.468345)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2)+2(\text{their } 1.86254+1.71830+1.56981+1.41994)+1.27165$ $(=14.61283)$	
	<u>Alternative method: Adding individual trapezia</u> $\text{Area} \approx 0.2\times\left[\frac{2+\text{"}1.86254\text{"}}{2}+\frac{\text{"}1.86254\text{"}+1.71830}{2}+\frac{1.71830+1.56981}{2}+\frac{1.56981+1.41994}{2}+\frac{1.41994+1.27165}{2}\right]$ $=1.641283$ **B1** 0.2 and a divisor of 2 on all terms inside brackets **M1** First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 **A1** anything that rounds to 1.6413	

(c)	**1st B1**	Must start from either • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(\mathrm{e}^x+2)}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ and state either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$ and end at $\int \frac{6}{u(u+2)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working.**
	Note	So, just writing $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **and** $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x=\int \frac{6}{u(u+2)}\,\mathrm{d}u$ is sufficient for 1st B1
	Note	Give 2nd B0 for $b=2.718...$, without reference to $a=1$ and $b=\mathrm{e}$ or $b=\mathrm{e}^1$
	Note	You can also give the 1st B1 mark for using a reverse process. i.e. Proceeding from $\int \frac{6}{u(u+2)}\,\mathrm{d}u$ to $\int \frac{6}{(\mathrm{e}^x+2)}\,\mathrm{d}x$, **with no incorrect working,** and stating either $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$
(d)	**Note**	Give final A0 for $3-3\ln(\mathrm{e}+2)+3\ln 3$ simplifying to $1-\ln(\mathrm{e}+2)+\ln 3$ (i.e. dividing their correct final answer by 3) Otherwise, you can ignore incorrect working (isw) following on from a correct exact value.
	Note	A decimal answer of 1.641502724... (without a correct **exact** answer) is final A0
	Note	$\left[-3\ln(u+2)+3\ln u\right]_1^{\mathrm{e}}$ followed by awrt 1.64 (without a correct **exact** answer) is final M1A0

Question	Notes Continued	
(d)	Note	**BE CAREFUL! Candidates will assign their own "A" and "B" for this question.**
	Note	*Writing down* $\dfrac{6}{(u+2)u}$ in the form $\dfrac{A}{(u+2)}+\dfrac{B}{u}$ with at least one of A or B correct is 1st M1
	Note	*Writing down* $\dfrac{6}{(u+2)u}$ as $\dfrac{-3}{(u+2)}+\dfrac{3}{u}$ is 1st M1 1st A1.
	Note	Condone $\displaystyle\int\left(\frac{3}{u}-\frac{3}{(u+2)}\right)\mathrm{d}u$ to give $3\ln u-3\ln u+2$ (poor bracketing) for 2nd A1.
	Note	**Award M0A0M1A1ft for a candidate who writes down** e.g. $\displaystyle\int\frac{6}{u(u+2)}\mathrm{d}u=\int\left(\frac{6}{u}+\frac{6}{(u+2)}\right)\mathrm{d}u=6\ln u+6\ln(u+2)$ **AS EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ **AS PARTIAL FRACTIONS.**
	Note	**Award M0A0M0A0 for a candidate who writes down** $\displaystyle\int\frac{6}{u(u+2)}\mathrm{d}u=6\ln u+6\ln(u+2)$ **or** $\displaystyle\int\frac{6}{u(u+2)}\mathrm{d}u=\ln u+6\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	**Award M1A1M1A1 for a candidate who writes down** $\displaystyle\int\frac{6}{u(u+2)}\mathrm{d}u=3\ln u-3\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	If they lose the "6" and find $\displaystyle\int_1^{e}\frac{1}{u(u+2)}\mathrm{d}u$ we can allow a maximum of M1A0M1A1ftM1A0

Question	Notes Continued		
(d) Way 2	$\left\{\displaystyle\int\frac{6}{u^2+2u}\mathrm{d}u=\int\frac{3(2u+2)}{u^2+2u}\mathrm{d}u-\int\frac{6u}{u^2+2u}\mathrm{d}u\right\}$		
	$=\displaystyle\int\frac{3(2u+2)}{u^2+2u}\mathrm{d}u-\int\frac{6}{u+2}\mathrm{d}u$	$\displaystyle\int\frac{\pm\alpha(2u+2)}{u^2+2u}\{\mathrm{d}u\}\pm\int\frac{\delta}{u+2}\{\mathrm{d}u\},\ \alpha,\beta,\delta\neq 0$	M1
		Correct expression	A1
	$=3\ln(u^2+2u)-6\ln(u+2)$	Integrates $\dfrac{\pm M(2u+2)}{u^2+2u}\pm\dfrac{N}{u\pm k}$, $M,N,k\neq 0$, to obtain any one of $\pm\lambda\ln(u^2+2u)$ **or** $\pm\mu\ln(\beta(u\pm k))$; $\lambda,\mu,\beta\neq 0$	M1
		Integration of both terms is **correctly followed through** from **their** M and from **their** N	A1 ft
	$\left\{\text{So, }\left[3\ln(u^2+2u)-6\ln(u+2)\right]_1^{e}\right\}$ $=\left(3\ln(e^2+2e)-6\ln(e+2)\right)-\left(3\ln 3-6\ln 3\right)$	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq 1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$=3\ln(e^2+2e)-6\ln(e+2)+3\ln 3$	$3\ln(e^2+2e)-6\ln(e+2)+3\ln 3$	A1 o.e.
			[6]
(d) Way 3	Applying $u=\theta-1$		
	$\left\{\displaystyle\int_1^{e}\frac{6}{u(u+2)}\mathrm{d}u=\right\}\displaystyle\int_2^{1+e}\frac{6}{(\theta-1)(\theta+1)}\mathrm{d}\theta=\int_2^{1+e}\frac{6}{\theta^2-1}\mathrm{d}u=\left[3\ln\left(\frac{\theta-1}{\theta+1}\right)\right]_2^{1+e}$		M1A1M1A1
	$=3\ln\left(\dfrac{1+e-1}{e+1+1}\right)-3\ln\left(\dfrac{2-1}{2+1}\right)=3\ln\left(\dfrac{e}{e+2}\right)-3\ln\left(\dfrac{1}{3}\right)$	3rd M mark is dependent on 2nd M mark	dM1A1
			[6]