The Newton-Raphson Method (From OCR 4726)

Q1, (Jan 2006, Q2)

2	Write as $f(x) = \pm(x - e^{-x})$	B1	Or equivalent
	So $f'(x) = \pm(1 + e^{-x})$	B1	Correct from their f(x)
	Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$	M1	Clear evidence of N-R on their f, f '
	Get $x_1 = 0.56631$, $x_2 = 0.56714$	A1√	At least one to 4d.p.
	Get $x_3 = 0.567(1)$	A1	cao to 3 d.p.

Q2, (Jan 2008, Q5)

(i)	Attempt use of product rule	M1	
	Clearly get $x = 1$	A1	Allow substitution of $x=1$
(ii)	Explain use of tangent for next approx.	B1	Not use of G.C. to show divergence
	Tangents at successive approx. give $x>1$	B1	Relate to crossing x-axis; allow diagram
(iii)	Attempt correct use of N-R with their derivative	M1	
	Get $x_2 = -1$	A1√	
	Get –0.6839, -0.5775, (-0.5672…)	A1	To 3 d.p. minimum
	Continue until correct to 3 d.p.	M1	May be implied
	Get –0.567	A1	cao

Q3, (Jun 2008, Q6i,ii)

(i)	Attempt to use N-R of correct form with clear $f'(x)$ used	M1	
	Get 2.633929, 2.645672	A1	For one correct to minimum of 6 d.p.
		A1√	For other correct from their x_2 in correct NR
		3	
(ii)	$\sqrt{7}$	B1	Allow ±
		1	

Q4, (Jan 2010, Q3i,ii)

(i)	Attempt gradient as $\pm f(x_1)/(x_2 - x_1)$	M1	Allow reasonable y-step/x-step
	Equate to gradient of curve at x_1	M1	Allow ±
	Clearly arrive at A.G.	A1	Beware confusing use of ±
	SC Attempt equation of tangent	M1	As $y - f(x_1) = f'(x_1)(x - x_1)$
	Put $(x_2, 0)$ into their equation	M1	
	Clearly arrive at A.G.	A1	
(ii)	Diagram showing at least one more tangent	B1	
	Description of tangent meeting x-axis, used as next starting value	B1	

Q5, (Jun 2010, Q7iii,iv)

(iii)	Reasonable attempt to use log/expo. rules	M1	Allow derivation either way
	Clearly get A.G.	A1	
	Attempt $f'(x)$ and use at least once in correct N-R formula	M1	
	Get answers that lead to 1.31	A1	Minimum of 2 answers; allow truncation/rounding to at least 3 d.p.
(iv)	Show $f'(\ln 36) = 0$	B1	
	Explain why N-R would not work	B1	Tangent parallel to *Ox* would not meet *Ox* again or divide by 0 gives an error

Q6, (Jan 2011, Q5)

(i)	$x_{n+1} = x_n - \dfrac{x_n^3 - 5x_n + 3}{3x_n^2 - 5} = \dfrac{2x_n^3 - 3}{3x_n^2 - 5}$	M1 A1 A1	 3	For attempt at N-R formula For correct N-R expression For correct answer (necessary details needed) **AG** Allow omission of suffixes
(ii)	$F'(x) =$ $\dfrac{6x^2(3x^2-5)-6x(2x^3-3)}{(3x^2-5)^2} = \dfrac{6x(x^3-5x+3)}{(3x^2-5)^2}$	M1 M1		For using quotient *OR* product rule to find $F'(x)$ For factorising numerator to show $k(x^3-5x+3)$
	$F'(\alpha) = \dfrac{6\alpha(\alpha^3-5\alpha+3)}{(3\alpha^2-5)^2} = 0$ since $\alpha^3 - 5\alpha + 3 = 0$	A1	3	For correct explanation of **AG**
(iii)	$x_1 = 2 \Rightarrow 1.85714,\ 1.83479,\ 1.83424,\ 1.83424$ $(\alpha =)\ 1.8342$ **SR** For starting value leading to another root allow up to B1 B1 B0	B1 B1 B1	 3	First iterate correct to at least 4 d.p. *OR* $\frac{13}{7}$ For 2 equal iterates to at least 4 d.p. For correct α to 4 d.p. Allow answer rounding to 1.8342 **SR** If not N-R, B0 B0 B0
			9	

Q7, (Jan 2013, Q8)

(i)	(a)	$x_1 = 4.15,\quad x_2 = 4.1474....$ $x_3 = 4.1465....,\quad x_4 = 4.1463....$ $\beta = 4.146$	M1 A1 [2]	Using iterative formula at least once using at least 4dp www	All iterates must be seen
(i)	(b)	Staircase diagram will always move to upper root	B1 B1 B1 [3]	Sketch showing an example $x_1 > \alpha$ Example with $x_1 < \alpha$ Statement Dep on 1st two B	Ignore any statement when $x_1 > \beta$
(ii)	(a)	$\ln(x-1) = x-3 \Rightarrow \ln(x-1)-(x-3) = 0$ $\Rightarrow \mathrm{f}(x) = \ln(x-1)-(x-3)$ $\Rightarrow \mathrm{f}'(x) = \dfrac{1}{x-1}-1$ $\Rightarrow x_{n+1} = x_n - \dfrac{\ln(x_n-1)-(x_n-3)}{\dfrac{1}{x_n-1}-1}$ $= x_n - \dfrac{(x_n-1)(\ln(x_n-1)-(x_n-3))}{1-(x_n-1)}$ $= \dfrac{x_n(2-x_n)+(x_n-1)(x_n-3)-(x_n-1)\ln(x_n-1)}{2-x_n}$ $= \dfrac{2x_n - x_n^2 + x_n^2 - 4x_n + 3 - (x_n-1)\ln(x_n-1)}{2-x_n}$ $\Rightarrow x_{n+1} = \dfrac{3-2x_n-(x_n-1)(\ln(x_n-1)}{2-x_n}$	M1 M1 M1 A1 A1 [5]	Get equation in correct form Differentiate Use correct formula Mult by $(x-1)$ **soi**	

(ii)	(b)	1.2 1.152(359) 1.152359 1.158448 1.158448 1.158594 1.158594 1.158594 Root = 1.159	B1 B1 B1 [3]	For x_2 For enough iterates to determine 3dp www	Allow 3 dp x_2 must be right for last B1. Any error is likely to be self-correcting

Q9, (Jun 2015, Q6i-iii,v)

(i)		$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3x_n^3 + 5x_n^2 - x_n - 1}{9x_n^2 + 10x_n - 1}$ $= \frac{x_n(9x_n^2 + 10x_n - 1) - (3x_n^3 + 5x_n^2 - x_n - 1)}{9x_n^2 + 10x_n - 1}$ $= \frac{9x_n^3 + 10x_n^2 - x_n - 3x_n^3 - 5x_n^2 + x_n + 1}{9x_n^2 + 10x_n - 1}$ $= \frac{6x_n^3 + 5x_n^2 + 1}{9x_n^2 + 10x_n - 1}$	**B1** **M1** **A1**	Correct derivative seen Combining terms seen as 1 fraction or 2 fractions with common denominator Line above seen ag Must contain suffices.	
			3		
(ii)		A suitable value is shown within range [0.1, 0.25]	**B1**	The point does not have to be labelled x_1	Accept a tangent which shows this.
			1		
(iii)		$\Rightarrow x_2 = 0 \Rightarrow x_3 = -1$, and statement that values alternate. Clear diagram with tangents from – 1 to 0 and back to –1	**B1** **B1**	Values seen either in words or on graph marked as these values	
(v)		Continuing the above to give root 0.47936	**M1** **A1**	Or any other starting point that converges to the positive root Cao	
			2		