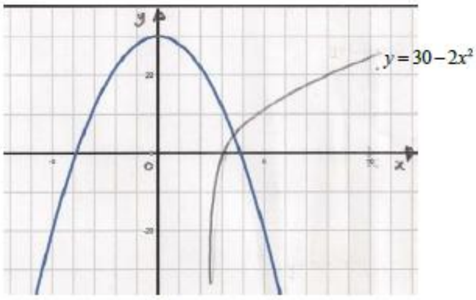
Newton Raphson Method Mark Scheme (Edexcel)

Q1.

Question	Scheme	Marks	AOs
(a)	$f(3.5) = -4.8,\ f(4) = (+)3.1$	M1	1.1b
	Change of sign and function continuous in interval $[3.5, 4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	Attempts $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} \Rightarrow \quad x_1 = 4 - \dfrac{3.099}{16.67}$	M1	1.1b
	$x_1 = 3.81$	A1	1.1b
	$y = \ln(2x - 5)$	(2)	
(c)	 Attempts to sketch both $y = \ln(2x - 5)$ and $y = 30 - 2x^2$	M1	3.1a
	States that $y = \ln(2x - 5)$ meets $y = 30 - 2x^2$ in just one place, therefore $y = \ln(2x - 5) = 30 - 2x$ has just one root $\Rightarrow f(x) = 0$ has just one root	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

M1: Attempts f(*x*) at both $x = 3.5$ and $x = 4$ with at least one correct to 1 significant figure

A1*: $f(3.5)$ and $f(4)$ correct to 1 sig figure (rounded or truncated) with a correct reason and conclusion. A reason could be change of sign, or $f(3.5) \times f(4) < 0$ or similar with f(*x*) being continuous in this interval. A conclusion could be 'Hence root' or 'Therefore root in interval'

(b)

M1: Attempts $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ evidenced by $x_1 = 4 - \dfrac{3.099}{16.67}$

A1: Correct answer only $x_1 = 3.81$

(c)

M1: For a valid attempt at showing that there is only one root. This can be achieved by

- Sketching graphs of $y = \ln(2x - 5)$ and $y = 30 - 2x^2$ on the same axes
- Showing that $f(x) = \ln(2x - 5) + 2x^2 - 30$ has no turning points
- Sketching a graph of $f(x) = \ln(2x - 5) + 2x^2 - 30$

A1: Scored for correct conclusion

Q2.

Question Number	Scheme	Notes	Marks
	$\mathrm{f}(x) = x^3 - \dfrac{5}{2x^{\frac{3}{2}}} + 2x - 3$		
(a)	$\mathrm{f}(1.1) = -1.6359604$, $\mathrm{f}(1.5) = 2.0141723$	Attempts to evaluate both $\mathrm{f}(1.1)$ and $\mathrm{f}(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a **root** / α is between $x = 1.1$ and $x = 1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. $-1.63.. < 0 < 2.014..$) and conclusion.	A1
			(2)
(b)	$\mathrm{f}(x) = x^3 - \frac{5}{2}x^{-\frac{3}{2}} + 2x - 3$ $\Rightarrow \mathrm{f}'(x) = 3x^2 + \frac{15}{4}x^{-\frac{5}{2}} + 2$	M1: $x^n \to x^{n-1}$ for at least one term A1: Correct derivative oe	M1A1
			(2)
(c)	$\mathrm{f}'(1.1) = 3(1.1)^2 + \frac{15}{4}(1.1)^{-\frac{5}{2}} + 2 (= 8.585)$	Attempt to find $\mathrm{f}'(1.1)$. Accept $\mathrm{f}'(1.1)$ seen and their value.	M1
	$\alpha_2 = 1.1 - \left(\dfrac{\text{"}-1.6359604\text{"}}{\text{"}8.585\text{"}}\right)$	Correct application of N-R	M1
	$\alpha_2 = 1.291$	cao	A1
			(3)
			Total 7

Q3.

Question Number	Scheme	Notes	Marks
	$f(x) = x^2 + \dfrac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$		
	$f(x) = x^2 + \dfrac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$f'(x) = 2x - \dfrac{3}{8}x^{-\frac{3}{2}} - 3\ \{+0\}$	M1: $x^n \to x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$f(4) = -2.625 = -\dfrac{21}{8} = -2\dfrac{5}{8}$ *or* $4^2 + \dfrac{3}{4\sqrt{4}} - 3\times4 - 7$	$f(4) = -2.625$ A correct evaluation of f(4) or a correct numerical expression for f(4). This can be implied by a correct answer below but in all other cases, f(4) must be seen explicitly evaluated or as an expression.	B1
	$f'(4) = 4.953125 = \dfrac{317}{64} = 4\dfrac{61}{64}$	Attempt to insert $x = 4$ into their $f'(x)$. Not dependent on the first M but must be what they think is $f'(x)$.	M1
	$\alpha_2 = 4 - \left(\dfrac{"-2.625"}{"4.953125"}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454\ldots \quad \left(= \dfrac{1436}{317} = 4\tfrac{168}{317}\right)$		
	$= 4.53$ (2 dp)	4.53 cso	A1 cao
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \dfrac{f(4)}{f'(4)} = 4.53$ can score full marks.		
			[6]
			6 marks

Q4.

Question	Scheme	Marks	AOs
(a)	$\{f(3.6)=\}\ 3.6+\tan\left(\frac{1}{2}(3.6)\right)=-0.686\ldots<0$ **and** $\{f(3.7)=\}\ 3.7+\tan\left(\frac{1}{2}(3.7)\right)=0.211\ldots>0$	M1	1.1b
	<u>Change of sign</u> and function is <u>continuous</u> in the interval $\Rightarrow$ <u>conclusion</u> e.g. "there is a root in [3.6, 3.7]" *	A1*	2.4
		(2)	
(b)	Use of $\tan\left(\frac{1}{2}x\right)\to\ldots\sec^2\left(\frac{1}{2}x\right)$	M1	1.1b
	$\{f'(x)=\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	A1	1.1b
		(2)	
(c)	Attempts $3.7-\dfrac{3.7+\tan\left(\frac{1}{2}(3.7)\right)}{"1+\frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)"}=\ldots$ (N.B. $f(3.7)=0.211\ldots$ and $f'(3.7)=7.58\ldots$)	M1	1.1b
	α = awrt 3.672	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Attempts both f(3.6) and f(3.7) or a narrower interval that contains the root 3.672…
(which may be implied by sight of f(3.6) = … and f(3.7) = … with at least one correct)
and obtains at least one correct to 1 significant figure (rounded or truncated) for their interval
and considers their signs. Use of degrees is M0.
Some examples for consideration of sign (which are also sufficient for the change of sign part of reasoning for the A1):

- $f(3.6)=-0.6<0$ and $f(3.7)=0.2>0$
- $f(3.6)\times f(3.7)<0$
- $f(3.6)=-0.7$, $f(3.7)=0.2$ "change in sign"

For reference $f(3.6)=-0.68626\ldots$ and $f(3.7)=0.21194\ldots$

A1*: This mark requires:

- both f(3.6) and f(3.7) correct to 1 significant figure (rounded or truncated) (or their values correct to 1 significant figure if using a narrower interval)
- a reference to sign change
- a reference to continuity {of f(x)}
- a (minimal) conclusion, e.g. "hence root", "proved", ✓, #, QED, $3.6<\alpha<3.7$

Accept as a minimum, "change of sign, continuous, root".
Do not condone "change in sign therefore continuous" or other incorrect statements such as "x is continuous", "the interval is continuous" – these score A0.
Condone "the graph is continuous".
Condone reference to x in place of α in their conclusion, e.g. "hence x lies in the interval".
Condone statements such as "there is at least one root" in place of their conclusion.

(b) **Note:** Their answer to (b) may be seen in part (c) provided that they have not clearly attempted part (b) incorrectly, e.g., an attempt at $f^{-1}(x)$ in (b).

M1: For $\tan\left(\frac{1}{2}x\right) \to ...\sec^2\left(\frac{1}{2}x\right)$ o.e. The brackets are not required. You may see attempts at the quotient rule but the method should be correct and they should reach something equivalent to $...\sec^2\left(\frac{1}{2}x\right)$.

e.g. $\tan\left(\frac{1}{2}x\right) = \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} \to \frac{k\cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right) - -k\sin\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ where k is a **positive** constant scores M1. If the formula is seen it must be correct.

A1: $\{f'(x) =\}\ 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ o.e. which may be unsimplified and apply isw.

The brackets are not required. There is no need for $f'(x) =$ just look for the expression.

Note that $\{f'(x) =\}\ \frac{3}{2} + \frac{1}{2}\tan^2\left(\frac{1}{2}x\right)$ is correct and appears occasionally.

$\{f'(x) =\}\ 1 + \frac{1}{2}\sec\frac{1}{2}x^2$ is condoned for M1A0 only but $1 + \frac{1}{2}\left(\sec\frac{1}{2}x\right)^2$ scores M1A1.

(c)

M1: Attempts $3.7 - \frac{f(3.7)}{f'(3.7)}$ and obtains a value following through on their $f'(x)$ as long as it is a "changed" function in terms of x.

Just stating $3.7 - \frac{f(3.7)}{f'(3.7)} = ...$ without evidence of use of 3.7 in $f(x)$ (note that this evidence might come from part (a)) **and** in their $f'(x)$ is M0 unless implied by a correct value for both $f(x)$ **and** $f'(x)$ **or** by their final answer.
Must be a correct N-R formula used – you may need to check their values – accuracy of at least 3s.f. rounded or truncated required.
Allow if attempted in degrees. For reference in degrees $f(3.7) = 3.73...$ and $f'(3.7) = 1.50...$ and gives $\alpha = 1.21...$
Note that the full N-R accuracy is 3.672051617.
For reference, the value of α is approximately 3.673194406… and scores M0A0 without other valid work.

A1: For awrt 3.672 Ignore any subsequent iterations.

Q5.

Question Number	Scheme	Marks
(a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f′ (1.1) = -6.37086…	B1 B1
	$x_1 = 1.1 - \frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) **[9]**

Q6.

Question	Scheme	Marks	AOs
(a)	Attempts $f(5) = -18$ and $f(6) = (+)\,4$	M1	1.1b
	States change of sign, function continuous so root between	A1	2.1
		(2)	
(b)	$p = \ln 2$ (so $f'(x) = 2^x \ln 2 - 10$)	B1	1.2
		(1)	
(c)	Attempts $x_1 = 6 - \frac{4}{64\ln 2 - 10}$	M1	1.1b
	Awrt 5.88	A1	1.1b
		(2)	
(d)	Sets $f'(x) = 2^x \ln 2 - 10 = 0 \Rightarrow 2^x = \frac{10}{\ln 2} \Rightarrow x = ...$	M1	1.1b
	$\Rightarrow x =$ awrt 3.85	A1	1.1b
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts both $\mathrm{f}(5)=-18$ and $\mathrm{f}(6)=(+)\,4$ with at least one correct.

A1: Completes the argument.

Requires

- both values correct
- gives full reason including "change of sign" (o.e) and "continuity"
- a minimal conclusion, e.g. root, tick, α lies in the interval $[5,6]$

Accept equivalent statements for sign change e.g. $\mathrm{f}(6)>0$, $\mathrm{f}(5)<0$ e.g. $\mathrm{f}(5)\times\mathrm{f}(6)<0$, $\mathrm{f}(5)<0<\mathrm{f}(6)$, "one negative one positive", "there is a change of sign"

A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(b)

B1: $p=\ln 2$ or $\mathrm{f}'(x)=2^x\ln 2-10$ Condone $p=\log 2$ unless $p=\log_{10}2$ is implied by subsequent work.

(c)

M1: Attempts $x_1=6-\dfrac{\mathrm{f}(6)}{\mathrm{f}'(6)}$ following through on their $\mathrm{f}(6)$ and $\mathrm{f}'(6)$

May be implied by their value if no working is shown.

A1: Awrt 5.88

(d)

M1: Sets $p\times 2^x-10=0$ with their positive numerical value of p (which may be 1) and uses correct processing to find a value for x e.g. $2^x=\dfrac{10}{p}\Rightarrow x=\log_2\dfrac{10}{p}$ or $2^x=\dfrac{10}{p}\Rightarrow x=\dfrac{\log\frac{10}{p}}{\log 2}$ or

$2^x=\dfrac{10}{p}\Rightarrow x=\dfrac{\ln\frac{10}{p}}{\ln 2}$

A1: awrt 3.85

Q7.

Question	Scheme	Marks	AOs
(a)	$(f'(x)=)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $f'(x)=4\cos\left(\frac{1}{2}x\right)-3=0 \Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $f(4)>0$, $f(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB $f(5)=-1.212...$ and $f'(5)=-6.204...$)	M1	1.1b
	$x_1=$ awrt 4.80	A1	1.1b
		(2)	
			(7 marks)
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $f'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $f'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $f'(x)=...$ or $\frac{dy}{dx}=...$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for x.

Look for

- $f'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\ \ a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0 \Rightarrow \cos\left(\frac{1}{2}x\right)=\pm k \Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0 \Rightarrow x=1.4...$ or $11.1...$ (or $82.8...$ or $637....$ or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. $14.011...$
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the y coordinate.
Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $f(4)>0$, $f(5)<0$ e.g. $f(4)\times f(5)<0$, "there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because x is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees $f(5) = -5.65\ldots$ and $f'(5) = 0.996\ldots$ and gives $x_1 = 10.67\ldots$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0