The Modulus Function Questions Sheet 2

Q1.

(a) Sketch the graph with equation

$$y = |4x - 3|$$

stating the coordinates of any points where the graph cuts or meets the axes.

(2)

Find the complete set of values of x for which

(b)

$$|4x - 3| > 2 - 2x$$

(4)

(c)

$$|4x - 3| > \tfrac{3}{2} - 2x$$

(2)

(Total 8 marks)

Q2.

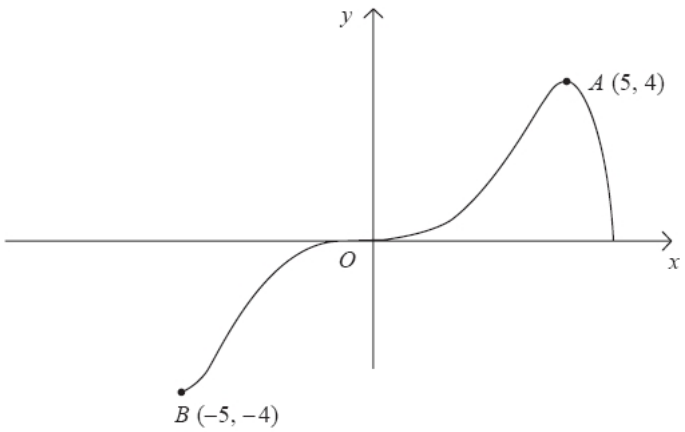


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$.
The curve passes through the origin O and the points $A(5, 4)$ and B(–5, –4).

In separate diagrams, sketch the graph with equation

(a) $y = | f(x) |$,

(3)

(b) $y = f(| x |)$,

(3)

(c) $y = 2f(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

(Total 10 marks)

Q3.

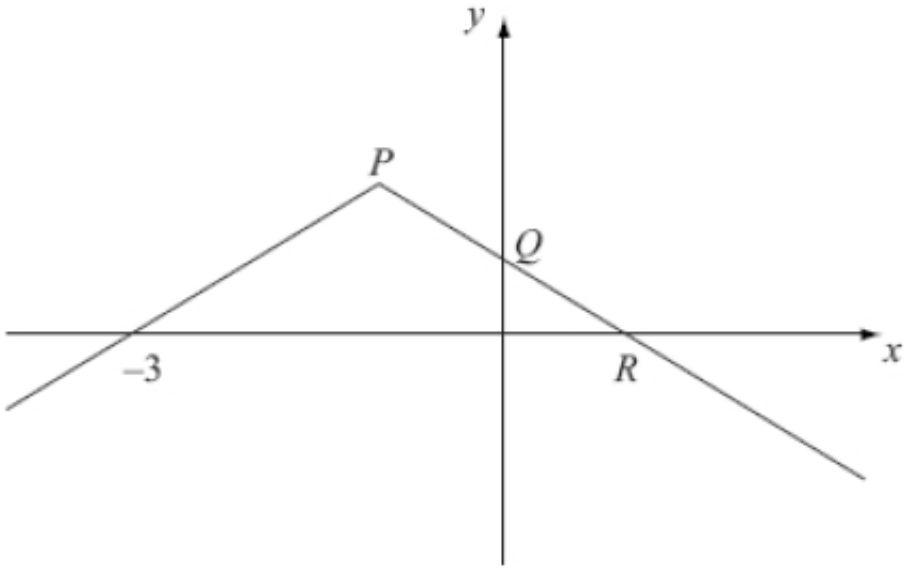


Figure 1

Figure 1 shows the graph of $y = f(x)$, $x \in \mathbb{R}$

The graph consists of two line segments that meet at the point P.

The graph cuts the y-axis at the point Q and the x-axis at the points $(–3, 0)$ and R.

Sketch, on separate diagrams, the graphs of

(a) $y = | f(x) |$,

(2)

(b) $y = f(– x)$.

(2)

Given that $f(x) = 2 – | x + 1 |$,

(c) find the coordinates of the points P, Q and R,

(3)

(d) solve $f(x) = \frac{1}{2}x$.

(5)

(Total 12 marks)

Q4.

The function f is defined by

$$f: x \mapsto |2x-5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g: x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find fg(2).

(2)

(d) Find the range of g.

(3)

(Total 10 marks)

Q5.

The functions f and g are defined by

$$f: x \mapsto \ln(2x-1), \qquad x \in \mathbb{R}, \ x > \frac{1}{2},$$

$$g: x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R}, \ x \neq 3.$$

(a) Find the exact value of fg(4).

(2)

(b) Find the inverse function $f^{-1}(x)$, stating its domain.

(4)

(c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.

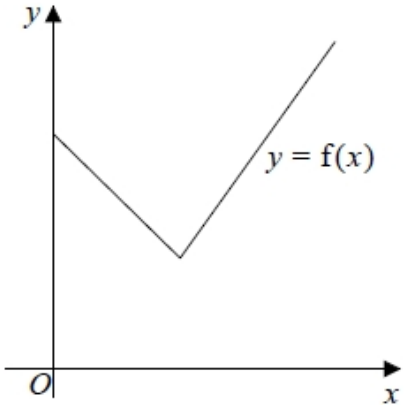
(3)

(d) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.

(3)

(Total 12 marks)

Q6.



The diagram shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

(Total for question = 6 marks)

Q7.

The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.

(2)

(b) Find fg(1).

(2)

(c) Find g^{-1}, the inverse function of g.

(2)

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

(Total 11 marks)

Q8. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$
(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

(b) find c in terms of a.

(4)

(Total for question = 8 marks)

Q9.

Figure 4 shows a sketch of the graph with equation

$$y = | 2x - 3k |$$

where k is a positive constant.

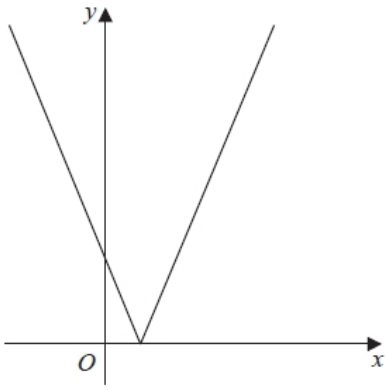


Figure 4

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - | 2x - 3k |$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - | 2x - 3k | > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

(Total for question = 10 marks)