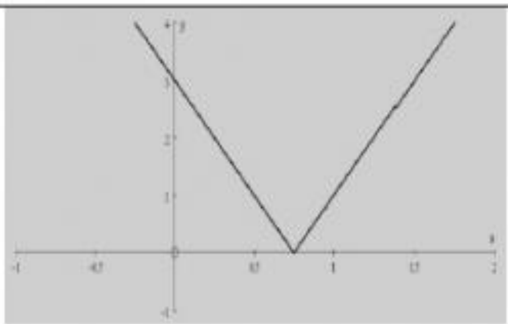
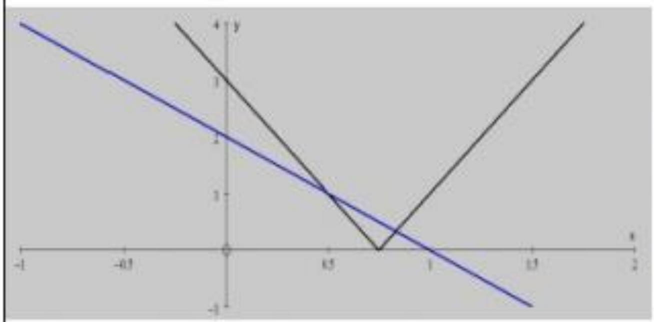
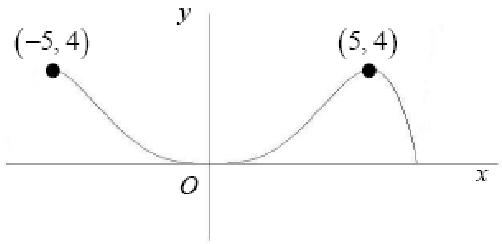
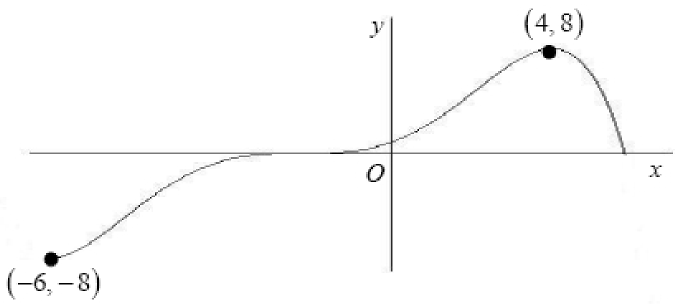
## The Modulus Function Questions Sheet 2

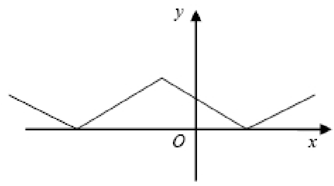
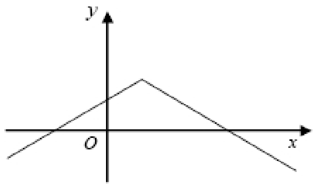
**Q1.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **(a)** |  | V shaped graph | B1 |
| | | Touches $x$ axis at ¾ and cuts y axis at 3 | B1 |
| | | | (2) |
| **(b)** |  | Solves $4x-3=2-2x$ or $3-4x=2-2x$ to give either value of $x$ | M1 |
| | | Both $x=\frac{5}{6}$ and $x=\frac{1}{2}$ | |
| | | or $x>\frac{5}{6}$ or $x<\frac{1}{2}$ | A1 |

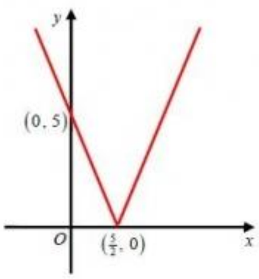
**Q2.**

| Question Number | Scheme | Marks |
|---|---|---|
| | (a)   | |
| | Shape | B1 |
| | $(5, 4)$ | B1 |
| | $(-5, 4)$ | B1 (3) |
| | (b) For the purpose of marking this paper, the graph is identical to (a) | |
| | Shape | B1 |
| | $(5, 4)$ | B1 |
| | $(-5, 4)$ | B1 (3) |
| | (c)   | |
| | General shape – unchanged | B1 |
| | Translation to left | B1 |
| | $(4, 8)$ | B1 |
| | $(-6, -8)$ | B1 (4) |
| | In all parts of this question ignore any drawing outside the domains shown in the diagrams above. | **[10]** |

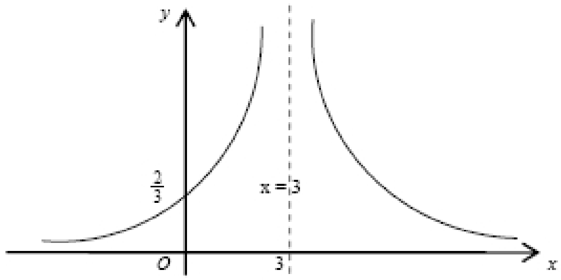
**Q3.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) |   shape | B1 |
| | Vertices correctly placed | B1 (2) |
| (b) |   shape | B1 |
| | Vertex and intersections with axes correctly placed | B1 (2) |
| (c) | $P:(-1, 2)$ | B1 |
| | $Q:(0, 1)$ | B1 |
| | $R:(1, 0)$ | B1 (3) |
| (d) | $x>-1$; $\quad 2-x-1=\frac{1}{2}x$ | M1 A1 |
| | Leading to $x=\frac{2}{3}$ | A1 |
| | $x<-1$; $\quad 2+x+1=\frac{1}{2}x$ | M1 |
| | Leading to $x=-6$ | A1 (5) |
| | | **(12 marks)** |

**Q4.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) |   | M1A1 (2) |
| (b) | $\underline{x = 20}$ | B1 |
| | $2x-5=-(15+x)$ ; $\Rightarrow \underline{x=-\frac{10}{3}}$ | M1;A1 oe. (3) |
| (c) | $fg(2)=f(-3)=\|2(-3)-5\|;=\|-11\|=11$ | M1;A1 (2) |
| (d) | $g(x)=x^2-4x+1=(x-2)^2-4+1=(x-2)^2-3$. Hence $g_{min}=-3$ | M1 |
| | Either $g_{min}=-3$ or $g(x)\geqslant -3$ or $g(5)=25-20+1=6$ | B1 |
| | $\underline{-3\leqslant g(x)\leqslant 6}$ or $\underline{-3\leqslant y\leqslant 6}$ | A1 (3) |
| | | [10] |

**Q5.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (*a*) | Finding g(4) = k and f(k) = …. or $fg(x) = \ln\left(\frac{4}{x-3} - 1\right)$ | | M1 |
| | [ f(2) = ln(2x2 − 1) fg(4) = ln(4 − 1)] = ln 3 | | A1 (2) |
| (*b*) | $y = \ln(2x-1) \Rightarrow e^y = 2x - 1$ or $e^x = 2y - 1$ | | M1, A1 |
| | $f^{-1}(x) = \frac{1}{2}(e^x + 1)$ Allow $y = \frac{1}{2}(e^x + 1)$ | | A1 |
| | Domain $x \in \Re$ [Allow $\Re$, all reals, $(-\infty, \infty)$] independent | | B1 (4) |
| (*c*) |  | Shape, and *x*-axis should appear to be asymptote | B1 |
| | | **Equation** $x = 3$ **needed,** may see in diagram (ignore others) | B1 ind. |
| | | Intercept $(0, \frac{2}{3})$ no other; accept y = ⅔ (0.67) or on graph | B1 ind (3) |
| (*d*) | $\frac{2}{x-3} = 3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv. | | B1 |
| | $\frac{2}{x-3} = -3$, $\Rightarrow x = 2\frac{1}{3}$ or exact equiv. | | M1, A1 (3) |
| | Note: $2 = 3(x+3)$ or $2 = 3(-x-3)$ o.e. is M0A0 | | |
| Alt: | Squaring to quadratic ($9x^2 - 54x + 77 = 0$) **and** solving M1; B1A1 | | **(12 marks)** |

**Q6.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) | $f(x) \geqslant 5$ | B1 | 1.1b |
| | | (1) | |
| (b) | Uses $-2(3-x)+5=\frac{1}{2}x+30$ | M1 | 3.1a |
| | Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$ | M1 | 1.1b |
| | $x=\frac{62}{3}$ only | A1 | 1.1b |
| | | (3) | |
| (c) | Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$ | M1 | 2.2a |
| | $\{k: k \in \mathbb{R}, 5<k \leqslant 11\}$ | A1 | 2.5 |
| | | (2) | |
| | | | **(6 marks)** |

**Notes:**

**(a)**

**B1:** $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

**(b)**

**M1:** Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

**M1:** Correct method used to solve their equation. Multiplies out bracket/ collects like terms

**A1:** $x=\frac{62}{3}$ only. Do not allow 20.6

**(c)**

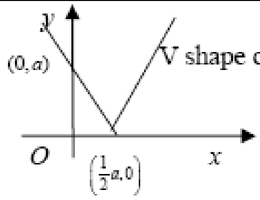
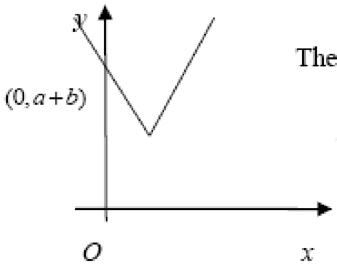
**M1:** Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

**A1:** Correct solution only $\{k: k \in \mathbb{R}, 5<k \leqslant 11\}$

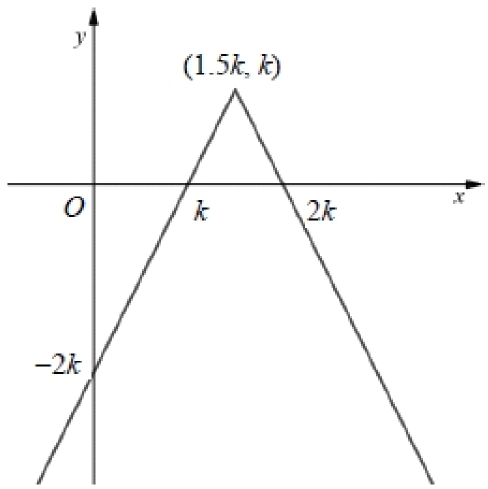
**Q7.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $f(x) \geqslant 3$ | M1A1 |
| | | (2) |
| (b) | An attempt to find $2\lvert 3-4x\rvert+3$ when $x=1$ | M1 |
| | Correct answer $fg(1)=5$ | A1 |
| | | (2) |
| (c ) | $y=3-4x \Rightarrow 4x=3-y \Rightarrow x=\dfrac{3-y}{4}$ | M1 |
| | $g^{-1}(x)=\dfrac{3-x}{4}$ | A1 |
| | | (2) |
| (d) | $[g(x)]^2=(3-4x)^2$ | B1 |
| | $gg(x)=3-4(3-4x)$ | M1 |
| | $gg(x)+[g(x)]^2=0 \Rightarrow -9+16x+9-24x+16x^2=0$ | |
| | $16x^2-8x=0$ | A1 |
| | $8x(2x-1)=0 \Rightarrow x=0, 0.5$ oe | M1A1 |
| | | (5 ) |
| | | (11 marks) |

**Q8.**

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)(i)** |  V shape on $x$ - axis **or** coordinates $\left(\frac{1}{2}a, 0\right)$ **and** $(0, a)$ | B1 |
| | Correct shape, position and coordinates | B1 |
| **(ii)** |  Their "V" shape translated up or $(0, a+b)$ | B1ft |
| | Correct shape, position and $(0, a+b)$ | B1 |
| | | **(4)** |
| **(b)** | States or uses $a+b=8$ | B1 |
| | Attempts to solve $\lvert 2x-a \rvert + b = \frac{3}{2}x+8$ in either $x$ or with $x = c$ | |
| | $2c-a+b=\frac{3}{2}c+8 \Rightarrow kc = \mathrm{f}(a,b)$ | M1 |
| | Combines $kc=\mathrm{f}(a,b)$ with $a+b=8 \quad \Rightarrow c=4a$ | dM1 A1 |
| | | (4) |
| | | **(8 marks)** |

**Q9.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| (a) |  | | |
| | ∧ shape in any position | B1 | 1.1b |
| | Correct $x$-intercepts or coordinates | B1 | 1.1b |
| | Correct $y$-intercept or coordinates | B1 | 1.1b |
| | Correct coordinates for the vertex of a ∧ shape | B1 | 1.1b |
| | | **(4)** | |
| (b) | $x = k$ | B1 | 2.2a |
| | $k-(2x-3k)=x-k \Rightarrow x=\ldots$ | M1 | 3.1a |
| | $x=\dfrac{5k}{3}$ | A1 | 1.1b |
| | **Set notation is required here for this mark** $\left\{x : x<\dfrac{5k}{3}\right\} \cap \{x : x>k\}$ | A1 | 2.5 |
| | | **(4)** | |
| (c) | $x = 3k$ **or** $y = 3 - 5k$ | B1ft | 2.2a |
| | $x = 3k$ **and** $y = 3 - 5k$ | B1ft | 2.2a |
| | | **(2)** | |
| | | | **(10 marks)** |

| Notes |
| --- |
| (a) **Note that the sketch may be seen on Figure 4**<br>B1: See scheme<br>B1: Correct $x$-intercepts. Allow as shown or written as $(k, 0)$ and $(2k, 0)$ and condone coordinates written as $(0, k)$ and $(0, 2k)$ as long as they are in the correct places.<br>B1: Correct $y$-intercept. Allow as shown or written as $(0, -2k)$ or $(-2k, 0)$ as long as it is in the correct place. Condone $k - 3k$ for $-2k$.<br>B1: Correct coordinates as shown<br>**Note that the marks for the intercepts and the maximum can be seen away from the sketch but the coordinates must be the right way round or e.g. as $y = 0, x = k$ etc. These marks can be awarded without a sketch but if there is a sketch, such points must not contradict the sketch.**<br>(b)<br>B1: Deduces the correct critical value of $x = k$. May be implied by e.g. $x > k$ or $x < k$<br>M1: Attempts to solve $k - (2x - 3k) = x - k$ or an equivalent equation/inequality to find the other critical value. Allow this mark for reaching $k = \dots$ or $x = \dots$ as long as they are solving the required equation.<br>A1: Correct value<br>A1: Correct answer using the correct set notation.<br>Allow e.g. $\left\{x : x \in \mathbb{R}, k < x < \frac{5k}{3}\right\}$, $\left\{x :\ k < x < \frac{5k}{3}\right\}$, $x \in \left(k, \frac{5k}{3}\right)$ and allow "\|" for ":"<br>But $\left\{x : x < \frac{5k}{3}\right\} \cup \{x : x > k\}$ scores A0 $\left\{x :\ k < x,\ x < \frac{5k}{3}\right\}$ scores A0<br>(c)<br>B1ft: Deduces one correct coordinate. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" or $y = 3 - 5\times$"$k$" but must be in terms of $k$.<br>Allow as coordinates or $x = .., y = \dots$<br>B1ft: Deduces both correct coordinates. Follow through their maximum coordinates from (a) so allow $x = 2\times$"$1.5k$" and $y = 3 - 5\times$"$k$" but must be in terms of $k$.<br>Allow as coordinates or $x = .., y = \dots$<br>If coordinates are given the wrong way round and not seen correctly as $x = \dots, y = \dots$<br>e.g. $(3 - 5k, 3k)$ this is B0B0 |

**Alternative to part (b) by squaring:**

$$k - |2x - 3k| = x - k \Rightarrow |2x - 3k| = 2k - x$$

$$4x^2 - 12kx + 9k^2 = 4k^2 - 4kx + x^2 \Rightarrow 3x^2 - 8kx + 5k^2 = 0$$

$$(3x - 5k)(x - k) = 0 \Rightarrow x = \frac{5k}{3},\ k$$

Score M1 for isolating the $|2x - 3k|$, squaring both sides to obtain 3 appropriate terms for each side, collects terms to obtain $Ax^2 + Bkx + Ck^2 = 0$ and solves for $x$

A1 for $x = \frac{5k}{3}$ and B1 for $x = k$

Then A1 as in the scheme.