Solving Equations Using Compound Angle Formulae Exam Questions (From OCR 4754A)

Q1, (Jun 2005, Q5)

$2\cos 2x = 2(2\cos^2 x - 1) = 4\cos^2 x - 2$	**M1**	Any double angle formula used
$\Rightarrow 4\cos^2 x - 2 = 1 + \cos x$		
$\Rightarrow 4\cos^2 x - \cos x - 3 = 0$	**M1**	*getting a quadratic in cos x*
$\Rightarrow (4\cos x + 3)(\cos x - 1) = 0$	**M1dep**	attempt to solve
	A1	for -3/4 and 1
$\Rightarrow \cos x = -3/4$ or 1		
$\Rightarrow x = 138.6°$ or $221.4°$	**B1 B1**	139,221 or better
or 0	**B1**	www
	[7]	-1 extra solutions in range

Q2, (Jan 2006, Q4)

(i) $2\sin 2\theta + \cos 2\theta = 1$		
$\Rightarrow 4\sin\theta\cos\theta + 1 - 2\sin^2\theta = 1$	M1	Using double angle formulae
$\Rightarrow 2\sin\theta(2\cos\theta - \sin\theta) = 0$ or $4\tan\theta - 2\tan^2\theta = 0$	A1	Correct simplification to factorisable or other form that leads to solutions
	A1	0° and 180°
$\Rightarrow \sin\theta = 0$ or $\tan\theta = 0$, $\theta = 0°, 180°$	M1	$\tan\theta = 2$
or $2\cos\theta - \sin\theta = 0$	A1,	(-1 for extra solutions in range)
$\Rightarrow \tan\theta = 2$	A1	
$\Rightarrow \theta = 63.43°, 243.43°$	[6]	
OR		
Using $R\sin(2\theta+\alpha)$	M1	
$R=\sqrt{5}$ and $\alpha=26.57°$	A1	
$2\theta + 26.57 = \arcsin 1/R$	M1	
$\theta=0°, 180°$	A1	(-1 for extra solutions in range)
$\theta=63.43°, 243.43°$	A1,A1	
	[6]	

Q3, (Jun 2006, Q3)

3 $\sin(\theta + \alpha) = 2\sin\theta$		Using correct Compound angle formula
$\Rightarrow \sin\theta\cos\alpha + \cos\theta\sin\alpha = 2\sin\theta$	M1	in a valid equation
$\Rightarrow \tan\theta\cos\alpha + \sin\alpha = 2\tan\theta$	M1	dividing by $\cos\theta$
$\Rightarrow \sin\alpha = 2\tan\theta - \tan\theta\cos\alpha$		
$= \tan\theta\ (2 - \cos\alpha)$	M1	collecting terms in $\tan\theta$ or $\sin\theta$ or dividing by $\tan\theta$ oe
$\Rightarrow \tan\theta = \dfrac{\sin\alpha}{2-\cos\alpha}$ *	E1	www (can be all achieved for the method in reverse)
$\sin(\theta + 40°) = 2\sin\theta$		
$\Rightarrow \tan\theta = \dfrac{\sin 40}{2-\cos 40} = 0.5209$	M1	
$\Rightarrow \theta = 27.5°, 207.5°$	A1 A1	$\tan\theta = \dfrac{\sin 40}{2-\cos 40}$
	[7]	-1 if given in radians -1 extra solutions in the range

Q4, (Jan 2007, Q3)

3(i) $\sin 60 = \sqrt{3}/2$, $\cos 60 = 1/2$, $\sin 45 = 1/\sqrt{2}$, $\cos 45 = 1/\sqrt{2}$	M1	
$\sin(105°) = \sin(60°+45°)$ $= \sin 60°\cos 45° + \cos 60°\sin 45°$	M1	splitting into 60° and 45°, and using the compound angle formulae
$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}}$	A1	
$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ *	E1 [4]	
(ii) Angle B = 105°		
By the sine rule: $\frac{AC}{\sin B} = \frac{1}{\sin 30}$	M1	Sine rule
$\Rightarrow$ AC $= \frac{\sin 105}{\sin 30} = \frac{\sqrt{3}+1}{2\sqrt{2}}.2$	A1	with exact values
$= \frac{\sqrt{3}+1}{\sqrt{2}}$ *	E1 [3]	www

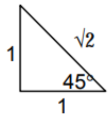
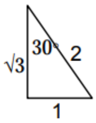
Q5, (Jan 2008, Q4)

(i) $\sin(\theta + 45°) = \cos\theta$		
$\Rightarrow$ $\sin\theta\cos 45 + \cos\theta\sin 45 = \cos\theta$	M1	compound angle formula
$\Rightarrow$ $(1/\sqrt{2})\sin\theta + (1/\sqrt{2})\cos\theta = \cos\theta$	B1	$\sin 45 = 1/\sqrt{2}$, $\cos 45 = 1/\sqrt{2}$
$\Rightarrow$ $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$	A1	
$\Rightarrow$ $\sin\theta = (\sqrt{2} - 1)\cos\theta$	M1	collecting terms
$\Rightarrow$ $\frac{\sin\theta}{\cos\theta} = \tan\theta = \sqrt{2} - 1$ *	E1 [5]	
(ii) $\tan\theta = \sqrt{2} - 1$		
$\Rightarrow$ $\theta = 22.5°$,	B1	
202.5°	B1 [2]	and no others in the range

Q6, (Jun 2012, Q5)

$\sin(x + 45°) = \sin x\cos 45° + \cos x\sin 45°$	M1	Use of **correct** compound angle formula
$= \sin x.1/\sqrt{2} + \cos x.\ 1/\sqrt{2}$	A1	
$= (1/\sqrt{2})(\sin x + \cos x) = 2\cos x$ $\Rightarrow$ $\sin x + \cos x = 2\sqrt{2}\cos x$ *	A1	**Since AG ,** $\sin x\cos 45° + \cos x\sin 45° = 2\cos x$ $\sin x + \cos x = 2\sqrt{2}\cos x$ only gets M1 need the second line or statement of $\cos 45° = \sin 45° = 1/\sqrt{2}$ oe as an intermediate step to get A1 A1
$\Rightarrow$ $\sin x = (2\sqrt{2} - 1)\cos x$ $\Rightarrow$ $\tan x = 2\sqrt{2} - 1$	M1	terms collected and $\tan x = \sin x/\cos x$ used
$\Rightarrow$ $x = 61.32°$,	A1	for first correct solution
241.32°	A1	for second correct solution and no others in the range 2dp but allow overspecification ignore solutions outside the range SC A1 for both 61.3° and 241.3° SC A1 for both 1.07 and 4.21 radians (or better) SC A1 for incorrect answers that round to 61.3° and 180° + their ans eg 61.33° and 241.33° Do not award SC marks if there are extra solutions in the range.
	[6]	

Q7, (Jun 2013, Q3)

 tan 45° = 1/1 = 1*  tan 30° = 1/√3*		For both B marks **AG** so need to be convinced and need triangles but further explanation need not be on their diagram. Any given lengths must be consistent.
	B1	Need √2 or indication that triangle is isosceles oe
	B1	Need all three sides oe
tan 75° = tan (45°+30°)	M1	use of **correct** compound angle formula with 45°,30° soi
$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1 + 1/\sqrt{3}}{1 - 1/\sqrt{3}}$	A1	substitution in terms of √3 in any **correct** form
$= \frac{1+\sqrt{3}}{-1+\sqrt{3}}$	M1	eliminating fractions within a fraction (or rationalising, whichever comes first) provided compound angle formula is used as tan(*A*+*B*)= tan(*A*±*B*)/(1±tan*A*tan*B*).
$= \frac{(1+\sqrt{3})^2}{3-1}$ (oe eg $\frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{(3+\sqrt{3})^2}{9-3}$)	M1	rationalising denominator (or eliminating fractions whichever comes second)
$= \frac{(3+2\sqrt{3}+1)}{3-1} = 2+\sqrt{3}$ *	A1	**correct** only, **AG** so need to see working
	[7]	

Q8, (Jun 2015, Q2)

$\cos 2\theta = 1 - 2\sin^2\theta$	M1*	$\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ (maybe implied in substitution)
$(6\cos 2\theta + \sin\theta =)\ 6 - 12\sin^2\theta + \sin\theta$	A1	
$6\cos 2\theta + \sin\theta = 0$ $\Rightarrow\quad 12\sin^2\theta - \sin\theta - 6 = 0$ $\Rightarrow\quad (4\sin\theta - 3)(3\sin\theta + 2) = 0$	M1dep*	Use of correct quadratic equation formula or factorising or comp. the square on their three term quadratic in $\sin\theta$ (see guidance in question 1 for awarding this method mark) provided $b^2 - 4ac \geq 0$
$\Rightarrow\quad \sin\theta = 3/4$ or $-2/3$	A1	www
	B1	First correct solution to 1 dp or better (eg 48.59° etc)
	B1	Three correct solutions
$\Rightarrow\quad \sin\theta = 3/4,\ \theta = 48.6°, 131.4°$ $\sin\theta = -2/3,\ \theta = 221.8°, 318.2°$	B1	All four correct solutions and no others in the range
		Ignore solutions outside the range
		SC Award max B1B1B0 for answers in radians (0.85, 2.29, 3.87, 5.55 or better – so one correct B1, three correct B1). Award max B1 if there are extra solutions in the range with radians
		SC If M1M1 awarded and both values of $\lvert\sin\theta\rvert \leq 1$ but B0B0B0 then award B1 only for evidence of using $\sin\theta \equiv \sin(180-\theta)$
	[7]	

Q9, (Jun 2016, Q4)

$4\sin\theta\cos\theta = 1 + 2\cos^2\theta - 1$	M1*	Use of correct double angle formulae: $\sin 2\theta \equiv 2\sin\theta\cos\theta$ **and** any one of $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$ or $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$
$2\cos\theta(2\sin\theta - \cos\theta) = 0$	A1	Correct equation in solvable form e.g. $2\sin\theta - \cos\theta = 0$ (oe) or $5\sin^4\theta - 6\sin^2\theta + 1 = 0$ or $5\cos^4\theta - 4\cos^2\theta = 0$ but not $4\sin\theta\cos\theta = 2\cos^2\theta$
$\Rightarrow \tan\theta = \frac{1}{2}$	M1dep*	Use of $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ on their $\alpha\sin\theta + \beta\cos\theta = 0$ or correct method for solving quadratic in either $\sin^2\theta$ or $\cos^2\theta$ (See guidance in question 2 for solving quadratics)
$\theta = 26.6^\circ$	A1	www (26.6 or better)
$\theta = 90^\circ$	B1	Not from incorrect working Ignore additional solutions outside the range. If any additional solutions given inside the range $0 \le \theta \le 180^\circ$ **and** full marks would have been awarded then remove last mark (so 4/5) Both answers in radians: 0.464 (or better) and $\pi / 2$ scores B1 Answers with no working scores B1 B1(so max 2/5)
	[5]	