Small Angle Approximations Exam Questions MS

Q1, (OCR H240/03, Sample Question Paper, Q4)

When θ is small $1+\cos\theta-3\cos^2\theta$ $\approx 1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\frac{1}{2}\theta^2\right)^2$	**M1**	**1.1a**	Attempt to use $\cos\theta\approx 1-\frac{1}{2}\theta^2$ or $=1+\left(1-\frac{1}{2}\theta^2+\ldots\right)$ $-3\left(1-\frac{1}{2}\theta^2+\ldots\right)^2$	**OR** **M1** Attempt to use $\cos\theta\approx 1-\frac{1}{2}\theta^2$
$=1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\theta^2+\frac{1}{4}\theta^4\right)$ $=1+1-\frac{1}{2}\theta^2-3+3\theta^2-\frac{3}{4}\theta^4$	**M1**	**1.1**	Multiply out	**M1** use trigonometric identity $1+\cos\theta-3\cos^2\theta$ $=1+\cos\theta-\frac{3}{2}-\frac{3}{2}\cos 2\theta$
Since θ is small, we can neglect the higher order terms	**E1**	**2.5**	For explanation of loss of θ^4 term and consistent use of notation throughout (Working need not be fully correct)	**E1** For showing clearly which identity has been used and consistent use of notation throughout
so $1+\cos\theta-3\cos^2\theta\approx -1+\frac{5}{2}\theta^2$ as required	**E1**	**2.1**	AG Clearly obtained www Condone θ^4 term missing without explanation and inconsistent notation	**E1** AG Clearly obtained www Condone inconsistent notation
	[4]			

Specimen

Q2, (OCR H240/03, Practice Paper Set 1, Q3)

(i)		$\dfrac{AC}{\sin\frac{3}{4}\pi} = \dfrac{1}{\sin\left(\pi - \frac{3}{4}\pi - \theta\right)}$	M1	2.1	Attempt sine rule
		$AC = \dfrac{\sin\frac{3}{4}\pi}{\sin\frac{1}{4}\pi\cos\theta - \cos\frac{1}{4}\pi\sin\theta}$	M1	2.1	For expanding $\sin\left(\frac{1}{4}\pi - \theta\right)$
		$\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ so $AC = \dfrac{1}{\cos\theta - \sin\theta}$	E1 [3]	2.2a	**AG**, so must show sufficient working; e.g. stating $\sin\frac{3}{4}\pi = \sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi$ or using $\dfrac{1}{\sqrt{2}}$ oe for each
(ii)		$AC = \left(1 + (-\theta - \frac{1}{2}\theta^2)\right)^{-1}$	B1	1.1	Using both small angle approximations
		$AC = 1 + (-1)\left(-\theta - \frac{1}{2}\theta^2\right) + \dfrac{(-1)(-2)}{2}\left(-\theta - \frac{1}{2}\theta^2\right)^2 + \dots$	M1	3.1a	Attempt binomial expansion of AC, with at least the first two terms present
		$AC \approx 1 + \theta + \frac{3}{2}\theta^2$	A1	1.1	$p = 1$
			A1 [4]	1.1	$q = \frac{3}{2}$

Q3, (OCR H240/02, Practice Paper Set 3, Q3)

$\dfrac{1 - \frac{1}{8}\theta^2}{1 + \theta} = 0.825$	M1	1.1a	
$0.125\theta^2 + 0.825\theta - 0.175 = 0$	A1	1.1	
$\theta = 0.206$ or -6.81 (3 sf)	A1	1.1	**BC**
Discard –6.81 as not small. $\theta = 0.206$ (3 sf)	A1 [4]	2.3	Statement needed and $\theta = 0.206$ alone

Q4, (Edexcel Practice Paper 5, Q2)

(a)	Shows that $2\cos 3\theta \approx 2\left(1-\frac{9\theta^2}{2}\right) = 2-9\theta^2$	**M1**	2.1	6th Understand small-angle approximations for sin, cos and tan (angle in radians).
	Shows that $2\cos 3\theta - 1 \approx 1-9\theta^2 = (1-3\theta)(1+3\theta)$	**M1**	1.1b	
	Shows $1+\sin\theta+\tan 2\theta = 1+\theta+2\theta = 1+3\theta$	**M1**	2.1	
	Recognises that $\frac{1+\sin\theta+\tan 2\theta}{2\cos 3\theta - 1} \approx \frac{1+3\theta}{(1-3\theta)(1+3\theta)} = \frac{1}{1-3\theta}$	**A1**	1.1b	
		(4)		
(b)	When θ is small, $\frac{1}{1-3\theta} \approx 1$	**A1**	1.1b	7th Use small-angle approximations to solve problems.
		(1)		
				(5 marks)

Q5, (Edexcel A Level Maths, Jun 2018, Paper 1, Q1)

Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
$= \frac{4}{3}$ oe	A1	1.1b
	(3)	

(3 marks)

Q6, (OCR H640/03, Jun 2018, Q3)

AC = [AO – OC] $=1-\cos\theta$ or $\cos\theta = 1 - AC$	**M1**	**1.1a**	**AG** Allow AC = AO – OC with $OC = \cos\theta$ for M1
θ small so AC $1-\left(1-\frac{\theta^2}{2}\right)=\frac{\theta^2}{2}$	**E1**	**2.1**	Convincing completion
	[2]		

Q7, (Edexcel Mock Paper 2, Q1)

1	$\cos\theta - \sin\left(\frac{1}{2}\theta\right) + 2\tan\theta = \frac{11}{10}$		
(a)	$1 - \frac{1}{2}\theta^2 - \frac{1}{2}\theta + 2\theta \approx \frac{11}{10}$	M1	1.2
		A1	1.1b
	$\Rightarrow \frac{1}{2}\theta^2 - \frac{3}{2}\theta + \frac{1}{10} \approx 0 \Rightarrow 5\theta^2 - 15\theta + 1 \approx 0$ *	A1*	2.1
		(3)	
(b)	$\theta = 0.068$ is valid because θ is small $\theta = 2.932$ is not valid because θ is large	B1	2.3
		(1)	
			(4 marks)