



**Further Trig and Reciprocal Trig Functions (From OCR 4723)**

**Q1, (Jun 2005, Q7)**

- (i) Write down the formula for  $\cos 2x$  in terms of  $\cos x$ . [1]
- (ii) Prove the identity  $\frac{4 \cos 2x}{1 + \cos 2x} \equiv 4 - 2 \sec^2 x$ . [3]
- (iii) Solve, for  $0 < x < 2\pi$ , the equation  $\frac{4 \cos 2x}{1 + \cos 2x} = 3 \tan x - 7$ . [5]
- 

**Q2, (Jun 2006, Q5)**

- (i) Write down the identity expressing  $\sin 2\theta$  in terms of  $\sin \theta$  and  $\cos \theta$ . [1]
- (ii) Given that  $\sin \alpha = \frac{1}{4}$  and  $\alpha$  is acute, show that  $\sin 2\alpha = \frac{1}{8}\sqrt{15}$ . [3]
- (iii) Solve, for  $0^\circ < \beta < 90^\circ$ , the equation  $5 \sin 2\beta \sec \beta = 3$ . [3]
- 

**Q3, (Jan 2007, Q2)**

It is given that  $\theta$  is the acute angle such that  $\sin \theta = \frac{12}{13}$ . Find the exact value of

- (i)  $\cot \theta$ , [2]
- (ii)  $\cos 2\theta$ . [3]
- 

**Q4, (Jun 2008, Q5)**

(a) Express  $\tan 2\alpha$  in terms of  $\tan \alpha$  and hence solve, for  $0^\circ < \alpha < 180^\circ$ , the equation

$$\tan 2\alpha \tan \alpha = 8. \quad [6]$$

(b) Given that  $\beta$  is the acute angle such that  $\sin \beta = \frac{6}{7}$ , find the exact value of

- (i)  $\operatorname{cosec} \beta$ , [1]
- (ii)  $\cot^2 \beta$ . [2]
- 

**Q5, (Jan 2009, Q3)**

(i) Express  $2 \tan^2 \theta - \frac{1}{\cos \theta}$  in terms of  $\sec \theta$ . [3]

(ii) Hence solve, for  $0^\circ < \theta < 360^\circ$ , the equation

$$2 \tan^2 \theta - \frac{1}{\cos \theta} = 4. \quad [4]$$

---



**Q6, (Jan 2010, Q2)**

The angle  $\theta$  is such that  $0^\circ < \theta < 90^\circ$ .

(i) Given that  $\theta$  satisfies the equation  $6 \sin 2\theta = 5 \cos \theta$ , find the exact value of  $\sin \theta$ . [3]

(ii) Given instead that  $\theta$  satisfies the equation  $8 \cos \theta \operatorname{cosec}^2 \theta = 3$ , find the exact value of  $\cos \theta$ . [5]

---

**Q7, (Jun 2010, Q3)**

(i) Express the equation  $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$  in the form  $a \sin^2 \theta + b \sin \theta + c = 0$ , where  $a$ ,  $b$  and  $c$  are constants. [3]

(ii) Hence solve, for  $-180^\circ < \theta < 180^\circ$ , the equation  $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$ . [3]

---

**Q8, (Jan 2012, Q4)**

The acute angles  $\alpha$  and  $\beta$  are such that

$$2 \cot \alpha = 1 \quad \text{and} \quad 24 + \sec^2 \beta = 10 \tan \beta.$$

(i) State the value of  $\tan \alpha$  and determine the value of  $\tan \beta$ . [4]

(ii) Hence find the exact value of  $\tan(\alpha + \beta)$ . [3]

---

**Q9, (Jan 2013, Q2)**

The acute angle  $A$  is such that  $\tan A = 2$ .

(i) Find the exact value of  $\operatorname{cosec} A$ . [2]

(ii) The angle  $B$  is such that  $\tan(A + B) = 3$ . Using an appropriate identity, find the exact value of  $\tan B$ . [3]

---

**Q10, (Jun 2013, Q2)**

Using an appropriate identity in each case, find the possible values of

(i)  $\sin \alpha$  given that  $4 \cos 2\alpha = \sin^2 \alpha$ , [3]

(ii)  $\sec \beta$  given that  $2 \tan^2 \beta = 3 + 9 \sec \beta$ . [4]

---

**Q11, (Jun 2014, Q2)**

By first using appropriate identities, solve the equation

$$5 \cos 2\theta \operatorname{cosec} \theta = 2$$

for  $0^\circ < \theta < 180^\circ$ .

[6]

---



**Q12, (Jun 2016, Q4)**

It is given that  $A$  and  $B$  are angles such that

$$\sec^2 A - \tan A = 13 \quad \text{and} \quad \sin B \sec^2 B = 27 \cos B \operatorname{cosec}^2 B.$$

Find the possible exact values of  $\tan(A - B)$ .

[8]

**Q13, (Jun 2016, Q9)**

(i) Show that  $\sin 2\theta(\tan \theta + \cot \theta) \equiv 2$ .

[4]

(ii) Hence

(a) find the exact value of  $\tan \frac{1}{12}\pi + \tan \frac{1}{8}\pi + \cot \frac{1}{12}\pi + \cot \frac{1}{8}\pi$ ,

[3]

(b) solve the equation  $\sin 4\theta(\tan \theta + \cot \theta) = 1$  for  $0 < \theta < \frac{1}{2}\pi$ ,

[3]

(c) express  $(1 - \cos 2\theta)^2 (\tan \frac{1}{2}\theta + \cot \frac{1}{2}\theta)^3$  in terms of  $\sin \theta$ .

[2]

**Q14, (Jun 2017, Q4)**

The angle  $\theta$ , where  $90^\circ < \theta < 180^\circ$ , satisfies the equation

$$3 \sec^2 \theta + 10 \tan \theta = 11.$$

(i) Find the value of  $\tan \theta$ .

[3]

(ii) Without using a calculator, determine the value of

(a)  $\tan 2\theta$ ,

[2]

(b)  $\cot(2\theta + 135^\circ)$ .

[3]