Further Trig and Reciprocal Trig Functions (From OCR 4723)

Q1, (Jun 2005, Q7)

(i)	State $2\cos^2 x - 1$	B1	**1**
(ii)	Attempt to express left hand side in terms of $\cos x$	M1	[using expression of form $a\cos^2 x + b$]
	Identify $\frac{1}{\cos x}$ as $\sec x$	M1	[maybe implied]
	Confirm result	A1	**3** [**AG**; necessary detail required]
(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$	B1	
	Attempt solution of quadratic equation in tan x	M1	[or equiv]
	Obtain $2\tan^2 x + 3\tan x - 9 = 0$ and hence tan $x = -3,\ \frac{3}{2}$	A1	
	Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of 0.313π, 1.31π, 0.602π, 1.60π)	A1	[allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi$, $1.89 + \pi$ allow degrees: 56, 236, 108, 288]
	Obtain all four solutions	A1	**5** [now with at least 3 s.f.; must be radians; no other solutions in the range 0 - 2π, ignore solutions outside range 0 - 2π]

Q2, (Jun 2006, Q5)

(i)	State $\sin 2\theta = 2\sin\theta\cos\theta$	B1	1	or equiv; any letter acceptable here (and in parts **(ii)** and **(iii)**)
(ii)	Attempt to find exact value of cos α	M1		using identity attempt or right-angled triangle
	Obtain $\frac{1}{4}\sqrt{15}$	A1		or exact equiv
	Substitute to confirm $\frac{1}{8}\sqrt{15}$	A1	3	AG
(iii)	State or imply $\sec\beta = \frac{1}{\cos\beta}$	B1		
	Use identity to produce equation involving $\sin\beta$	M1		
	Obtain $\sin\beta = 0.3$ and hence 17.5	A1	3	and no other values between 0 and 90; allow 17.4 or value rounding to 17.4 or 17.5

Q3, (Jan 2007, Q2)

(i)	Attempt complete method for finding cot θ	M1		rt-angled triangle, identities, calculator, …
	Obtain $\frac{5}{12}$	A1	**2**	or exact equiv
(ii)	Attempt relevant identity for cos 2θ	M1		$\pm 2\cos^2\theta \pm 1$ or $\pm 1 \pm 2\sin^2\theta$ or $\pm(\cos^2\theta - \sin^2\theta)$
	State correct identity with correct value(s) substituted	A1		
	Obtain $-\frac{119}{169}$	A1	**3**	correct answer only earns 3/3

Q4, (Jun 2008, Q5)

(a)	Obtain expression of form $\dfrac{a\tan\alpha}{b + c\tan^2\alpha}$	**M1**	any non-zero constants a, b, c
	State correct $\dfrac{2\tan\alpha}{1-\tan^2\alpha}$	**A1**	or equiv
	Attempt to produce polynomial equation in $\tan\alpha$	**M1**	using sound process
	Obtain at least one correct value of $\tan\alpha$	**A1**	$\tan\alpha = \pm\sqrt{\frac{4}{5}}$
	Obtain 41.8	**A1**	allow 42 or greater accuracy; allow 0.73
	Obtain 138.2 and no other values between 0 and 180	**A1**	allow 138 or greater accuracy
	[SC: Answers only 41.8 or … B1; 138.2 or … and no others B1]	**6**	
(b)(i)	State $\frac{7}{6}$	**B1**	
		1	
(ii)	Attempt use of identity linking $\cot^2\beta$ and $\text{cosec}^2\beta$	**M1**	or equiv retaining exactness; condone sign errors
	Obtain $\frac{13}{36}$	**A1**	or exact equiv
		2	

Q5, (Jan 2009, Q3)

(i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta \pm 1$; or equiv
	Replace $\dfrac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta - 1) - \sec\theta$	A1	**3**	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	**4**	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			7	

Q6, (Jan 2010, Q2)

(i)	Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1	
	Attempt value of $\sin\theta$ from $k\sin\theta\cos\theta = 5\cos\theta$	M1	any constant k; or equiv
	Obtain $\frac{5}{12}$	A1 3	or exact equiv; ignore subsequent work
(ii)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ or $\operatorname{cosec}^2\theta = 1+\cot^2\theta$	B1	or equiv
	Attempt to produce equation involving $\cos\theta$ only	M1	using $\sin^2\theta = \pm 1 \pm \cos^2\theta$ or equiv
	Obtain $3\cos^2\theta + 8\cos\theta - 3 = 0$	A1	or equiv
	Attempt solution of 3-term quadratic equation	M1	using formula or factorisation or equiv
	Obtain $\frac{1}{3}$ as only final value of $\cos\theta$	A1 5	or exact equiv; ignore subsequent work
		8	

Q7, (Jun 2010, Q3)

(i)	Use $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	
	Attempt to express equation in terms of $\sin\theta$	M1	using $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or equiv
	Obtain or clearly imply $6\sin^2\theta - 11\sin\theta - 10 = 0$	A1 3	or $-6\sin^2\theta + 11\sin\theta + 10 = 0$
(ii)	Attempt solution to obtain at least one value of $\sin\theta$	M1	should be $s = -\frac{2}{3}, \frac{5}{2}$
	Obtain -41.8	A1	allow -42 or greater accuracy
	Obtain -138	A1 3	or greater accuracy; and no others between -180 and 180
	[Answer(s) only: award 0 out of 3.]		
		6	

Q8, (Jan 2012, Q4)

(i)		State $\tan\alpha = 2$	B1	ignoring subsequent work to find angle
		Use identity $\sec^2\beta = 1+\tan^2\beta$	B1	
		Attempt solution of quad eqn for $\tan\beta$	M1	3 term quad eqn; using reasonable attempt at factorisation to find value or use of quadratic formula (with no more than one slip)
		Obtain $\tan\beta = 5$	A1	ignoring subsequent work to find angle; value 5 must be obtained legitimately
			[4]	

(ii)	Substitute their values of $\tan\alpha$ and $\tan\beta$ in formula …	M1	… of form $\frac{\pm\tan\alpha\pm\tan\beta}{\pm1\pm\tan\alpha\tan\beta}$
	Obtain $\frac{2+5}{1-2\times5}$	A1ft	following their values from part **(i)**
	Obtain $-\frac{7}{9}$	A1	or correct simplified exact equiv including $\frac{7}{-9}$; A0 if $\tan\beta=5$ obtained incorrectly in part **(i)** SC: use of calculator for $\tan(\tan^{-1}2+\tan^{-1}5)$ to give $-\frac{7}{9}$ earns all 3 marks (but 0 out of 3 if answer is not exact); with either or both of 2 and 5 wrong, 2 out of 3 available for this approach if result is exact and correct given their two values
		[3]	

Q9, (Jan 2013, Q2)

(i)	Either Attempt to find exact value of $\sin A$	M1	using right-angled triangle or identity or …
	Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
		[2]	
	Or Attempt use of identity $1+\cot^2 A=\operatorname{cosec}^2 A$	M1	using $\cot A=\frac{1}{2}$; allow sign error in attempt at identity
	Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
(ii)	State or imply $\frac{2+\tan B}{1-2\tan B}=3$	B1	
	Attempt solution of equation of form $\frac{\text{linear in } t}{\text{linear in } t}=3$	M1	by sound process at least as far as $k\tan B=c$
	Obtain $\tan B=\frac{1}{7}$	A1	answer must be exact; ignore subsequent attempt to find angle B
		[3]	

Q10, (Jun 2013, Q2)

(i)	Use $2\cos^2\alpha-1$ or $\cos^2\alpha-\sin^2\alpha$ or $1-2\sin^2\alpha$	B1	
	Obtain equation in which $\sin^2\alpha$ appears once	M1	condoning sign slips or arithmetic slips; for solution which gives equation involving $\tan^2\alpha$, M1 is not earned until valid method for reaching $\sin\alpha$ is used; attempt involving $4(1-s^2)=s^2$ is M0
	Obtain $\pm\frac{2}{3}$	A1	both values needed; ±0.667 is A0; $\pm\sqrt{\frac{4}{9}}$ is A0; ignore subsequent work to find angle(s)
		[3]	
(ii)	Either Attempt use of identity	M1	of form $\tan^2\beta=\pm\sec^2\beta\pm1$
	Obtain $2\sec^2\beta-9\sec\beta-5=0$	A1	condone absence of $=0$
	Attempt solution of 3-term quadratic in $\sec\beta$ to obtain at least one value of $\sec\beta$	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
	Obtain 5 with no errors in solution	A1	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$
		[4]	
	Or Attempt to express equation in terms of $\cos\beta$	M1	using identities which are correct apart maybe for sign slips
	Obtain $5\cos^2\beta+9\cos\beta-2=0$	A1	condone absence of $=0$
	Attempt solution of 3-term quadratic and show switch at least once to a secant value	M1	if factorising, factors must be such that expansion gives their first and third terms; if using formula, this must be correct for their values
	Obtain 5 with no errors in solution	A1 [4]	and, finally, no other value; no need to justify rejection of $-\frac{1}{2}$

Q11, (Jun 2014, Q2)

State or imply $\text{cosec}\theta = 1 \div \sin\theta$	B1	allow $\text{cosec} = 1 \div \sin$	
Attempt to express equation in terms of $\sin\theta$ only	M1	using identity of form $\pm 1 \pm 2\sin^2\theta$ for $\cos 2\theta$	
Obtain $10\sin^2\theta + 2\sin\theta - 5 = 0$	A1	or unsimplified equiv involving $\sin\theta$ only but with no $\sin\theta$ remaining in denominator	
Attempt use of formula to find $\sin\theta$ from 3-term quadratic equation involving $\sin\theta$ (using formula or completing square even if their equation can be solved by factorisation)	M1	use implied by at least one correct value of $\sin\theta$ or θ; if correct quadratic formula quoted, condone one sign error for M1; if formula not first quoted, any error leads to M0	if completion of square used to solve equation, this must be correct for M1 to be earned
Obtain 37.9°	A1	or greater accuracy 37.8896…	
Obtain 142°	A1	or greater accuracy 142.1103…; and no others between 0 and 180; ignore any answers, right or wrong, outside 0 - 180	no working and answers only (max 2/6): 37.9 (or greater accuracy) B1 142 (or greater accuracy) and no others … B1
	[6]		

Q12, (Jun 2016, Q4)

Use identity $\sec^2 A = 1 + \tan^2 A$	B1		
Attempt solution of three-term quadratic equation to obtain two values of $\tan A$	M1	Implied if correct values obtained; allow M1 for incorrect factorisation provided expansion would give correct first and third terms; allow M1 for incorrect use of formula if only one error present	
Obtain $\tan A = -3$ and $\tan A = 4$	A1	And no others; implied by $A = \tan^{-1} -3$ and $\tan^{-1} 4$;	$A = -3, 4$ is A0 here unless subsequent work shows values used correctly
Use correct identities to produce equation in $\tan B$ only	M1	Equation might be $t^3 = 27$ …	… or $t^5 + t^3 - 27t^2 - 27 = 0$
State $\tan B = 3$	A1	And no others	
Substitute at least one pair of non-zero numerical values into $\dfrac{\tan A - \tan B}{1 + \tan A \tan B}$	M1	Must be the correct identity	
Obtain one of $\frac{1}{13}$ and $\frac{3}{4}$ or exact equiv	A1		
Obtain the other exact value or equiv	A1	And no others	
	[8]		

Q13, (Jun 2016, Q9)

i		Use $\sin 2\theta = 2\sin\theta\cos\theta$	B1		
		State $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ or $\tan\theta + \frac{1}{\tan\theta}$	B1	Perhaps as part of expression	
		Simplify using correct identities	M1		Note that going directly from $2\sin^2\theta + 2\cos^2\theta$ to 2 is M0 but $2(\sin^2\theta + \cos^2\theta)$ to 2 is M1A1
		Obtain 2 correctly	A1	AG; necessary detail needed	
			[4]		
ii	a	Obtain expression involving at least one of $\sin\frac{1}{6}\pi$ and $\sin\frac{1}{4}\pi$	M1		
		Obtain $\frac{2}{\sin\frac{1}{6}\pi} + \frac{2}{\sin\frac{1}{4}\pi}$	A1	Or equiv involving cosecant	
		Obtain $4 + 2\sqrt{2}$ or exact equiv	A1	Answer only is 0/3	
			[3]		
	b	Use $\sin 4\theta = 2\sin 2\theta\cos 2\theta$	B1		
		Obtain $\cos 2\theta = \frac{1}{4}$ or $\cos^2\theta = \frac{5}{8}$ or $\sin^2\theta = \frac{3}{8}$	B1		
		Obtain 0.659 or 0.66	B1	Or greater accuracy; and no others between 0 and $\frac{1}{2}\pi$; allow 0.21π but not 0.659π; answer only earns 0/3	
			[3]		
	c	Express in form $k_1\sin^4\theta \times \frac{k_2}{\sin^3\theta}$	M1		
		Obtain $4\sin^4\theta \times \frac{8}{\sin^3\theta}$ and hence $32\sin\theta$	A1	A0 if $(-2\sin^2\theta)^2$ involved in simplification	
			[2]		

Q14, (Jun 2017, Q4)

i		Use identity $\sec^2\theta = 1 + \tan^2\theta$	B1	Identity must be used not merely quoted
		Attempt solution of 3-term quadratic equation in $\tan\theta$	M1	If using factorisation, M1 earned if their factors correct; if using formula, M1 earned if substitution of their values into correct formula correct; for incorrect equation and two values produced with no working, check that values are correct given their equation so that M1 can be awarded
		Obtain at least $\tan\theta = -4$ from the correct equation	A1	Ignore second value given provided no error at this stage is involved; so $\frac{2}{3}$ and -4 is A1, -4 only is A1, $\frac{2}{3}$ only is A0, $\frac{3}{2}$ and -4 is A0 ; allow solution such as $y = -4$ when clear that y is $\tan\theta$; ignore subsequent work with angles
			[3]	
ii	a	Attempt substitution into $\frac{2\tan\theta}{1-\tan^2\theta}$	M1	Using any value from (i)
		Use -4 to obtain $\frac{8}{15}$ and no other value	A1	Or exact equiv; full details to be shown; indication of use of calculator is M0; finding $\tan 2\theta$ for both angles is M1A0; answer $\frac{8}{15}$ with no working is M0A0; final answer $\frac{-8}{-15}$ is A0
			[2]	
	b	State or imply $\cot(2\theta + 135°)$ is $1 \div \tan(2\theta + 135°)$	B1	Either at beginning of solution or towards the end
		Attempt substitution of their value from (a) into $\frac{1-\tan 2\theta\tan 135°}{\tan 2\theta + \tan 135°}$ or into $\frac{\tan 2\theta + \tan 135°}{1-\tan 2\theta\tan 135°}$	M1	Allow with $\tan 135°$ still present
		Obtain $-\frac{23}{7}$ and no other value	A1	Or exact equiv; full details to be shown; allow $\frac{23}{-7}$
			[3]	