Reciprocal Trig Functions and Compound Angles Exam Questions Sheet 2

Q1.

(a) Prove that

$$2\cot 2x + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total for question = 10 marks)

Q2.

(a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \qquad A \neq \frac{(2n+1)\pi}{4}, \; n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)

(Total for question = 9 marks)

Q3.

(a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

(Total for question = 9 marks)

Q4.

(i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4 \sin x = \sec x$$

(4)

(ii) Solve, for $0 \leq \theta < 360°$, the equation

$$5 \sin \theta - 5 \cos \theta = 2$$

giving your answers to one decimal place.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 9 marks)

Q5.

(a) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \quad x \neq 90n°, \quad n \in \mathbb{R}.$$

(5)

(b) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec} (4\theta + 10°) + \cot (4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total 10 marks)

Q6.

(a) Prove that

$\sin 2x - \tan x \equiv \tan x \cos 2x, \quad x \neq (2n + 1)90°, \quad n \in \mathbb{Z}$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leq x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

(Total for question = 9 marks)

Q7.

(i) Use an appropriate double angle formula to show that

$$\text{cosec}2x = \lambda \text{ cosec } x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

(Total 9 marks)

Q8.

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \; n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\text{cosec } 4x - \cot 4x = 1$$

(5)

(Total 12 marks)

Q9.

Find all the solutions

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leqslant \theta < 360°$

(6)

(Total 6 marks)

Q10.

(a) Prove that

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}2\theta, \qquad \theta \neq 90n°.$$

(4)

(b) On the axes below, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

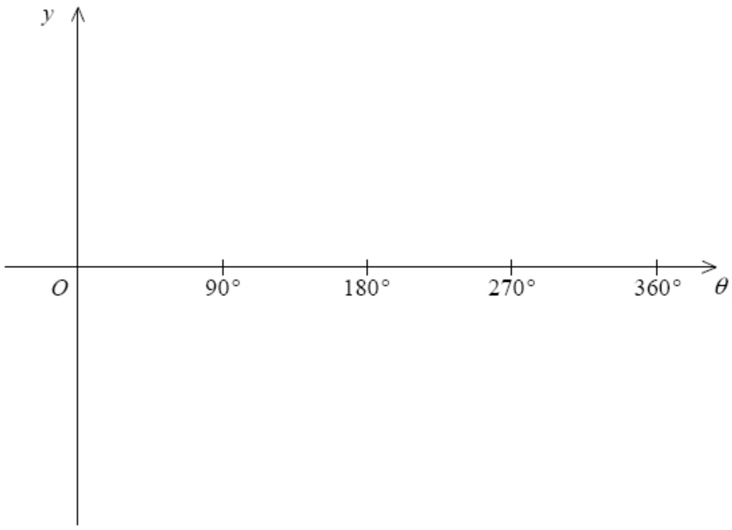
(2)

(c) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=3,$$

giving your answers to 1 decimal place.

(6)



(Total 12 marks)