Reciprocal Trig Functions and Compound Angles Exam Questions Sheet 2 MS

Q1.

Question	Scheme	Marks
(a)	$2\cot 2x + \tan x \equiv \frac{2}{\tan 2x} + \tan x$	B1
	$\equiv \frac{(1-\tan^2 x)}{\tan x} + \frac{\tan^2 x}{\tan x}$	M1
	$\equiv \frac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \text{cosec}^2 x - 2 \Rightarrow 3\cot x = \text{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \frac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \frac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)
(a)alt 1	$2\cot 2x + \tan x \equiv \frac{2\cos 2x}{\sin 2x} + \tan x$	B1
	$\equiv 2\frac{\cos^2 x - \sin^2 x}{2\sin x\cos x} + \frac{\sin x}{\cos x}$	M1
	$\equiv \frac{\cos^2 x - \sin^2 x}{\sin x\cos x} + \frac{\sin^2 x}{\sin x\cos x} \equiv \frac{\cos^2 x}{\sin x\cos x}$	M1
	$\equiv \frac{\cos x}{\sin x}$	
	$\equiv \cot x$	A1*
(a)alt 2	$2\cot 2x + \tan x \equiv 2\frac{(1-\tan^2 x)}{2\tan x} + \tan x$	B1M1
	$\equiv \frac{2}{2\tan x} - \frac{2\tan^2 x}{2\tan x} + \tan x$ or $\frac{(1-\tan^2 x) + \tan^2 x}{\tan x}$	
	$\equiv \frac{2}{2\tan x} = \cot x$	M1A1*

Alt (b)	$6\cot 2x + 3\tan x = \text{cosec}^2 x - 2 \Rightarrow \frac{3\cos x}{\sin x} = \frac{1}{\sin^2 x} - 2$	
	$(\times \sin^2 x) \Rightarrow 3\sin x\cos x = 1 - 2\sin^2 x$	M1
	$\Rightarrow \frac{3}{2}\sin 2x = \cos 2x$	M1A1
	$\Rightarrow \tan 2x = \frac{2}{3} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)

Q2.

Question Number	Scheme	Marks
(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1 (4)
		(9 marks)

Question Number	Scheme	Marks
Alt I **From RHS**	$\dfrac{\cos A+\sin A}{\cos A-\sin A} = \dfrac{\cos A+\sin A}{\cos A-\sin A}\times\dfrac{\cos A+\sin A}{\cos A+\sin A}$	
	$= \dfrac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$	(Pythagoras) M1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	(Double Angle) M1
	$= \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	(Single Fraction) M1
	$= \sec 2A + \tan 2A$	B1 (Identity), A1*
Alt II **Both sides**	Assume true $\sec 2A + \tan 2A = \dfrac{\cos A+\sin A}{\cos A-\sin A}$	
	$\dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A} = \dfrac{\cos A+\sin A}{\cos A-\sin A}$	B1 (identity)
	$\dfrac{1+\sin 2A}{\cos 2A} = \dfrac{\cos A+\sin A}{\cos A-\sin A}$	M1 (single fraction)
	$\dfrac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A} = \dfrac{\cos A+\sin A}{\cos A-\sin A}$	M1 (double angles)
	$\times(\cos A-\sin A) \Rightarrow \dfrac{1+2\sin A\cos A}{\cos A+\sin A} = \cos A + \sin A$	
	$1+2\sin A\cos A = \cos^2 A + 2\sin A\cos A + \sin^2 A = 1 + 2\sin A\cos A$ True	M1 (Pythagoras) A1*
Alt 111 **Very difficult**	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \tan 2A$	(Identity) B1
	$= \dfrac{1}{\cos 2A} + \dfrac{2\tan A}{1-\tan^2 A}$	
	$= \dfrac{1-\tan^2 A + 2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$	(Single fraction) M1
	$= \dfrac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$	
	$= \dfrac{1-\dfrac{\sin^2 A}{\cos^2 A}+2\dfrac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\dfrac{\sin^2 A}{\cos^2 A}\right)}$	(Double Angle and in just sin and cos) M1
	$\times\cos^2 A = \dfrac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$	
	$= \dfrac{(\cos^2 A-\sin^2 A)(1+2\sin A\cos A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$	
	Final two marks as in main scheme	M1A1*

Q3.

Question	Scheme	Marks	AOs
	$1-\cos 2\theta \equiv \tan\theta \sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n \in \mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$= \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ or $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x = 0$ or $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ or $1+\tan^2 x-5 = 3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x = ...$	M1	1.1b
	$x = -\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Q4.

Question	Scheme	Marks	AOs
	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}$ $\left\{= \frac{2\pm\sqrt{3}}{4}\text{ or } 0.933\ldots, 0.066\ldots\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta = \text{awrt } 61.4°,\ \text{awrt } 208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme		Marks	AOs
	(ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$			
(ii) Alt 1	Complete strategy, i.e. • Attempts to apply $(5\sin\theta)^2 = (2+5\cos\theta)^2$ or $(5\sin\theta-2)^2 = (5\cos\theta)^2$ followed by applying $\cos^2\theta + \sin^2\theta = 1$ and solving a quadratic equation in either $\sin\theta$ or $\cos\theta$ to give at least one of $\sin\theta = k$ or $\cos\theta = k,\ \|k\| < 1,\ k \ne 0$		M1	3.1a
	e.g. $25\sin^2\theta = 4 + 20\cos\theta + 25\cos^2\theta$ $\Rightarrow 25(1-\cos^2\theta) = 4 + 20\cos\theta + 25\cos^2\theta$ or e.g. $25\sin^2\theta - 20\sin\theta + 4 = 25\cos^2\theta$ $\Rightarrow 25\sin^2\theta - 20\sin\theta + 4 = 25(1-\sin^2\theta)$		M1	1.1b
	$50\cos^2\theta + 20\cos\theta - 21 = 0$	$50\sin^2\theta - 20\sin\theta - 21 = 0$		
	$\cos\theta = \frac{-20 \pm \sqrt{4600}}{100}$, o.e.	$\sin\theta = \frac{20 \pm \sqrt{4600}}{100}$, o.e.	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arccos\left(\frac{-2+\sqrt{46}}{10}\right)$	e.g. $\theta = \arcsin\left(\frac{2+\sqrt{46}}{10}\right)$	dM1	1.1b
	$\theta =$ awrt $61.4°$, awrt $208.6°$		A1	2.1
			(5)	

Notes for Question

(i)	
B1:	For recalling that $\sec x = \frac{1}{\cos x}$
M1:	Correct strategy of • **Way 1:** applying $\sin 2x = 2\sin x\cos x$ and proceeding to $\sin 2x = k,\ \|k\| \le 1,\ k \ne 0$ • **Way 2:** squaring both sides, applying $\cos^2 x + \sin^2 x = 1$ and solving a quadratic equation in either $\sin^2 x$ or $\cos^2 x$ to give $\sin^2 x = k$ or $\cos^2 x = k,\ \|k\| \le 1,\ k \ne 0$
dM1:	Uses the correct order of operations to find at least one value for x in either radians or degrees
A1:	Clear reasoning to achieve both $x = \frac{\pi}{12}, \frac{5\pi}{12}$ and no other values in the range $0 \le x < \frac{\pi}{2}$
Note:	Give dM1 for $\sin 2x = \frac{1}{2} \Rightarrow$ any of $\frac{\pi}{12}, \frac{5\pi}{12}$, 15°, 75°, awrt 0.26 or awrt 1.3
Note:	Give special case, SC B1M0M0A0 for writing down any of $\frac{\pi}{12}, \frac{5\pi}{12}$, 15° or 75° with no working

Q5.

Question Number	Scheme	Marks
(a)	$\operatorname{cosec}2x+\cot 2x=\dfrac{1}{\sin 2x}+\dfrac{\cos 2x}{\sin 2x}$	M1
	$=\dfrac{1+\cos 2x}{\sin 2x}$	M1
	$=\dfrac{1+2\cos^2 x-1}{2\sin x\cos x}$	
	$=\dfrac{2\cos^2 x}{2\sin x\cos x}$	M1 A1
	$=\dfrac{\cos x}{\sin x}=\cot x$	A1*
		(5)
(b)	$\operatorname{cosec}(4\theta+10°)+\cot(4\theta+10°)=\sqrt{3}$	
	$\cot(2\theta\pm..°)=\sqrt{3}$	M1
	$2\theta\pm...=30°\Rightarrow\theta=12.5°$	dM1, A1
	$2\theta\pm...=180+PV°\Rightarrow\theta=..°$	dM1
	$\theta=102.5°$	A1
		(5)
		(10 marks)

Question Number	Scheme	Marks
(a)Alt 1	$\text{cosec}2x+\cot 2x=\frac{1}{\sin 2x}+\frac{1}{\tan 2x}$	1^{ST} M1
	$=\frac{1}{2\sin x\cos x}+\frac{1-\tan^2 x}{2\tan x}$	
	$=\frac{\tan x+(1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $=\frac{2\tan x+2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$	2^{nd} M1
	$=\frac{\tan x+\sin x\cos x-\tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x+\sin x\cos x-\tan x\sin^2 x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x(1-\sin^2 x)+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\tan x\cos^2 x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{\sin x\cos x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\frac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe	3^{rd} M1A1
	$=\frac{1}{\tan x}=\cot x$	A1* (5)
(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\text{cosec}2x+\cot 2x=\cot x \Leftrightarrow \frac{1}{\sin 2x}+\frac{1}{\tan 2x}=\frac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x\tan x+\sin 2x\tan x=\sin 2x\tan 2x$	2^{nd} M1
	$\Leftrightarrow \frac{2\tan x}{1-\tan^2 x}\times\tan x+2\sin x\cos x\times\frac{\sin x}{\cos x}=2\sin x\cos x\times\frac{2\tan x}{1-\tan^2 x}$	
	$\Leftrightarrow \frac{2\tan^2 x}{1-\tan^2 x}+2\sin^2 x=\frac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x)\Leftrightarrow 2\tan^2 x+2\sin^2 x(1-\tan^2 x)=4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x-2\sin^2 x\tan^2 x=2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x)=2\sin^2 x$	3^{rd} M1
	$\div 2\tan^2 x\Leftrightarrow 1-\sin^2 x=\cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

Q6.

Question Number	Scheme	Marks
(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x\cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = \ldots$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

Q7.

Question Number	Scheme	Marks
(i)	$\text{cosec}\,2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x\cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}\,x\sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

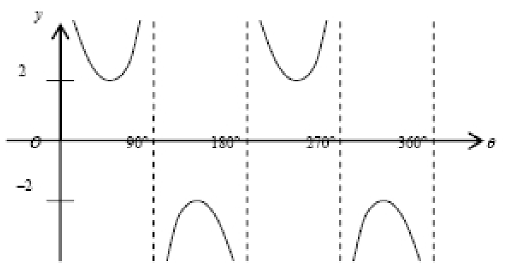
Q8.

Question Number	Scheme	Marks
(a)	$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$= \frac{\sin\theta}{\cos\theta} = \tan\theta$ cso	A1*
		(4)
(b)(i)	$\tan 15° = \frac{1}{\sin 30°} - \frac{\cos 30°}{sin 30°}$	M1
	$\tan 15° = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ cso	dM1 A1*
		(3)
(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1
		(5)
		12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°)$ *or* $\tan(45° - 30°)$	
	$\tan 15° = \frac{\tan 60 - \tan 45}{1 + \tan 60 tan 45}$ *or* $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15° = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$ *or* $\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2 - \sqrt{3}$	A1*

Q9.

Question Number	Scheme		Marks
	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2(1-2\sin^2\theta) = 1 - 2\sin\theta$	Substitutes either $1-2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \dfrac{2 \pm \sqrt{4-4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54°$ or $\alpha_2 = -18°$	Any one correct answer	A1
	$\theta = \{54, 126, 198, 342\}$	180-their pv	dM1(*)
		All four solutions correct.	A1
			[6]

Q10.

Question Number	Scheme		Marks
(a)	$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$		M1
	M1 Use of common denominator to obtain single fraction		
	$= \dfrac{1}{\cos\theta\sin\theta}$		M1
	M1 Use of appropriate trig identity (in this case $\sin^2\theta + \cos^2\theta = 1$)		
	$= \dfrac{1}{\frac{1}{2}\sin 2\theta}$	Use of $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$= 2\text{cosec}2\theta$ (✱)		A1 cso (4)
Alt.*(a)*	$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \tan\theta + \dfrac{1}{\tan\theta} = \dfrac{\tan^2\theta + 1}{\tan\theta}$	M1	
	$= \dfrac{\sec^2\theta}{\tan\theta}$	M1	
	$= \dfrac{1}{\cos\theta\sin\theta} = \dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	
	$= 2\text{cosec}2\theta$ (✱) (cso)	A1	
	If show two expressions are equal, need conclusion such as QED, tick, true.		
(b)		Shape (May be translated but need to see 4"sections")	B1
		T.P.s at $y = \pm 2$, asymptotic at correct x-values (dotted lines not required)	B1 dep. (2)

(c)	$2\text{cosec}2\theta = 3$	
	$\sin 2\theta = \frac{2}{3}$ Allow $\frac{2}{\sin 2\theta} = 3$ [M1 for equation in $\sin 2\theta$]	M1, A1
	(2θ) = [41.810…°, 138.189…° ; 401.810…°, 498.189…°] 1st M1 for $\alpha, 180 - \alpha$; 2nd M1 adding 360° to at least one of values	M1; M1
	$\theta = 20.9°, 69.1°, 200.9°, 249.1°$ (1 d.p.) awrt	
Note	1st A1 for any two correct, 2nd A1 for other two Extra solutions in range lose final A1 only SC: Final 4 marks: $\theta = 20.9°$, after M0M0 is B1; record as M0M0A1A0	A1,A1 (6)
Alt.(c)	$\tan\theta + \frac{1}{\tan\theta} = 3$ and form quadratic , $\tan^2\theta - 3\tan\theta + 1 = 0$ M1, A1 (M1 for attempt to multiply through by tanθ, A1 for correct equation above) Solving quadratic [$\tan\theta = \frac{3 \pm \sqrt{5}}{2}$ = 2.618… or = 0.3819…] M1 θ = 69.1°, 249.1° θ = 20.9°, 200.9° (1 d.p.) M1, A1, A1 (M1 is for one use of 180° + α°, A1A1 as for main scheme)	**(12 marks)**