Question 1 (Jun 2005, Q1)

Worked Solution

Find the quotient and remainder when $x^4 + 3x^3 + 5x^2 + 4x - 1$ is divided by $x^2 + x + 1$.

Polynomial long division algorithm:

At each step, divide the leading term of the current dividend by the leading term of the divisor (x^2), multiply the whole divisor by the result, and subtract.

$$\begin{array}{r r}
 & x^2 + 2x + 2 \\
x^2 + x + 1 \,\big) & x^4 + 3x^3 + 5x^2 + 4x - 1 \\
 & \underline{x^4 + x^3 + x^2} \qquad\qquad \\
 & 2x^3 + 4x^2 + 4x - 1 \\
 & \underline{2x^3 + 2x^2 + 2x} \quad\;\; \\
 & 2x^2 + 2x - 1 \\
 & \underline{2x^2 + 2x + 2} \\
 & -3
\end{array}$$

Quotient $= x^2 + 2x + 2$, Remainder $= -3$

Question 2 (Jan 2006, Q1)

Worked Solution

Simplify $\dfrac{x^3 - 3x^2}{x^2 - 9}$.

Factorise numerator and denominator.

Numerator: $x^3 - 3x^2 = x^2(x-3)$.

Denominator: $x^2 - 9$ is a difference of two squares, so use the grid to verify the factorisation:

$\times$	x	$+3$
x	x^2	$3x$
-3	$-3x$	-9

So $x^2 - 9 = (x-3)(x+3)$.

Cancel the common factor $(x-3)$:

$$\frac{x^2(x-3)}{(x-3)(x+3)} = \frac{x^2}{x+3}$$

$$\frac{x^2}{x+3}$$

Question 3 (Jun 2009, Q1)

Worked Solution

Find the quotient and remainder when $3x^4 - x^3 - 3x^2 - 14x - 8$ is divided by $x^2 + x + 2$.

Polynomial long division algorithm:

$$\begin{array}{r}
3x^2 - 4x - 5 \\
x^2 + x + 2 \;\big)\; 3x^4 - x^3 - 3x^2 - 14x - 8 \\
\underline{3x^4 + 3x^3 + 6x^2 \qquad\qquad\quad} \\
-4x^3 - 9x^2 - 14x - 8 \\
\underline{-4x^3 - 4x^2 - 8x \quad\;\;} \\
-5x^2 - 6x - 8 \\
\underline{-5x^2 - 5x - 10} \\
-x + 2
\end{array}$$

Quotient $= 3x^2 - 4x - 5$, Remainder $= -x + 2$

Question 4 (Jun 2007, Q7)

Worked Solution

Part (i): Divide $2x^3 + 3x^2 + 9x + 12$ by $x^2 + 4$.

Note that the divisor has no x term; keep a placeholder column for it.

Polynomial long division algorithm:

$$\begin{array}{r} 2x + 3 \\ x^2 + 0x + 4 \;\big)\; \overline{2x^3 + 3x^2 + 9x + 12} \\ \underline{2x^3 + 0x^2 + 8x } \\ 3x^2 + x + 12 \\ \underline{3x^2 + 0x + 12} \\ x \end{array}$$

Quotient $= 2x + 3$, Remainder $= x$

Part (ii): Express as $Ax + B + \dfrac{Cx + D}{x^2 + 4}$.

Using the quotient and remainder from part (i):

$$\frac{2x^3 + 3x^2 + 9x + 12}{x^2 + 4} = 2x + 3 + \frac{x}{x^2 + 4}$$

$A = 2, \; B = 3, \; C = 1, \; D = 0$

Question 5 (Jan 2008, Q3)

Worked Solution

When $x^4 - 2x^3 - 7x^2 + 7x + a$ is divided by $x^2 + 2x - 1$, the quotient is $x^2 + bx + 2$ and the remainder is $cx + 7$. Find a, b, c.

Use the grid method to multiply the quotient by the divisor, then add the remainder.

Write the identity:

$$x^4 - 2x^3 - 7x^2 + 7x + a \equiv (x^2 + 2x - 1)(x^2 + bx + 2) + cx + 7$$

Use the grid to expand $(x^2 + 2x - 1)(x^2 + bx + 2)$:

$\times$	x^2	$+bx$	$+2$
x^2	x^4	bx^3	$2x^2$
$+2x$	$2x^3$	$2bx^2$	$4x$
-1	$-x^2$	$-bx$	-2

Collecting columns:

$$\begin{aligned} &x^4 : x^4 \\ &x^3 : bx^3 + 2x^3 = (b+2)x^3 \\ &x^2 : 2x^2 + 2bx^2 - x^2 = (2b+1)x^2 \\ &x^1 : 4x - bx = (4-b)x \\ &x^0 : -2 \end{aligned}$$

After adding remainder $cx + 7$, the full right-hand side is:

$$x^4 + (b+2)x^3 + (2b+1)x^2 + (4 - b + c)x + 5$$

Compare coefficients with $x^4 - 2x^3 - 7x^2 + 7x + a$:

x^3: $b + 2 = -2 \implies \mathbf{b = -4}$

x^2: $2(-4) + 1 = -7$ ✓

x^1: $4 - (-4) + c = 7 \implies 8 + c = 7 \implies \mathbf{c = -1}$

constant: $\mathbf{a = 5}$

$a = 5, \quad b = -4, \quad c = -1$

Question 6 (Jun 2008, Q1)

Worked Solution

Part (a): Simplify $\dfrac{(2x^2-7x-4)(x+1)}{(3x^2+x-2)(x-4)}$**.**

Factorise each quadratic using the grid method.

Factorise $2x^2-7x-4$**:** $ac=-8$; numbers: -8 and $+1$.

$\times$	$2x$	$+1$
x	$2x^2$	x
-4	$-8x$	-4

$\implies 2x^2-7x-4=(2x+1)(x-4)$

Factorise $3x^2+x-2$**:** $ac=-6$; numbers: $+3$ and -2.

$\times$	$3x$	-2
x	$3x^2$	$-2x$
$+1$	$3x$	-2

$\implies 3x^2+x-2=(3x-2)(x+1)$

Cancel the common factors $(x+1)$ **and** $(x-4)$**:**

$$\frac{(2x+1)(x-4)(x+1)}{(3x-2)(x+1)(x-4)}=\frac{2x+1}{3x-2}$$

$$\frac{2x+1}{3x-2}$$

Part (b): Divide x^3+2x^2-6x-5 **by** x^2+4x+1**.**

Polynomial long division algorithm:

$$\begin{array}{r}
x-2 \\
x^2+4x+1\ \overline{)\ x^3+2x^2-6x-5} \\
\underline{x^3+4x^2+x} \\
-2x^2-7x-5 \\
\underline{-2x^2-8x-2} \\
x-3
\end{array}$$

Quotient $= x - 2$, Remainder $= x - 3$

Question 7 (Jun 2009, Q1)

Worked Solution

Find the quotient and remainder when $x^4+11x^3+28x^2+3x+1$ is divided by x^2+5x+2.

Polynomial long division algorithm:

$$\begin{array}{r} x^2 + 6x - 4 \\ x^2+5x+2 \enclose{longdiv}{x^4 + 11x^3 + 28x^2 + 3x + 1} \\ \underline{x^4 + 5x^3 + 2x^2 } \\ 6x^3 + 26x^2 + 3x + 1 \\ \underline{6x^3 + 30x^2 + 12x } \\ -4x^2 - 9x + 1 \\ \underline{-4x^2 - 20x - 8} \\ 11x + 9 \end{array}$$

Quotient $= x^2 + 6x - 4$, Remainder $= 11x + 9$

Question 8 (Jun 2014, Q1)

Worked Solution

Express $x + \dfrac{1}{1-x} + \dfrac{2}{1+x}$ as a single fraction, simplifying your answer.

The common denominator of all three terms is $(1-x)(1+x) = 1 - x^2$.

Write $x = \dfrac{x(1-x)(1+x)}{(1-x)(1+x)}$, then combine:

$$\frac{x(1-x^2) + (1+x) + 2(1-x)}{(1-x)(1+x)}$$

Expand the numerator:

$$x - x^3 + 1 + x + 2 - 2x = 3 - x^3$$

$$\frac{3 - x^3}{1 - x^2}$$

Question 9 (Jun 2015, Q1)

Worked Solution

Part (i): Express $\frac{2}{3-x} + \frac{3}{1+x}$ as a single fraction.

Common denominator: $(3-x)(1+x)$.

$$\frac{2(1+x)+3(3-x)}{(3-x)(1+x)} = \frac{2+2x+9-3x}{(3-x)(1+x)} = \frac{11-x}{(3-x)(1+x)}$$

$$\frac{11-x}{(3-x)(1+x)}$$

Part (ii): Express $\left(\frac{2}{3-x} + \frac{3}{1+x}\right) \times \frac{x^2+8x-33}{121-x^2}$ in lowest terms.

Using the result of part (i), the expression becomes:

$$\frac{11-x}{(3-x)(1+x)} \times \frac{x^2+8x-33}{121-x^2}$$

Factorise $x^2+8x-33$ using the grid method.

Numbers multiplying to -33 and adding to $+8$: $+11$ and -3.

$\times$	x	$+11$
x	x^2	$11x$
-3	$-3x$	-33

$\implies x^2+8x-33 = (x+11)(x-3)$

Factorise $121-x^2$: this is a difference of two squares:

$$121-x^2 = (11-x)(11+x)$$

Substitute and cancel. Note that $x-3 = -(3-x)$ and $x+11 = 11+x$:

$$\frac{(11-x)}{(3-x)(1+x)} \times \frac{(x+11)(x-3)}{(11-x)(11+x)} = \frac{(11-x)(x+11)(x-3)}{(3-x)(1+x)(11-x)(11+x)}$$

Cancel $(11-x)$ and $(x+11) = (11+x)$, then use $x-3 = -(3-x)$:

$$= \frac{x-3}{(3-x)(1+x)} = \frac{-(3-x)}{(3-x)(1+x)} = \frac{-1}{1+x}$$

$$\frac{-1}{1+x}$$

End of Worked Solutions