Question 1

Worked Solution

Express $\dfrac{2(3x+2)}{9x^2-4} - \dfrac{2}{3x+1}$ as a single fraction in its simplest form.

Step 1: Factorise the denominator using difference of two squares.

$$9x^2 - 4 = (3x-2)(3x+2)$$

Step 2: Cancel the common factor $(3x+2)$ in the first fraction.

$$\frac{2(3x+2)}{(3x-2)(3x+2)} = \frac{2}{3x-2}$$

Step 3: Combine over the common denominator $(3x-2)(3x+1)$.

$$\frac{2}{3x-2} - \frac{2}{3x+1} = \frac{2(3x+1)-2(3x-2)}{(3x-2)(3x+1)} = \frac{6x+2-6x+4}{(3x-2)(3x+1)} = \frac{6}{(3x-2)(3x+1)}$$

$$\frac{6}{(3x-2)(3x+1)}$$

Question 2

Worked Solution

Express $\dfrac{4x}{x^2-9} - \dfrac{2}{x+3}$ as a single fraction in its simplest form.

Step 1: Factorise the denominator using difference of two squares.

$$x^2 - 9 = (x+3)(x-3)$$

Step 2: Combine over the common denominator $(x+3)(x-3)$.

$$\frac{4x}{(x+3)(x-3)} - \frac{2(x-3)}{(x+3)(x-3)} = \frac{4x-2(x-3)}{(x+3)(x-3)} = \frac{2x+6}{(x+3)(x-3)}$$

Step 3: Factorise the numerator and cancel.

$$\frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$$

$$\frac{2}{x-3}$$

Question 3

Worked Solution

Express $\dfrac{3x+5}{x^2+x-12} - \dfrac{2}{x-3}$ as a single fraction in its simplest form.

Step 1: Factorise the denominator x^2+x-12 using the grid method.

We need two numbers that multiply to -12 and add to $+1$: these are $+\mathbf{4}$ and $-\mathbf{3}$.

$\times$	x	$+4$
x	x^2	$4x$
-3	$-3x$	-12

So $x^2+x-12=(x+4)(x-3)$.

Step 2: Combine over the common denominator $(x+4)(x-3)$.

$$\frac{3x+5}{(x+4)(x-3)} - \frac{2(x+4)}{(x+4)(x-3)} = \frac{3x+5-2(x+4)}{(x+4)(x-3)} = \frac{x-3}{(x+4)(x-3)}$$

Step 3: Cancel the common factor $(x-3)$.

$$= \frac{1}{x+4}$$

$$\frac{1}{x+4}$$

Question 4

Worked Solution

Express $\dfrac{x+1}{3x^2-3} - \dfrac{1}{3x+1}$ as a single fraction in its simplest form.

Step 1: Factorise the denominator of the first fraction.

$$3x^2 - 3 = 3(x^2-1) = 3(x+1)(x-1)$$

Step 2: Cancel the common factor $(x+1)$.

$$\frac{x+1}{3(x+1)(x-1)} = \frac{1}{3(x-1)}$$

Step 3: Combine over the common denominator $3(x-1)(3x+1)$.

$$\frac{1}{3(x-1)} - \frac{1}{3x+1} = \frac{(3x+1)-3(x-1)}{3(x-1)(3x+1)} = \frac{3x+1-3x+3}{3(x-1)(3x+1)} = \frac{4}{3(x-1)(3x+1)}$$

$$\frac{4}{3(x-1)(3x+1)}$$

Question 5

Worked Solution

Show that $\mathrm{h}(x) = \dfrac{2}{x+2} + \dfrac{4}{x^2+5} - \dfrac{18}{(x^2+5)(x+2)} = \dfrac{2x}{x^2+5}$ for $x \geq 0$.

Step 1: Combine all three fractions over the common denominator $(x+2)(x^2+5)$.

$$\mathrm{h}(x) = \frac{2(x^2+5) + 4(x+2) - 18}{(x+2)(x^2+5)}$$

Step 2: Expand and simplify the numerator.

$$2x^2 + 10 + 4x + 8 - 18 = 2x^2 + 4x = 2x(x+2)$$

Step 3: Cancel the common factor $(x+2)$.

$$\frac{2x(x+2)}{(x+2)(x^2+5)} = \frac{2x}{x^2+5}$$

$\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ ✓

Question 6

Worked Solution

Show that $\mathrm{f}(x) = \dfrac{2(x-1)}{x^2-2x-3} - \dfrac{1}{x-3} = \dfrac{1}{x+1}$ for $x > 3$.

Step 1: Factorise $x^2 - 2x - 3$ using the grid method.

Numbers multiplying to -3 and adding to -2: $\mathbf{-3}$ and $\mathbf{+1}$.

$\times$	x	-3
x	x^2	$-3x$
$+1$	x	-3

So $x^2 - 2x - 3 = (x-3)(x+1)$.

Step 2: Combine over the common denominator $(x-3)(x+1)$.

$$\frac{2(x-1)}{(x-3)(x+1)} - \frac{(x+1)}{(x-3)(x+1)} = \frac{2x-2-x-1}{(x-3)(x+1)} = \frac{x-3}{(x-3)(x+1)}$$

Step 3: Cancel the common factor $(x-3)$.

$\mathrm{f}(x) = \dfrac{1}{x+1}$ ✓

Question 7

Worked Solution

Show that $f(x) = \dfrac{3(x+1)}{2x^2+7x-4} - \dfrac{1}{x+4} = \dfrac{1}{2x-1}$ for $x > \frac{1}{2}$.

Step 1: Factorise $2x^2+7x-4$ using the grid method.

For ax^2+bx+c with $a=2$, $c=-4$: find two numbers multiplying to $ac=-8$ and adding to $b=7$. These are $+\mathbf{8}$ and $-\mathbf{1}$.

Arrange these as the cross-terms in the grid. We factor out from columns to get the row and column headers:

$\times$	$2x$	-1
x	$2x^2$	$-x$
$+4$	$8x$	-4

Reading off: $2x^2+7x-4=(2x-1)(x+4)$.

Step 2: Combine over the common denominator $(2x-1)(x+4)$.

$$\frac{3(x+1)}{(2x-1)(x+4)} - \frac{(2x-1)}{(2x-1)(x+4)} = \frac{3x+3-2x+1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)}$$

Step 3: Cancel the common factor $(x+4)$.

$$f(x) = \frac{1}{2x-1} \quad ✓$$

Question 8

Worked Solution

Part (a): Simplify $\dfrac{2x^2+9x-5}{x^2+2x-15}$.

Factorise the numerator $2x^2+9x-5$ **using the grid method.**

Find two numbers multiplying to $ac = 2 \times (-5) = -10$ and adding to 9: these are $+\mathbf{10}$ and $-\mathbf{1}$.

$\times$	$2x$	-1
x	$2x^2$	$-x$
$+5$	$10x$	-5

So $2x^2+9x-5 = (2x-1)(x+5)$.

Factorise the denominator $x^2+2x-15$ **using the grid method.**

Numbers multiplying to -15 and adding to $+2$: $+\mathbf{5}$ and $-\mathbf{3}$.

$\times$	x	$+5$
x	x^2	$5x$
-3	$-3x$	-15

So $x^2+2x-15 = (x+5)(x-3)$.

Cancel the common factor $(x+5)$:

$$\frac{(2x-1)(x+5)}{(x+5)(x-3)} = \frac{2x-1}{x-3}$$

$$\frac{2x-1}{x-3}$$

Part (b): Find x **in terms of** e**, given** $\ln(2x^2+9x-5) = 1 + \ln(x^2+2x-15)$**,** $x \neq -5$**.**

Rearrange using log laws:

$$\ln(2x^2+9x-5) - \ln(x^2+2x-15) = 1 \implies \ln\left(\frac{2x^2+9x-5}{x^2+2x-15}\right) = 1$$

Substituting the simplified form from part (a):

$$\ln\left(\frac{2x-1}{x-3}\right) = 1 \implies \frac{2x-1}{x-3} = e$$

Solve for x:

$$2x - 1 = e(x-3) \implies 2x - ex = 1 - 3e \implies x(2-e) = 1 - 3e$$

$$x = \frac{1-3e}{2-e} = \frac{3e-1}{e-2}$$

End of Worked Solutions