Rational Functions Exam Questions Sheet 2 Mark Scheme

Q1.

Question Number	Scheme	Marks
	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of $(3x+2)$ at any stage $\frac{2(3x+2)}{(3x-2)(3x+2)}=\frac{2}{3x-2}$	B1
	Use of a common denominator $\frac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\frac{2(3x+1)}{(3x-2)(3x+1)}-\frac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\frac{6}{(3x-2)(3x+1)}$ or $\frac{6}{9x^2-3x-2}$	A1
		(4 marks)

Notes

B1 For factorising $9x^2-4=(3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark

B1 For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.

M1 For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\frac{2(3x+2)}{(9x^2-4)(3x+1)}-\frac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ Only one numerator adapted, separate fractions

$\frac{2\times3x+1-2\times3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction

A1 $\frac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative method

$\frac{2(3x+2)}{(9x^2-4)}-\frac{2}{(3x+1)}=\frac{2(3x+2)(3x+1)-2(9x^2-4)}{(9x^2-4)(3x+1)}=\frac{18x+12}{(9x^2-4)(3x+1)}$ has scored 0,0,1,0 so far

$=\frac{6(3x+2)}{(3x+2)(3x-2)(3x+1)}$ is now 1,1,1,0

$=\frac{6}{(3x-2)(3x+1)}$ and now 1,1,1,1

Q2.

Question Number	Scheme	Marks
	$x^2-9=(x+3)(x-3)$	B1
	$\frac{4x}{x^2-9}-\frac{2}{(x+3)}=\frac{4x-2(x-3)}{(x+3)(x-3)}$	M1
	$=\frac{2x+6}{(x+3)(x-3)}$	A1
	$=\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$=\frac{2}{(x-3)}$	A1
		(4)

B1 $x^2-9=(x+3)(x-3)$ This can occur anywhere.

M1 For combining the two fractions with a common denominator. The denominator must be correct and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

For example accept $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2(x^2-9)}{(x+3)(x^2-9)}$

accept separately $\frac{4x}{(x+3)(x-3)}-\frac{2}{(x+3)}=\frac{4x}{(x+3)(x-3)}-\frac{2x-3}{(x+3)(x-3)}$ condoning missing bracket

condone $\frac{4x}{x^2-9}-\frac{2}{x+3}=\frac{4x(x+3)-2}{(x+3)(x^2-9)}$ .....as only one numerator has been adapted

A1 A correct intermediate form of $\frac{\text{simplified linear}}{\text{simplified quadratic}}$

Accept $\frac{2x+6}{(x+3)(x-3)}$, $\frac{2x+6}{x^2-9}$, and even $\frac{(2x+6)\cancel{(x+3)}}{(x^2-9)\cancel{(x+3)}}$,

A1 Further factorises and cancels (which may be implied) to reach the answer $\frac{2}{x-3}$

Do not penalise correct solutions that include incomplete lines Eg $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{4x-2x+6}{\ldots}=\frac{2x+6}{(x+3)(x-3)}=\frac{2}{x-3}$

This is not a "show that" question.

Note: Watch out for an answer of $\frac{2}{x+3}$ probably scored from $\frac{4x-2(x-3)}{(x+3)(x-3)}=\frac{2x-6}{(x+3)(x-3)}=\frac{2(x-3)}{(x+3)(x-3)}$

This would score B1 M1 A0 A0

Q3.

Question Number	Scheme	Marks
	(a) $x^2+x-12=(x+4)(x-3)$	B1
	Attempt as a single fraction $\dfrac{(3x+5)(x-3)-2(x^2+x-12)}{(x^2+x-12)(x-3)}$ or $\dfrac{3x+5-2(x+4)}{(x+4)(x-3)}$	M1
	$=\dfrac{x-3}{(x+4)(x-3)}$, $=\dfrac{1}{(x+4)}$ cao	A1, A1
		(4 marks)

Notes for Question

B1 For correctly factorising $x^2+x-12=(x+4)(x-3)$. It could appear anywhere in their solution

M1 For an attempt to combine two fractions. The denominator must be correct for 'their' fractions.
The terms could be separate but one term must have been modified.
Condone invisible brackets.
Examples of work scoring this mark are;

$\dfrac{(3x+5)(x-3)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Two separate terms

$\dfrac{3x+5-2x+4}{(x+4)(x-3)}$ Single term, invisible bracket

$\dfrac{(3x+5)}{(x^2+x-12)(x-3)}-\dfrac{2(x^2+x-12)}{(x^2+x-12)(x-3)}$ Separate terms, only one numerator modified

A1 Correct un simplified answer $\dfrac{x-3}{(x+4)(x-3)}$

If $\dfrac{x^2-6x-9}{(x^2+x-12)(x-3)}$ scored M1 the fraction must be subsequently be reduced to a correct $\dfrac{x-3}{x^2+x-12}$ or $\dfrac{(x-3)(x-3)}{(x+4)(x-3)(x-3)}$ to score this mark.

A1 cao $\dfrac{1}{(x+4)}$

Do Not isw in this question.

The method of partial fractions is perfectly acceptable and can score full marks

$$\underbrace{\frac{3x+5}{(x+4)(x-3)}}_{B1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}+\frac{2}{x-3}}_{M1A1}-\frac{2}{x-3}=\underbrace{\frac{1}{x+4}}_{A1}$$

Q4.

Question Number	Scheme	Marks
	$\frac{x+1}{3x^2-3}-\frac{1}{3x+1}$	
	$=\frac{x+1}{3(x^2-1)}-\frac{1}{3x+1}$	
	$=\frac{x+1}{3(x+1)(x-1)}-\frac{1}{3x+1}$ $x^2-1 \rightarrow (x+1)(x-1)$ or $3x^2-3 \rightarrow (x+1)(3x-3)$ or $3x^2-3 \rightarrow (3x+3)(x-1)$ seen or implied anywhere in candidate's working.	**Award below**
	$=\frac{1}{3(x-1)}-\frac{1}{3x+1}$	
	$=\frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$ Attempt to combine.	M1
	or $\frac{3x+1}{3(x-1)(3x+1)}-\frac{3(x-1)}{3(x-1)(3x+1)}$ Correct result.	A1
	Decide to award M1 here!!	**M1**
	$=\frac{4}{3(x-1)(3x+1)}$ Either $\frac{4}{3(x-1)(3x+1)}$ or $\frac{\frac{4}{3}}{(x-1)(3x+1)}$ or $\frac{4}{(3x-3)(3x+1)}$ or $\frac{4}{9x^2-6x-3}$	A1 **aef**
		[4]

Q5.

Question Number	Scheme	Marks
	(a) $\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\frac{2x}{(x^2+5)}$	A1*
		(4)

Q6.

Question Number	Scheme	Marks
(a)	$x^2-2x-3=(x-3)(x+1)$	B1
	$f(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)}\right)$	M1 A1
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ *	A1 cso (4)

Q7.

Question No	Scheme	Marks
	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1* (4)

Q8.

Question Number	Scheme	Marks
(a)	$\dfrac{(x+5)(2x-1)}{(x+5)(x-3)}=\dfrac{(2x-1)}{(x-3)}$	M1 B1 A1 aef (3)
(b)	$\ln\left(\dfrac{2x^2+9x-5}{x^2+2x-15}\right)=1$	M1
	$\dfrac{2x^2+9x-5}{x^2+2x-15}=e$	dM1
	$\dfrac{2x-1}{x-3}=e \Rightarrow \quad 3e-1=x(e-2)$	M1
	$\Rightarrow\ x=\dfrac{3e-1}{e-2}$	A1 aef cso (4) [7]