Question 1 (Jun 2006, Q7)

Worked Solution

$AB = 11$ cm, $BC = 8$ cm, $\angle ABC = 0.8$ rad, $\angle DAC = 1.7$ rad, centre A.

Part (i): Show $AC = 7.90$ cm

Using the cosine rule in triangle ABC:

$$AC^2 = AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos(\angle ABC) = 11^2 + 8^2 - 2(11)(8)\cos(0.8)$$

$$= 121 + 64 - 176\cos(0.8) = 185 - 176 \times 0.6967 \approx 185 - 122.62 = 62.38$$

$$AC = \sqrt{62.38} \approx 7.898 \approx 7.90 \text{ cm} \quad \checkmark$$

Part (ii): Area of shaded segment

The shaded segment is bounded by chord DC and arc DC, where the sector ACD has radius $AC = 7.90$ cm and angle $\angle DAC = 1.7$ rad.

Area of sector ACD:

$$= \frac{1}{2}r^2\theta = \frac{1}{2} \times 7.90^2 \times 1.7 = \frac{1}{2} \times 62.41 \times 1.7 \approx 53.0 \text{ cm}^2$$

Area of triangle ACD (using $\frac{1}{2}r^2\sin\theta$):

$$= \frac{1}{2} \times 7.90^2 \times \sin(1.7) \approx \frac{1}{2} \times 62.41 \times 0.9917 \approx 30.9 \text{ cm}^2$$

Area of shaded segment $= 53.0 - 30.9 = 22.1$ cm^2

Area of shaded segment $= 22.1$ cm^2 (3 s.f.)

Part (iii): Perimeter of shaded segment

The perimeter consists of arc DC and chord DC.

Arc $DC = r\theta = 7.90 \times 1.7 = 13.4$ cm.

Chord DC by cosine rule in triangle ACD ($AC = AD = 7.90$ cm, angle $= 1.7$ rad):

$$DC^2 = 7.90^2 + 7.90^2 - 2(7.90)^2\cos(1.7) = 2(62.41)(1 - \cos 1.7) \approx 2(62.41)(1 - (-0.1288)) \approx 140.7$$

$$DC \approx 11.86 \text{ cm}$$

Perimeter $= 13.4 + 11.86 \approx 25.3$ cm.

Perimeter of shaded segment ≈ 25.3 cm (3 s.f.)

Question 2 (Jan 2007, Q2)

Worked Solution

Sector OAB: centre O, radius 8 cm, $\angle AOB = 46°$.

Part (i): Express 46° in radians

$$46 \times \frac{\pi}{180} = \frac{46\pi}{180} = \frac{23\pi}{90} \approx 0.803 \text{ rad}$$

$46° = 0.803$ rad (3 s.f.)

Part (ii): Length of arc AB

$$\text{Arc } AB = r\theta = 8 \times 0.803 = 6.42 \text{ cm}$$

Arc $AB = 6.4$ cm (2 s.f.)

Part (iii): Area of sector OAB

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 64 \times 0.803 \approx 25.7 \text{ cm}^2$$

Area of sector $OAB \approx 25.7 \text{ cm}^2$

Question 3 (Jun 2008, Q3)

Worked Solution

Sector AOB: centre O, radius 8 cm, area = 48 cm^2.

Part (i): Find angle AOB

$$\frac{1}{2}r^2\theta = 48 \implies \frac{1}{2}(64)\theta = 48 \implies 32\theta = 48 \implies \theta = \frac{48}{32} = 1.5 \text{ rad}$$

$\angle AOB = 1.5$ rad

Part (ii): Area of segment

Area of segment = area of sector − area of triangle OAB:

$$= 48 - \frac{1}{2}r^2 \sin\theta = 48 - \frac{1}{2}(64)\sin(1.5) = 48 - 32\sin(1.5) \approx 48 - 32(0.9975) \approx 48 - 31.9 = 16.1 \text{ cm}^2$$

Area of segment ≈ 16.1 cm^2

Question 4 (Jun 2010, Q5)

Worked Solution

Two congruent triangles BCD and BAE; $BD = 8$ cm, $CD = 11$ cm, $\angle CBD = 65°$. Arc ED: centre B, radius 8 cm.

Part (i): Find angle BCD

Using the sine rule in triangle BCD:

$$\frac{\sin(\angle BCD)}{BD} = \frac{\sin(\angle CBD)}{CD} \implies \sin(\angle BCD) = \frac{8\sin 65°}{11} = \frac{8 \times 0.9063}{11} \approx 0.6591$$

$$\angle BCD = \arcsin(0.6591) \approx 41.2°$$

$\angle BCD \approx 41.2°$

Part (ii)(a): Show $\angle EBD = 0.873$ rad

$\angle BDC = 180° - 65° - 41.2° = 73.8°$

Since triangles BCD and BAE are congruent with A, B, C collinear: $\angle EBA = \angle DBC = 65°$

$\angle EBD = 180° - 2 \times 65° = 50°$

Converting: $50 \times \frac{\pi}{180} = \frac{5\pi}{18} \approx 0.873$ rad ✓

Part (ii)(b): Area of shaded segment

The shaded segment is bounded by chord ED and arc ED.

Area of sector BED:

$$= \frac{1}{2}r^2\theta = \frac{1}{2}(64)(0.873) \approx 27.9 \text{ cm}^2$$

Area of triangle BED (isosceles, $BE = BD = 8$ cm, angle 0.873 rad):

$$= \frac{1}{2}(64)\sin(0.873) \approx 32 \times 0.7672 \approx 24.5 \text{ cm}^2$$

Area of segment $= 27.9 - 24.5 = 3.41$ cm^2

Area of shaded segment ≈ 3.41 cm^2 (3 s.f.)

Question 5 (Jan 2013, Q7)

Worked Solution

Two circles of radius 7 cm with centres A and B; $AB = 12$ cm; C lies on both circles.

Part (i): Show $\angle CAB = 0.5411$ rad

In triangle CAB: $CA = 7$ cm (radius), $AB = 12$ cm, $CB = 7$ cm.

Using the cosine rule:

$$\cos(\angle CAB) = \frac{CA^2 + AB^2 - CB^2}{2 \cdot CA \cdot AB} = \frac{49 + 144 - 49}{2(7)(12)} = \frac{144}{168} = \frac{6}{7}$$

$$\angle CAB = \arccos\left(\frac{6}{7}\right) = 0.5411 \text{ rad} \quad \checkmark$$

Part (ii): Perimeter of shaded region

By symmetry, the shaded region is a lens shape. There are two arcs, each of radius 7 and angle $2 \times 0.5411 = 1.0822$ rad.

Arc length $= r\theta = 7 \times 1.0822 = 7.575$ cm each.

Perimeter $= 2 \times 7.575 = 15.2$ cm.

Perimeter of shaded region $= 15.2$ cm (3 s.f.)

Part (iii): Area of shaded region

Each half of the shaded region is a circular segment from each circle.

Area of one segment = area of sector − area of triangle:

$$= \frac{1}{2}(7^2)(1.0822) - \frac{1}{2}(7^2)\sin(1.0822) = \frac{49}{2}(1.0822 - \sin(1.0822))$$

$$= 24.5(1.0822 - 0.8819) \approx 24.5 \times 0.2003 \approx 4.907 \text{ cm}^2$$

Total shaded area $= 2 \times 4.907 \approx 9.81$ cm^2.

Area of shaded region ≈ 9.81 cm^2 (3 s.f.)

Question 6 (Jun 2014, Q3)

Worked Solution

Sector OAB: centre O, radius 12 cm, $\angle AOB = \frac{2\pi}{3}$ rad.

Part (i): Exact length of arc AB

$$\text{Arc } AB = r\theta = 12 \times \frac{2\pi}{3} = 8\pi \text{ cm}$$

Arc $AB = 8\pi$ cm

Part (ii): Exact area of shaded segment

Area of sector:

$$= \frac{1}{2}r^2\theta = \frac{1}{2}(144)\left(\frac{2\pi}{3}\right) = 48\pi \text{ cm}^2$$

Area of triangle OAB:

$$= \frac{1}{2}r^2 \sin\theta = \frac{1}{2}(144)\sin\left(\frac{2\pi}{3}\right) = 72 \times \frac{\sqrt{3}}{2} = 36\sqrt{3} \text{ cm}^2$$

Area of segment $= 48\pi - 36\sqrt{3}$ cm^2

Area of segment $= 48\pi - 36\sqrt{3} \approx 88.4$ cm^2

Question 7 (Jun 2016, Q2)

Worked Solution

Sector AOB: centre O, radius r cm, $\angle AOB = 54°$, perimeter = 60 cm.

Part (i): Express 54° in radians exactly

$$54° \times \frac{\pi}{180} = \frac{54\pi}{180} = \frac{3\pi}{10} \text{ rad}$$

$$54° = \frac{3\pi}{10} \text{ rad}$$

Part (ii): Find r

Perimeter of sector $= 2r + r\theta = r(2+\theta)$:

$$r\left(2 + \frac{3\pi}{10}\right) = 60 \implies r = \frac{60}{2 + \frac{3\pi}{10}} = \frac{60}{\frac{20+3\pi}{10}} = \frac{600}{20 + 3\pi}$$

$$r \approx \frac{600}{20 + 9.425} = \frac{600}{29.425} \approx 20.4 \text{ cm}$$

$r \approx 20.4$ cm (3 s.f.)

End of Worked Solutions