Radians, Circle Sectors and Triangles Exam Questions (Sheet 2)

Q1.

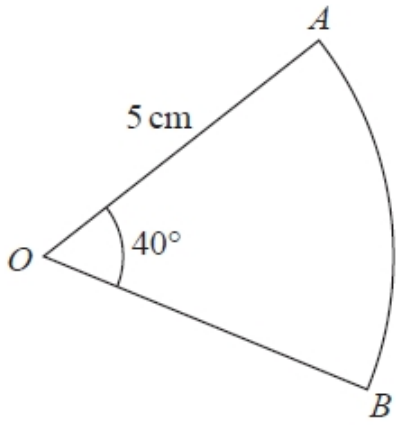

Figure 1

Figure 1 shows a sector *AOB* of a circle with centre *O*, radius 5 cm and angle *AOB* = 40°

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times 5^2 \times 40$$

$$= 500\,\text{cm}^2$$

(a) Explain the error made by this student.

(1)

(b) Write out a correct solution.

(2)

(Total for question = 3 marks)

Q2.

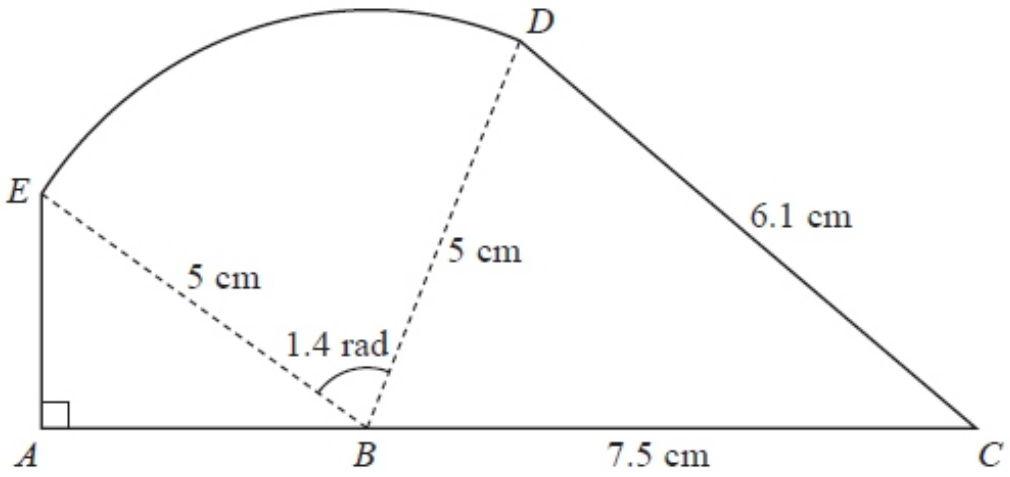

Figure 2

The shape *ABCDEA*, as shown in Figure 2, consists of a right-angled triangle *EAB* and a triangle *DBC* joined to a sector *BDE* of a circle with radius 5 cm and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *BC* = 7.5 cm.

Angle *EAB* = $\pi/2$ radians, angle *EBD* = 1.4 radians and *CD* = 6.1 cm.

(a) Find, in cm^2, the area of the sector *BDE*.

(2)

(b) Find the size of the angle *DBC*, giving your answer in radians to 3 decimal places.

(2)

(c) Find, in cm^2, the area of the shape *ABCDEA*, giving your answer to 3 significant figures.

(5)

(Total 9 marks)

Q3.

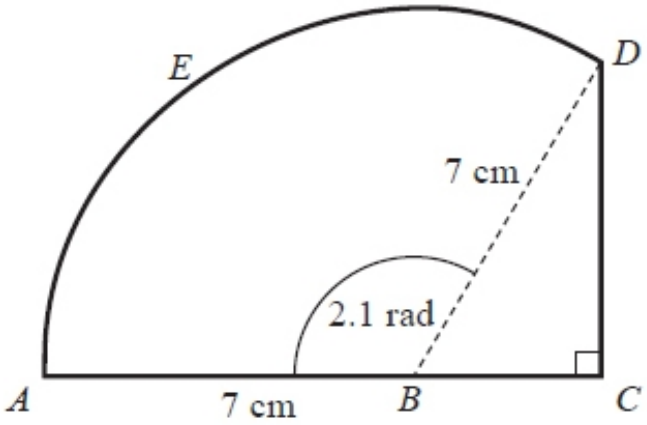

Figure 2

Figure 2 shows the shape *ABCDEA* which consists of a right-angled triangle *BCD* joined to a sector *ABDEA* of a circle with radius 7 cm and centre *B*.

A, *B* and *C* lie on a straight line with *AB* = 7 cm.

Given that the size of angle *ABD* is exactly 2.1 radians,

(a) find, in cm, the length of the arc *DEA*,

(2)

(b) find, in cm, the perimeter of the shape *ABCDEA*, giving your answer to 1 decimal place.

(4)

(Total 6 marks)

Q4.

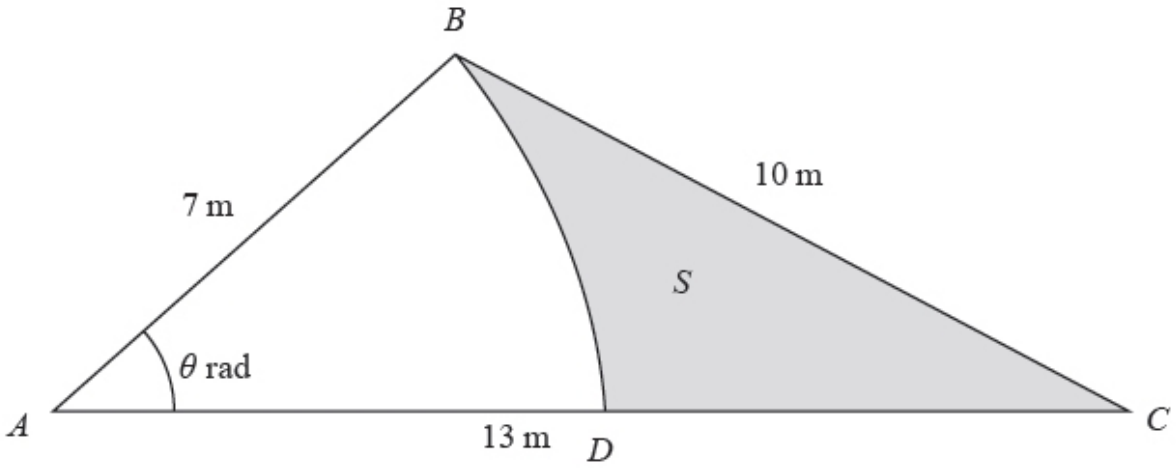

Figure 2

Figure 2 shows the design for a triangular garden ABC where $AB = 7$ m, $AC = 13$ m and $BC = 10$ m.

Given that angle $BAC = \theta$ radians,

(a) show that, to 3 decimal places, $\theta = 0.865$

(3)

The point D lies on AC such that BD is an arc of the circle centre A, radius 7 m.

The shaded region S is bounded by the arc BD and the lines BC and DC. The shaded region S will be sown with grass seed, to make a lawned area.

Given that 50 g of grass seed are needed for each square metre of lawn,

(b) find the amount of grass seed needed, giving your answer to the nearest 10 g.

(7)

(Total 10 marks)

Q5.

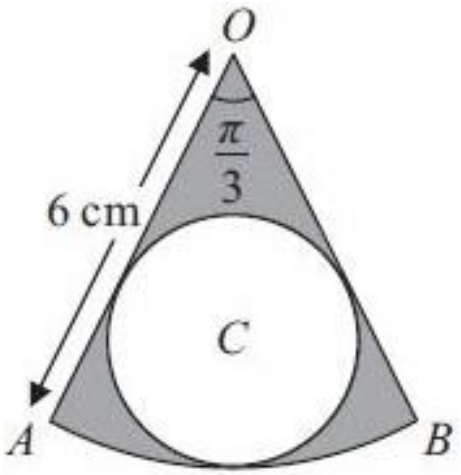

Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(a) the area of the sector OAB,

(2)

(b) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

(Total 7 marks)

Q6.

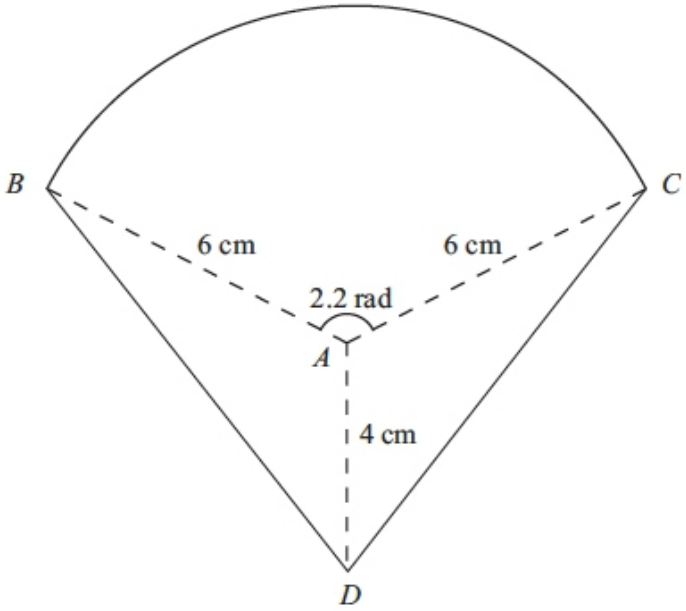

Figure 3

The shape *BCD* shown in Figure 3 is a design for a logo.
The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm.
The size of $\angle BAC$ is 2.2 radians and *AD* = 4 cm.
Find
(a) the area of the sector *BAC*, in cm^2,

(2)

(b) the size of $\angle DAC$, in radians to 3 significant figures,

(2)

(c) the complete area of the logo design, to the nearest cm^2.

(4)

(Total 8 marks)

Q7.

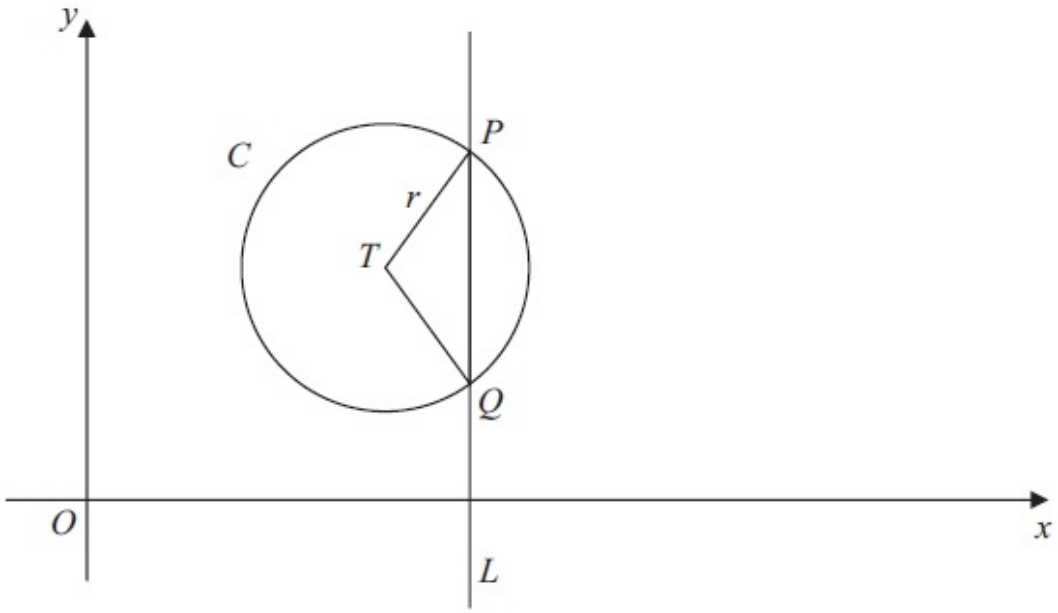

Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,(d) find the perimeter of the sector PTQ.

(3)

(Total 11 marks)

Q8.

A circle C has centre M (6, 4) and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

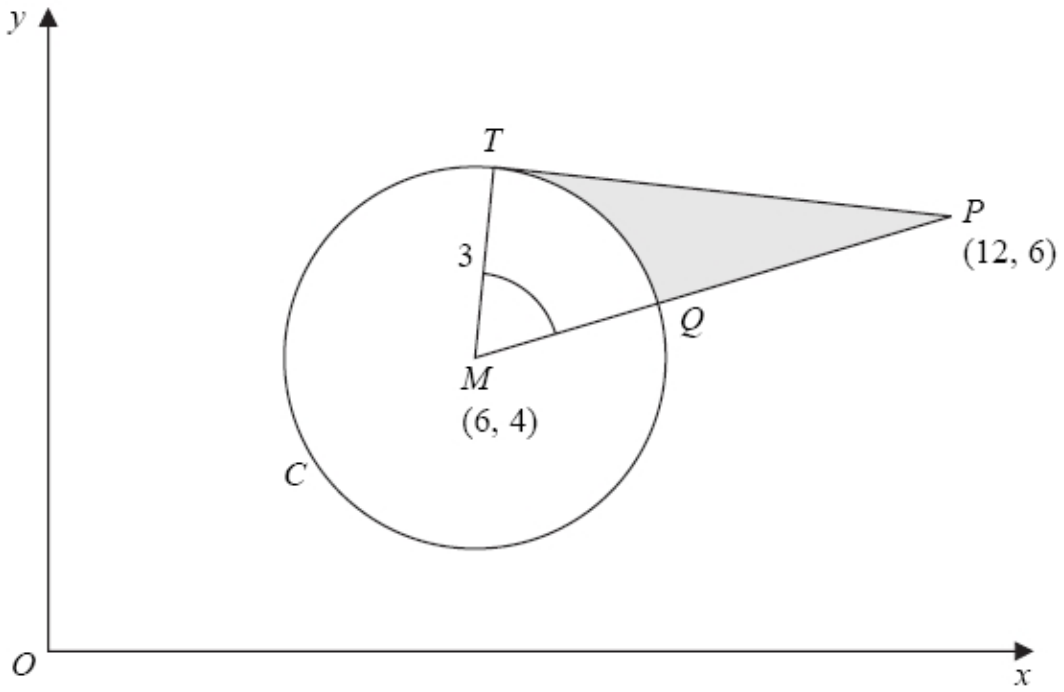

Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point P (12, 6). The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

(Total for question = 11 marks)

Q9.

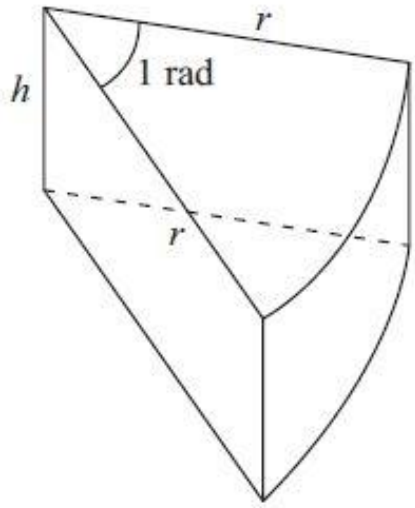

Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)

(Total 13 marks)