Question 1

Worked Solution

Part (a): The formula Area $= \frac{1}{2}r^2\theta$ requires θ to be in **radians**. The student used $\theta = 40$ (degrees) directly without converting to radians first.

Part (b): Convert $40°$ to radians first:

$$\theta = 40 \times \frac{\pi}{180} = \frac{2\pi}{9} \text{ rad}$$

Then apply the sector area formula:

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 5^2 \times \frac{2\pi}{9} = \frac{25\pi}{9}$$

$$\text{Area of sector} = \frac{25\pi}{9} \approx 8.73 \text{ cm}^2$$

Question 2

Worked Solution

Part (a): Area of sector BDE

Using Area $= \frac{1}{2}r^2\theta$ with $r = 5$ cm and $\theta = 1.4$ rad:

$$\text{Area}_{BDE} = \frac{1}{2} \times 5^2 \times 1.4 = \frac{1}{2} \times 25 \times 1.4$$

Area of sector $BDE = 17.5\text{ cm}^2$

Part (b): Angle DBC

Using the cosine rule in triangle DBC with $BD = DC = 5$ cm, $BC = 7.5$ cm, $CD = 6.1$ cm:

$$\cos(\angle DBC) = \frac{BD^2 + BC^2 - CD^2}{2 \cdot BD \cdot BC} = \frac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5} = \frac{25 + 56.25 - 37.21}{75} = \frac{44.04}{75}$$

$$\angle DBC = \arccos\left(\frac{44.04}{75}\right) \approx 0.943\text{ rad}$$

$\angle DBC = 0.943$ rad (3 d.p.)

Part (c): Area of shape ABCDEA

The shape consists of three regions: sector BDE, triangle CBD, and triangle EAB.

Area of triangle CBD:

$$\text{Area}_{CBD} = \frac{1}{2} \times BC \times BD \times \sin(\angle DBC) = \frac{1}{2} \times 7.5 \times 5 \times \sin(0.943) \approx 15.177\text{ cm}^2$$

Angle EBA: Since A, B, C are collinear and $\angle EBD = 1.4$ rad:

$$\angle EBA = \pi - 1.4 - 0.943 = \pi - 2.343 \approx 0.7986\text{ rad}$$

Side AB: $AB = BE\cos(\angle EBA) = 5\cos(0.7986) \approx 3.489$ cm

Side AE: $AE = BE\sin(\angle EBA) = 5\sin(0.7986) \approx 3.581$ cm

Area of triangle EAB:

$$\text{Area}_{EAB} = \frac{1}{2} \times AB \times AE = \frac{1}{2} \times 5\cos(0.7986) \times 5\sin(0.7986) \approx 6.24\text{ cm}^2$$

Total area:

$$\text{Area}_{ABCDEA} = 15.177 + 17.5 + 6.24 \approx 38.9\text{ cm}^2$$

Area of $ABCDEA \approx 38.9\text{ cm}^2$ (3 s.f.)

Question 3

Worked Solution

Part (a): Length of arc DEA

Arc length $= r\theta$ with $r = 7$ cm and $\theta = 2.1$ rad:

$$\text{Arc } DEA = 7 \times 2.1 = 14.7 \text{ cm}$$

Length of arc $DEA = 14.7$ cm

Part (b): Perimeter of shape ABCDEA

The perimeter consists of: arc DEA, side AB, side BC, and side CD.

Since A, B, C lie on a straight line with $AB = 7$ cm, and $\angle ABD = 2.1$ rad, the angle $\angle CBD = \pi - 2.1$ rad.

In right-angled triangle BCD:

$$BC = 7\cos(\pi - 2.1) = 7\cos(1.0416\ldots) \approx 3.50 \text{ cm}$$

$$CD = 7\sin(\pi - 2.1) = 7\sin(1.0416\ldots) \approx 6.00 \text{ cm}$$

(Note: 2.1 rad $= 120.3°$, so $\angle CBD = 180 - 120.3 = 59.7° \approx 1.042$ rad. Since angle $BCD = 90°$:)

$$BC = 7\cos(\pi - 2.1) \approx 3.488 \text{ cm}, \qquad CD = 7\sin(\pi - 2.1) \approx 6.062 \text{ cm}$$

Perimeter:

$$P = AB + \text{arc } DEA + CD + BC = 7 + 14.7 + 6.062 + 3.488 \approx 31.3 \text{ cm}$$

Perimeter of $ABCDEA \approx 31.3$ cm (1 d.p.)

Question 4

Worked Solution

Part (a): Show $\theta = 0.865$ rad (3 d.p.)

Using the cosine rule in triangle ABC with $AB = 7$ m, $AC = 13$ m, $BC = 10$ m, and $\angle BAC = \theta$:

$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos\theta$$

$$100 = 49 + 169 - 2(7)(13)\cos\theta \implies 100 = 218 - 182\cos\theta$$

$$\cos\theta = \frac{118}{182} = \frac{59}{91} \implies \theta = \arccos\left(\frac{59}{91}\right) = 0.8653\ldots \approx 0.865 \text{ rad} \quad \checkmark$$

Part (b): Amount of grass seed

The shaded region S = (area of triangle ABC) − (area of sector ABD).

Area of triangle ABC:

$$\text{Area}_{ABC} = \frac{1}{2} \times AB \times AC \times \sin\theta = \frac{1}{2} \times 7 \times 13 \times \sin(0.865) \approx \frac{1}{2} \times 91 \times 0.7638 \approx 34.7 \text{ m}^2$$

Area of sector ABD (radius = $AB = 7$ m, angle = $\theta = 0.865$ rad):

$$\text{Area}_{ABD} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 49 \times 0.865 \approx 21.2 \text{ m}^2$$

Area of shaded region S:

$$\text{Area}_S = 34.7 - 21.2 = 13.4 \text{ m}^2 \quad \text{(allow 13.4 or 13.5)}$$

Amount of grass seed at 50 g per m^2:

$$\text{Seed} = 13.4 \times 50 = 670 \text{ g}$$

Amount of grass seed needed = **670** g (to the nearest 10 g)

Question 5

Worked Solution

The sector OAB has $r = 6$ cm and $\angle AOB = \frac{\pi}{3}$ rad.

Part (a): Area of sector OAB

$$\text{Area}_{OAB} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 36 \times \frac{\pi}{3} = 6\pi \approx 18.85 \text{ cm}^2$$

Area of sector $OAB = 6\pi \approx 18.85$ cm^2

Part (b): Radius of circle C

Let the radius of circle C be r_c. The circle touches both straight edges OA and OB, so its centre lies on the angle bisector of $\angle AOB$, at distance d from O, where:

$$\sin\left(\frac{\pi}{6}\right) = \frac{r_c}{d} \implies \frac{1}{2} = \frac{r_c}{d} \implies d = 2r_c$$

The circle also touches the arc AB (radius 6 from O), so:

$$d + r_c = 6 \implies 2r_c + r_c = 6 \implies r_c = 2$$

Radius of circle $C = 2$ cm

Part (c): Area of shaded region

$$\text{Shaded area} = \text{Area of sector} - \text{Area of circle } C = 6\pi - \pi(2)^2 = 6\pi - 4\pi = 2\pi$$

Area of shaded region $= 2\pi \approx 6.28$ cm^2

Question 6

Worked Solution

The sector BAC has $r = 6$ cm, $\angle BAC = 2.2$ rad, and $AD = 4$ cm.

Part (a): Area of sector BAC

$$\text{Area}_{BAC} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 36 \times 2.2 = 39.6 \text{ cm}^2$$

Area of sector $BAC = 39.6$ cm^2

Part (b): Size of $\angle DAC$

By symmetry of the isosceles shape, D lies on the axis of symmetry, so:

$$\angle DAC = \frac{2\pi - 2.2}{2} = \pi - 1.1 = 2.04 \text{ rad}$$

$\angle DAC = 2.04$ rad (3 s.f.)

Part (c): Complete area of logo design

The logo consists of the sector BAC plus two congruent triangles DAC (and DBC by symmetry). We need the area of triangle DAC.

Using Area $= \frac{1}{2}ab\sin C$ with sides $AC = 6$ cm, $AD = 4$ cm, and included angle $\angle DAC = 2.04$ rad:

$$\text{Area}_{\triangle DAC} = \frac{1}{2} \times 6 \times 4 \times \sin(2.04) \approx 12 \times 0.8961 \approx 10.75 \text{ cm}^2$$

Total area = sector $+2\times$ triangle:

$$= 39.6 + 2 \times 10.75 = 39.6 + 21.5 = 61.1 \approx 61 \text{ cm}^2$$

Complete area of logo design ≈ 61 cm^2 (nearest cm^2)

Question 7

Worked Solution

The circle C: $x^2 + y^2 - 20x - 16y + 139 = 0$.

Part (a): Centre of C

Complete the square:

$$(x^2 - 20x) + (y^2 - 16y) = -139$$
$$(x - 10)^2 - 100 + (y - 8)^2 - 64 = -139$$
$$(x - 10)^2 + (y - 8)^2 = 25$$

Centre $T = (10, 8)$

Part (b): Show $r = 5$

From the completed-square form $(x - 10)^2 + (y - 8)^2 = 25$:

$$r^2 = 25 \implies r = 5 \quad \checkmark$$

Part (c): y-coordinates of P and Q

Substitute $x = 13$ into the circle equation:

$$(13 - 10)^2 + (y - 8)^2 = 25 \implies 9 + (y - 8)^2 = 25 \implies (y - 8)^2 = 16$$

$$y - 8 = \pm 4 \implies y = 12 \text{ or } y = 4$$

P has y-coordinate 12; Q has y-coordinate 4

Part (d): Perimeter of sector PTQ

With $r = 5$, $\theta = 1.855$ rad, arc length $PQ = r\theta = 5 \times 1.855 = 9.275$ cm.

Perimeter $= 2r + \text{arc } PQ = 2(5) + 9.275 = 19.275$

Perimeter of sector $PTQ \approx 19.3$ cm (3 s.f.)

Question 8

Worked Solution

Circle C has centre $M(6,4)$ and radius 3.

Part (a): Equation of circle

$$(x-6)^2+(y-4)^2=9$$

Part (b): Show $\angle TMQ = 1.0766$ rad

Find MP:

$$MP=\sqrt{(12-6)^2+(6-4)^2}=\sqrt{36+4}=\sqrt{40}$$

Since T lies on the circle and TP is a tangent at T, $\angle MTQ = 90°$. Therefore:

$$\cos(\angle TMQ)=\frac{MT}{MP}=\frac{3}{\sqrt{40}}$$

$$\angle TMQ=\arccos\left(\frac{3}{\sqrt{40}}\right)=1.0766\text{ rad}\quad\checkmark$$

Part (c): Area of shaded region TPQ

Area of triangle TMP:

$$TP=\sqrt{MP^2-MT^2}=\sqrt{40-9}=\sqrt{31}$$

$$\text{Area}_{\triangle TMP}=\frac{1}{2}\times MT\times TP=\frac{1}{2}\times 3\times\sqrt{31}=\frac{3\sqrt{31}}{2}\approx 8.352\text{ cm}^2$$

Area of sector MTQ (radius = 3, angle = 1.0766 rad):

$$\text{Area}_{\text{sector}}=\frac{1}{2}\times 3^2\times 1.0766=\frac{1}{2}\times 9\times 1.0766\approx 4.845\text{ cm}^2$$

Area of shaded region TPQ:

$$\text{Area}_{TPQ}=\text{Area}_{\triangle TMP}-\text{Area}_{\text{sector}}=8.352-4.845\approx 3.507\text{ cm}^2$$

Area of shaded region $TPQ \approx 3.507\text{ cm}^2$ (3 d.p.)

Question 9

Worked Solution

The box is a right prism with a sector cross-section: radius r, angle $\theta = 1$ rad, height h, volume $= 300$ cm^3.

Part (a): Show $S = r^2 + \dfrac{1800}{r}$

Volume: $V = \frac{1}{2}r^2\theta \cdot h = \frac{1}{2}r^2(1)h = \frac{r^2h}{2} = 300$, so $h = \dfrac{600}{r^2}$.

Surface area (2 sector faces + 2 rectangular radial faces + 1 curved rectangular face):

$$\begin{aligned} S &= 2 \times \frac{1}{2}r^2\theta + 2 \times rh + r\theta \cdot h \\ &= r^2 + 2rh + rh = r^2 + 3rh \end{aligned}$$

Substituting $h = \dfrac{600}{r^2}$:

$$S = r^2 + 3r \times \frac{600}{r^2} = r^2 + \frac{1800}{r} \quad \checkmark$$

Part (b): Value of r for stationary S

$$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$$

Setting $\dfrac{dS}{dr} = 0$:

$$2r = \frac{1800}{r^2} \implies r^3 = 900 \implies r = \sqrt[3]{900} \approx 9.65 \text{ cm}$$

$r = \sqrt[3]{900} \approx 9.65$ cm

Part (c): Prove this is a minimum

$$\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3}$$

Since $r > 0$, we have $\dfrac{3600}{r^3} > 0$, so $\dfrac{d^2S}{dr^2} = 2 + \dfrac{3600}{r^3} > 0$ for all valid r.
Therefore the stationary point is a **minimum**.

Part (d): Minimum value of S

$$S_{\text{min}} = (9.65)^2 + \frac{1800}{9.65} \approx 93.12 + 186.53 \approx 279.65$$

Minimum surface area ≈ 280 cm^2 (nearest cm^2)

End of Worked Solutions