Radians, Circle Sectors and Triangles Exam Questions Mark Scheme (Sheet 2)

Q1.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	The formula is only valid when the angle AOB is given in radians	B1	This mark is given for a correct explanation
(b)	$\frac{40}{360} \times \pi \times 5^2$	M1	This mark is given for a correct method to find the area of the sector
	$\frac{25\pi}{9}$ cm^2	A1	This mark is given for a correct value for the area of the sector
			(Total 3 marks)

Q2.

Question Number	Scheme		Marks
(a)	Area BDE $= \frac{1}{2}(5)^2(1.4)$	M1: Use of the correct formula or method for the area of the sector	M1A1
	$= 17.5$ (cm^2)	A1: 17.5 oe	
			[2]
(b)	**Parts (b) and (c) can be marked together**		
	$6.1^2 = 5^2 + 7.5^2 - (2 \times 5 \times 7.5 \cos DBC)$ or $\cos DBC = \frac{5^2 + 7.5^2 - 6.1^2}{2 \times 5 \times 7.5}$ (or equivalent)		M1
	M1: A correct statement involving the angle DBC		
	Angle $DBC = 0.943201...$	awrt 0.943	A1
	Note that work for (b) may be seen on the diagram or in part (c)		
			[2]
(c)	**Note that candidates may work in degrees in (c)** (Angle $DBC = 54.04....$degrees)		
	Area $CBD = \frac{1}{2}5(7.5)\sin(0.943)$		
	Angle $EBA = \pi - 1.4 - "0.943"$ **(Maybe seen on the diagram)**	Area $CBD = \frac{1}{2}5(7.5)\sin(\text{their } 0.943)$ or awrt 15.2. (Note area of $CBD = 15.177...$) A correct method for the area of triangle CBD which can be implied by awrt 15.2	M1
	$\pi - 1.4 -$ "their 0.943" A value for angle EBA of awrt 0.8 (from 0.7985926536... or 0.7983916536...) or value for angle EBA of $(1.74159... -$ their angle $DBC)$ would imply this mark.		M1
	$AB = 5\cos(\pi - 1.4 - "0.943")$ or $AE = 5\sin(\pi - 1.4 - "0.943")$		

		$AB = 5\cos(\pi - 1.4 - \text{their } 0.943)$ $AB = 5\cos(0.79859...) = 3.488577938...$ Allow M1 for $AB =$ awrt 3.49 Or $AE = 5\sin(\pi - 1.4 - \text{their } 0.943)$ $AE = 5\sin(0.79859...) = 3.581874365688...$ Allow M1 for $AE =$ awrt 3.58 **It must be clear that $\pi - 1.4 - "0.943"$ is being used for angle EBA.** **Note that some candidates use the sin rule here but it must be used correctly – do not allow mixing of degrees and radians.**	M1
	Area $EAB = \frac{1}{2}5\cos(\pi - 1.4 - "0.943") \times 5\sin(\pi - 1.4 - "0.943")$		
	This is dependent on the previous M1 and there must be no other errors in finding the area of triangle EAB		dM1
	Allow M1 for area $EAB =$ awrt 6.2		
	Area $ABCDE = 15.17... + 17.5 + 6.24... = 38.92...$		
		awrt 38.9	A1cso
			[5]
	Note that a sign error in (b) can give the obtuse angle (2.198....) and could lead to the correct answer in (c) – this would lose the final mark in (c)		**Total 9**

Q3.

Question Number	Scheme		Marks
(a)	Length $DEA = 7(2.1) = 14.7$	M1: 7×2.1 only A1: 14.7	M1A1
			[2]
(b)	Angle $CBD = \pi - 2.1$	**May be seen on the diagram** (allow awrt 1.0 and allow 180 – 120). Could score for sight of **Angle CBD = awrt 60 degrees.**	M1
	Both $7\cos(\pi - 2.1)$ **and** $7\sin(\pi - 2.1)$ or **Both** $7\cos(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\cos(\pi - 2.1))^2}$ or **Both** $7\sin(\pi - 2.1)$ **and** $\sqrt{7^2 - (7\sin(\pi - 2.1))^2}$ **Or equivalents to these**	A correct attempt to find BC and BD. You can ignore how the candidate assigns BC and CD. $7\cos(\pi - 2.1)$ can be implied by awrt 3.5 and $7\sin(\pi - 2.1)$ can be implied by awrt 6. Note if the sin rule is used, do not allow mixing of degrees and radians unless their answer implies a correct interpretation. **Dependent on the previous method mark.**	dM1
	Note that 2.1 radians is 120 degrees (to 3sf) which if used gives angle CBD as 60 degrees. If used this gives a correct perimeter of 31.3 and could score full marks.		
	$P = 7\cos(\pi - 2.1) + 7\sin(\pi - 2.1) + 7 + 14.7$	their BC + their CD + 7 + their DEA **Dependent on both previous method marks**	ddM1
	$= 31.2764...$	Awrt 31.3	A1
			[4]
			Total 6

Q4.

Question Number	Scheme	Marks
(a)	**Way 1**: $10^2 = 7^2 + 13^2 - 2\times7\times13\cos\theta$ or $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$	M1
	$\cos\theta = \dfrac{59}{91}$ or $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$ or $\cos\theta = 0.6483$ or 0.8644	A1 o.e
	$(\theta = 0.8653789549\ldots\ ) = 0.865$ * (to 3 dp)	A1* cso (3)
	Way 2: Uses $\cos\theta = \dfrac{x}{7}$, where $7^2 - x^2 = 10^2 - (13-x)^2$ and finds x $(=59/13)$	M1
	$\cos\theta = \dfrac{59}{91}$ and $(\theta = 0.8653789549\ldots\ ) = 0.865$ * (to 3 dp) – as in Way 1	A1, A1 (3)
(b)	Area triangle $ABC = \dfrac{1}{2}\times13\times7\sin0.865$ or $\dfrac{1}{2}\times13\times7\sin49.6$ or $20\sqrt{3}$	M1
	Area sector $ABD = \dfrac{1}{2}\times7^2\times0.865$ or $\dfrac{49.6}{360}\times\pi\times7^2$	M1
	=34.6 (triangle) or 21.2 (Sector)	A1
	Area of $S = \dfrac{1}{2}\times13\times7\sin0.865 - \dfrac{1}{2}\times7^2\times0.865$ $(=13.4)$	M1 A1
	(Amount of seed =) 13.4 x 50 = 670g or 680g (need one of these two answers)	M1 A1 (7)
		Total 10

	Notes for Question
(a)	M1: use correct cosine **formula in any form** A1: give a value for $\cos\theta$ NB $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times7\times13}$ earns M1A1 A1: deduce and state the printed answer θ = 0.865
(b)	M1: Uses Correct method for area of the correct triangle i.e. ABC M1: Uses Correct method for the area of the sector A1: This is earned for one of the correct answers. May be implied if these answers are not calculated but the final answer is correct with no errors (or shaded area is 13.4 or13.5) M1: Their area of Triangle ABC– **Area** of Sector (may have $kr^2\theta$ but not $k\theta$) A1: **Correct expression** or awrt 13.4 or 13.5 (may be implied by final answer) M1: Multiply their previous answer by 50 A1: 670g or 680 g (There is an argument for rounding answer up to provide enough seed)

N.B. $(\frac{1}{2}\times13\times7\sin0.865 - \frac{1}{2}\times7^2\times0.865\ )\times50$=670 or 680 earns full marks

$(\frac{1}{2}\times13\times7\sin0.865 - \frac{1}{2}\times7^2\times0.865\ )\times50$ = awrt 670 or 680 just loses last mark

$(\frac{1}{2}\times13\times7\sin0.865 - \frac{1}{2}\times7^2\times0.865\ )\times50$ = wrong answer M1M1A0M1A1M1A0

Q5.

Question Number	Scheme		Marks
(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right) = 6\pi$ or 18.85 or awrt 18.8 $(\text{cm})^2$	Using $\frac{1}{2}r^2\theta$ (See notes)	M1
		6π or 18.85 or awrt 18.8	A1
			[2]
(b)	$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$	$\sin\left(\frac{\pi}{6}\right)$ or $\sin 30^\circ = \frac{r}{6-r}$	M1
	$\frac{1}{2} = \frac{r}{6-r}$	Replaces sin by numeric value	dM1
	$6 - r = 2r \Rightarrow r = 2$	$r = 2$	A1 cso
			[3]
(c)	Area $= 6\pi - \pi(2)^2 = 2\pi$ or awrt 6.3 $(\text{cm})^2$	their area of sector $- \pi r^2$	M1
		2π or awrt 6.3	A1 cao
			[2]
			7

(a)
M1: Needs θ in radians for this formula.
Candidate could convert to degrees and use the degrees formula.
A1: Does not need units. Answer should be either 6π or 18.85 or awrt 18.8
Correct answer with no working is M1A1.
This M1A1 can only be awarded in part (a).

(b)
M1: Also allow $\cos\left(\frac{\pi}{3}\right)$ or $\cos 60^\circ = \frac{r}{6-r}$.

1st M1: Needs correct trigonometry method. Candidates could state $\sin\left(\frac{\pi}{6}\right) = \frac{r}{x}$ and $x + r = 6$ or equivalent in their working to gain this method mark.

dM1: Replaces sin by numerical value. $0.009... = \frac{r}{6-r}$ from working "incorrectly" in degrees is fine here for dM1.
A1: For $r = 2$ from correct solution only.
<u>Alternative:</u> 1st M1 for $\frac{r}{OC} = \sin 30$ or $\frac{r}{OC} = \cos 60$. 2nd M1 for $OC = 2r$ and then A1 for $r = 2$.
<u>Note</u> seeing $OC = 2r$ is M1M1.
<u>Special Case:</u> If a candidate states an answer of $r = 2$ (must be in part (b)) as a guess or from an incorrect method then award SC: M0M0B1. Such a candidate could then go on to score M1A1 in part (c).

(c)
M1: For "their area of sector – their area of circle", where $r > 0$ is ft from their answer to part (b).
Allow the method mark if "their area of sector" < "their area of circle". The candidate must show somewhere in their working that they are subtracting the correct way round, even if their answer is negative.
Some candidates in part (c) will either use their value of r from part (b) or even introduce a value of r in part (c). You can apply the scheme to award either M0A0 or M1A0 or M1A1 to these candidates.
<u>Note:</u> Candidates can get M1 by writing "their part (a) answer $-\pi r^2$", where the radius of the circle is not substituted.
A1: cao – accept exact answer or awrt 6.3
Correct answer only with no working in (c) gets M1A1
<u>**Beware: The answer in (c) is the same as the arc length of the pendant**</u>

Q6.

Question Number	Scheme	Marks
(a)	$\frac{1}{2}r^2\theta = \frac{1}{2}\times 6^2 \times 2.2 = 39.6 \quad (\text{cm}^2)$	M1 A1 (2)
(b)	$\left(\frac{2\pi - 2.2}{2} = \right)\ \pi - 1.1 = 2.04 \quad (\text{rad})$	M1 A1 (2)
	(c) $\Delta DAC = \frac{1}{2}\times 6 \times 4\sin 2.04 \qquad (\approx 10.7)$	M1 A1ft
	Total area = sector + 2 triangles = 61 $\quad (\text{cm}^2)$	M1 A1 (4) **[8]**
(a)	**M1**: Needs θ in radians for this formula. Could convert to degrees and use degrees formula. **A1**: Does not need units. Answer should be 39.6 exactly. Answer with no working is **M1 A1**. This **M1A1** can only be awarded in part (a).	
(b)	**M1**: Needs full method to give angle in radians **A1**: Allow answers which round to 2.04 (Just writes 2.04 – no working is 2/2)	
(c)	**M1**: Use $\frac{1}{2}\times 6 \times 4\sin A$ (if any other triangle formula e.g. $\frac{1}{2}b\times h$ is used the method must be complete for this mark) (No value needed for A, but should not be using 2.2) **A1**: ft the value obtained in part (b) – need not be evaluated- could be in degrees **M1**: Uses Total area = sector + 2 triangles or other complete method **A1**: Allow answers which round to 61. (Do not need units) Special case degrees: Could get M0A0, M0A0, M1A1M1A0 Special case: Use $\Delta\, BDC - \Delta\, BAC$ Both areas needed for first **M1** Total area = sector + area found is second **M1** NB Just finding lengths BD, DC, and angle BDC then assuming area BDC is a sector to find area BDC is 0/4	

Q7.

Question number	Scheme	Marks
(a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	Obtain $(x-10)^2$ **and** $(y-8)^2$	A1
	Centre is (10, 8). N.B. This may be indicated on **diagram only** as (10, 8)	A1 (3)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25 (= r^2)$ or $(r^2 =)$"100"+"64"−139	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	**A1** (2)
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	**M1**
	e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y =$ or $13^2 + y^2 - 20\times13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$	
	$y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**A1, A1** (3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	**M1**
	Perimeter $PTQ = 2r +$ their **arc** PQ (Finding perimeter of triangle is M0 here)	**M1**
	= 19.275 or 19.28 or 19.3	**A1** (3)
		11 marks
Alternatives (a)	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
	Centre is $(-g, -f)$, **and so centre is** (10, 8).	A1, A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	*Method 2:* Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)$"100"+"64"−139	M1
	$r = 5$ *	A1
OR	*Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$	**M1**
	$r = 5$ *	**A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate "$8 \pm h$" - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**
Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be <u>implied</u> by $(\pm10, \pm8)$. **Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 =$ "100"+"64"−139 (not 139 − "100" − "64") or for using equation of circle in $(x \pm 10)^2 + (y \pm 8)^2 = k^2$ form to identify $r=$ **3rd A1** $r = 5$ **(NB This is a given answer so should follow** $k^2 = 25$ or $r^2 = 100 + 64 - 139$) **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

Q8.

Question Number	Scheme	Marks
(a)	$(x-\ 6)^2 + (y-\ 4)^2 =\ ;\ 3^2$	B1; B1 (2)
(b)	Complete method for MP: $=\sqrt{(12-6)^2+(6-4)^2}$	M1
	$=\sqrt{40}$ or awrt 6.325	A1
	[These first two marks can be scored if seen as part of solution for (c)]	
	Complete method for $\cos\theta$, $\sin\theta$ or $\tan\theta$	M1
	e.g. $\cos\theta = \frac{MT}{MP} = \frac{3}{candidate's\ \sqrt{40}}$ $(= 0.4743)$ $(\theta = 61.6835^{\circ})$	
	[If TP = 6 is used, then M0]	
	$\theta = 1.0766$ rad **AG**	A1 (4)
(c)	Complete method for area TMP; e.g. $= \frac{1}{2}\times 3\times\sqrt{40}\sin\theta$	M1
	$= \frac{3}{2}\sqrt{31}$ $(= 8.3516..)$ allow awrt 8.35	A1
	Area (sector) MTQ $= 0.5\times 3^2\times 1.0766$ $(= 4.8446...)$	M1
	Area TPQ = candidate's $(8.3516.. - 4.8446..)$	M1
	$= 3.507$ awrt [Note: 3.51 is A0]	A1 (5) **[11]**
Notes	(a) Allow 9 for 3^2. (b) First M1 can be implied by $\sqrt{40}$ or $\sqrt{31}$ For second M1: May find TP $= \sqrt{(\sqrt{40})^2 - 3^2} = \sqrt{31}$, then either $\sin\theta = \frac{TP}{MP} = \frac{\sqrt{31}}{\sqrt{40}}$ $(= 0.8803...)$ *or* $\tan\theta = \frac{\sqrt{31}}{3}$ $(1.8859..)$ or cos rule **NB. Answer is given, but allow final A1 if all previous work is correct.** (c) First M1: (alternative) $\frac{1}{2}\times 3\times\sqrt{40-9}$ Second M1: allow even if candidate's value of θ used. (Despite being given !)	

Q9.

Question Number	Scheme	Marks
Q (a)	(Arc length =) $r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).	B1
	(Sector area =) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).	B1
	Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)	M1
	Volume = 300 = $\frac{1}{2}r^2h$	B1
	Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)	A1cso (5)
(b)	$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\frac{dS}{dr} = 0 \Rightarrow r^3 = \ldots$, $r = \sqrt[3]{900}$, or AWRT 9.7 (NOT -9.7 or ± 9.7)	M1, A1 (4)
(c)	$\frac{d^2S}{dr^2} = \ldots$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum	M1, A1ft (2)
(d)	$S_{min} = (9.65\ldots)^2 + \frac{1800}{9.65\ldots}$	
	(Using their value of r, however found, in the <u>given</u> S formula)	M1
	$= 279.65\ldots$ (AWRT: 280) (Dependent on full marks in part (b))	A1 (2) [13]
(a)	M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.	
(b)	<u>In parts (b), (c) and (d), ignore labelling of parts</u> 1st M1 for attempt at differentiation (one term is sufficient) $r^n \to kr^{n-1}$ 2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \ldots$ (depending upon their 'changed function', this could be $r = \ldots$ or $r^2 = \ldots$, etc., but the algebra <u>must deal with a negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \ldots$ is consistent with their derivative).	
(c)	M1 for attempting second derivative (one term is sufficient) $r^n \to kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>). A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum. <u>Alternative</u>: M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign. A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum. <u>Alternative</u>: M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65. A1ft: Indicate that both values are more than 279.65, and conclude minimum.	