Question 1 (OCR 4724, Jun 2005, Q8i [Modified])

Worked Solution

Given $\dfrac{3x+4}{(1+x)(2+x)^2} \equiv \dfrac{A}{1+x} + \dfrac{B}{2+x} + \dfrac{C}{(2+x)^2}$, find A, B and C.

Multiply both sides by $(1+x)(2+x)^2$:

$$3x + 4 \equiv A(2+x)^2 + B(1+x)(2+x) + C(1+x)$$

Set $x = -1$:

$$-3 + 4 = A(1)^2 \implies A = 1$$

Set $x = -2$:

$$-6 + 4 = C(-1) \implies C = 2$$

Compare x^2 coefficients: $0 = A + B$, so $B = -A = -1$.

Check with x terms: $3 = 4A + 3B + C = 4 - 3 + 2 = 3$ ✓

$A = 1, \ B = -1, \ C = 2$

$$\frac{3x+4}{(1+x)(2+x)^2} = \frac{1}{1+x} - \frac{1}{2+x} + \frac{2}{(2+x)^2}$$

Question 2 (OCR 4724, Jan 2006, Q7i [Modified])

Worked Solution

Express $\frac{11+8x}{(2-x)(1+x)^2}$ in the form $\frac{A}{2-x}+\frac{B}{1+x}+\frac{C}{(1+x)^2}$.

$$11+8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$$

Set $x=2$: $11+16=A(9) \implies 27=9A \implies A=3$.

Set $x=-1$: $11-8=C(3) \implies 3=3C \implies C=1$.

Compare x^2 coefficients: $0=A-B \implies B=A=3$.

Check with constant terms: $11=A\cdot 1+B\cdot 2+C\cdot 2=3+6+2=11$ ✓

$A=3,\ B=3,\ C=1$

$$\frac{11+8x}{(2-x)(1+x)^2}=\frac{3}{2-x}+\frac{3}{1+x}+\frac{1}{(1+x)^2}$$

Question 3 (OCR 4724, Jun 2007, Q1i [Modified])

Worked Solution

Express $\dfrac{3x+1}{(x+2)(x-3)}$ in partial fractions.

$$\frac{3x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$
$$3x+1 \equiv A(x-3) + B(x+2)$$

Set $x = 3$**:** $10 = 5B \implies B = 2.$

Set $x = -2$**:** $-5 = -5A \implies A = 1.$

$$\frac{3x+1}{(x+2)(x-3)} = \frac{1}{x+2} + \frac{2}{x-3}$$

Question 4 (OCR 4724, Jan 2008, Q2i)

Worked Solution

Express $\dfrac{x}{(x+1)(x+2)}$ in partial fractions.

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$
$$x \equiv A(x+2) + B(x+1)$$

Set $x = -1$**:** $-1 = A(1) \implies A = -1.$

Set $x = -2$**:** $-2 = B(-1) \implies B = 2.$

$$\frac{x}{(x+1)(x+2)} = -\frac{1}{x+1} + \frac{2}{x+2}$$

Question 5 (OCR 4724, Jun 2010, Q3)

Worked Solution

Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions.

Since the denominator has a repeated linear factor:

$$\frac{x^2}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$$

$$x^2 \equiv A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

Set $x = 1$: $1 = B(1-2) = -B \implies B = -1$.

Set $x = 2$: $4 = C(1)^2 = C \implies C = 4$.

Compare x^2 coefficients: $1 = A + C \implies A = 1 - 4 = -3$.

Check constant terms: $0 = 2A - 2B + C = -6 + 2 + 4 = 0$ ✓

$A = -3, \;\; B = -1, \;\; C = 4$

$$\frac{x^2}{(x-1)^2(x-2)} = -\frac{3}{x-1} - \frac{1}{(x-1)^2} + \frac{4}{x-2}$$

Question 6 (OCR 4724, Jun 2013, Q1)

Worked Solution

Express $\dfrac{(x-7)(x-2)}{(x+2)(x-1)^2}$ in partial fractions.

$$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$(x-7)(x-2) \equiv A(x-1)^2 + B(x+2)(x-1) + C(x+2)$$

Set $x = 1$: $(1-7)(1-2) = (-6)(-1) = 6 = C(3) \implies C = 2$.

Set $x = -2$: $(-9)(-4) = 36 = A(-3)^2 = 9A \implies A = 4$.

Compare x^2 coefficients: $1 = A + B \implies B = 1 - 4 = -3$.

Check constant terms: $14 = A \cdot 1 + B \cdot (-2) + C \cdot 2 = 4 + 6 + 4 = 14$ ✓

$A = 4,\ \ B = -3,\ \ C = 2$

$$\frac{(x-7)(x-2)}{(x+2)(x-1)^2} = \frac{4}{x+2} - \frac{3}{x-1} + \frac{2}{(x-1)^2}$$

Question 7 (Edexcel 6666, Jan 2013, Q3)

Worked Solution

Express $\dfrac{9x^2+20x-10}{(x+2)(3x-1)}$ in partial fractions.

The degree of the numerator equals the degree of the denominator (both degree 2 after expanding), so we first divide:

$(x+2)(3x-1) = 3x^2+5x-2$.

$\dfrac{9x^2+20x-10}{3x^2+5x-2}$: since leading coefficient ratio is 3:

$9x^2+20x-10 = 3(3x^2+5x-2)+(5x-4)$.

So:

$$\frac{9x^2+20x-10}{(x+2)(3x-1)} = 3+\frac{5x-4}{(x+2)(3x-1)}$$

Now decompose $\dfrac{5x-4}{(x+2)(3x-1)} = \dfrac{B}{x+2}+\dfrac{C}{3x-1}$:

$$5x-4 \equiv B(3x-1)+C(x+2)$$

Set $x=-2$**:** $-10-4=-14=B(-7) \implies B=2$.

Set $x=\frac{1}{3}$**:** $\frac{5}{3}-4=-\frac{7}{3}=C\cdot\frac{7}{3} \implies C=-1$.

$$\frac{9x^2+20x-10}{(x+2)(3x-1)} = 3+\frac{2}{x+2}-\frac{1}{3x-1}$$

Question 8 (Edexcel IAL, C34, Jun 2016, Q4)

Worked Solution

$\mathrm{g}(x) = \dfrac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12}, \;\; x > 3, \, x \in \mathbb{R}.$

Part (a): Given $\dfrac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} \equiv x^2 + A + \dfrac{B}{x-3}$, find A and B.

Note $x^2 + x - 12 = (x+4)(x-3)$.

Perform polynomial long division of $x^4 + x^3 - 7x^2 + 8x - 48$ by $x^2 + x - 12$:

$x^4 + x^3 - 7x^2 + 8x - 48 = (x^2 + x - 12)(x^2) + \text{remainder}$

$x^2(x^2 + x - 12) = x^4 + x^3 - 12x^2$

Remainder: $(x^4 + x^3 - 7x^2 + 8x - 48) - (x^4 + x^3 - 12x^2) = 5x^2 + 8x - 48$

$5x^2 + 8x - 48 = 5(x^2 + x - 12) + (3x + 12) = 5(x^2 + x - 12) + 3(x + 4)$

So:

$$\frac{x^4 + x^3 - 7x^2 + 8x - 48}{x^2 + x - 12} = x^2 + 5 + \frac{3(x+4)}{(x+4)(x-3)} = x^2 + 5 + \frac{3}{x-3}$$

(a) $A = 5, \;\; B = 3$

$$\mathrm{g}(x) = x^2 + 5 + \frac{3}{x-3}$$

Part (b): Find the equation of the tangent to $y = \mathrm{g}(x)$ at $x = 4$ in the form $y = mx + c$.

$\mathrm{g}'(x) = 2x - \dfrac{3}{(x-3)^2}$

At $x = 4$: $\mathrm{g}'(4) = 8 - \dfrac{3}{1} = 5.$

$\mathrm{g}(4) = 16 + 5 + 3 = 24.$

Tangent: $y - 24 = 5(x - 4) \implies y = 5x + 4.$

(b) $y = 5x + 4$

Question 9 (Edexcel IAL, C34, Jun 2017, Q5)

Worked Solution

$$\frac{6-5x-4x^2}{(2-x)(1+2x)} \equiv A + \frac{B}{2-x} + \frac{C}{1+2x}$$

Part (a): Find A, B and C.

Since the degree of numerator (2) equals degree of denominator (2), we check the leading coefficient: numerator $-4x^2$, denominator $-2x^2$, so $A = \frac{-4}{-2} = 2$.

Alternatively, multiply out:

$$6-5x-4x^2 \equiv A(2-x)(1+2x) + B(1+2x) + C(2-x)$$

Set $x = 2$: $6 - 10 - 16 = -20 = B(5) \implies B = -4$.
Set $x = -\frac{1}{2}$: $6 + \frac{5}{2} - 1 = \frac{15}{2} = C(\frac{5}{2}) \implies C = 3$.
Compare x^2 coefficients: $-4 = -2A \implies A = 2$.

(a) $A = 2,\ B = -4,\ C = 3$

$$\frac{6-5x-4x^2}{(2-x)(1+2x)} = 2 - \frac{4}{2-x} + \frac{3}{1+2x}$$

Part (b): Given $f(x) = \frac{6-5x-4x^2}{(2-x)(1+2x)}$, $x > 2$, find $f'(x)$.

$$f(x) = 2 - \frac{4}{2-x} + \frac{3}{1+2x} = 2 - 4(2-x)^{-1} + 3(1+2x)^{-1}$$

$$f'(x) = -\frac{4}{(2-x)^2} - \frac{6}{(1+2x)^2}$$

(b) $f'(x) = -\frac{4}{(2-x)^2} - \frac{6}{(1+2x)^2}$

Part (c): Prove that $f(x)$ is a decreasing function.

For $x > 2$: $(2-x)^2 > 0$ and $(1+2x)^2 > 0$, so both terms are negative.

Therefore $f'(x) = -\frac{4}{(2-x)^2} - \frac{6}{(1+2x)^2} < 0$ for all $x > 2$.

Hence $f(x)$ is a decreasing function. ✓

(c) Since $(2-x)^2 > 0$ and $(1+2x)^2 > 0$ for $x > 2$, we have $f'(x) < 0$, so $f(x)$ is decreasing.

End of Worked Solutions