Partial Fractions Exam Questions Sheet 2 Mark Scheme

Q1.

Question Number	Scheme		Marks
	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2,\ \ C = 2$	Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.	B1 cso
	$5x+3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 cso
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		**[4]** **4**

Notes for Question

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both "A" $= 2$ and "C" $= 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:
- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Q2.

Question Number	Scheme		Marks	
	$9x^2 = A(x-1)(2x+1)+B(2x+1)+C(x-1)^2$		B1	
	$x \to 1$ $\quad 9 = 3B \Rightarrow B = 3$		M1	
	$x \to -\frac{1}{2}$ $\quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$	Any two of A, B, C	A1	
	x^2 terms $\quad 9 = 2A + C \Rightarrow A = 4$	All three correct	A1	**(4)** **[4]**
	Alternatives for finding A.			
	x terms $\quad 0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $\quad 0 = -A + B + C \Rightarrow A = 4$			

Q3.

Question Number	Scheme	Marks
By Division	$$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$$	
	$a=3$	**B1**
	Long division as far as $$\begin{array}{r} 3x^2-2x\ldots\ldots \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+\ldots\ldots\ldots\ldots \\ \underline{-2x^3+\ldots\ldots\ldots\ldots} \end{array}$$	**M1**
	Two of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
	All four of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
		(4 marks)

Notes for Question

B1 Stating $a=3$. This can also be scored by the coefficient of x^2 in $3x^2-2x+7$

M1 Using long division by x^2-4 and getting as far as the 'x' term. The coefficients need not be correct. Award if you see the whole number part as $\ldots x^2+\ldots x$ following some working. You may also see this in a table/ grid.

Long division by $(x+2)$ will not score anything until $(x-2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b=-2\ c=7\ d=-8\ e=24$.

The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b=-2\ c=7\ d=-8$ and $e=24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see a=…, b=…, or the values embedded in the rhs for all 4 marks

Q4.

Question Number	Scheme	Marks
	(a) $A = 2$	B1
	$2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$	
	$x \to 1$ $\quad -3 = 3B \Rightarrow B = -1$	M1 A1
	$x \to -2$ $\quad -12 = -3C \Rightarrow C = 4$	A1 (4)

Q5.

Question Number	Scheme	Marks
Q (a)	$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$	
	$4 - 2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x \to -\frac{1}{2}$, $5 = A(\frac{1}{2})(\frac{5}{2}) \Rightarrow A = 4$ any one correct constant	A1
	$x \to -1$, $6 = B(-1)(2) \Rightarrow B = -3$	
	$x \to -3$, $10 = C(-5)(-2) \Rightarrow C = 1$ all three constants correct	A1 (4)

Q6.

Question	Scheme	Marks	AOs
	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$		
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3) + B(1-2x) + C(x-3) \Rightarrow B = ..., C = ...$	M1	2.1
	$A = 3$	B1	1.1b
	Uses substitution or compares terms to find either $B = ...$ or $C = ...$	M1	1.1b
	$B = 4$ and $C = -2$ which have been found using a correct identity	A1	1.1b
		(4)	

Q7.

Question Number	Scheme		Marks
	Method 1: Using one identity		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$A = 3$	their constant term = 3	B1
	$9x^2 + 20x - 10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either x^2: $9 = 3A$, x: $20 = 5A + 3B + C$ constant: $-10 = -2A - B + 2C$ or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using a correct identity.	A1
			[4]
	Method 2: Long Division		
	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{5x-4}{(x+2)(3x-1)}$	their constant term = 3	B1
	So, $\frac{5x-4}{(x+2)(3x-1)} \equiv \frac{B}{(x+2)} + \frac{C}{(3x-1)}$		
	$5x - 4 \equiv B(3x-1) + C(x+2)$	Forming a correct identity.	B1
	Either x: $5 = 3B + C$, constant: $-4 = -B + 2C$ or $x = -2 \Rightarrow -10 - 4 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	Attempts to find the value of either one of their B or their C from their identity.	M1
	$x = \frac{1}{3} \Rightarrow \frac{5}{3} - 4 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$	Correct values for their B and their C, which are found using $5x - 4 \equiv B(3x-1) + C(x+2)$	A1
	So, $\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv 3 + \frac{2}{(x+2)} - \frac{1}{(3x-1)}$		[4]
			4