Question 1 (OCR 4724, Jun 2005, Q7)

Worked Solution

$x = t^2, \ \ y = \dfrac{1}{t}.$

Part (i): Find $\dfrac{dy}{dx}$ in terms of t.

$$\frac{dx}{dt} = 2t, \qquad \frac{dy}{dt} = -\frac{1}{t^2}$$
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-1/t^2}{2t} = -\frac{1}{2t^3}$$

(i) $\dfrac{dy}{dx} = -\dfrac{1}{2t^3}$

Part (ii): Show that the tangent at $P\left(4, -\frac{1}{2}\right)$ is $x - 16y = 12$.

At P: $x = 4 = t^2 \Rightarrow t = \pm 2$. Also $y = \frac{1}{t} = -\frac{1}{2} \Rightarrow t = -2$.

$$m = -\frac{1}{2(-2)^3} = -\frac{1}{2(-8)} = \frac{1}{16}$$

Tangent: $y - \left(-\frac{1}{2}\right) = \frac{1}{16}(x - 4)$

$16y + 8 = x - 4 \implies x - 16y = 12$ ✓

(ii) Shown: $x - 16y = 12$.

Part (iii): Find the value of t at the point where the tangent meets the curve again.

The tangent is $x - 16y = 12$. Substitute $x = t^2$, $y = \frac{1}{t}$:

$$t^2 - \frac{16}{t} = 12 \implies t^3 - 12t - 16 = 0$$

We know $t = -2$ is a root. Factor:

$$(t + 2)(t^2 - 2t - 8) = (t + 2)(t - 4)(t + 2) = (t + 2)^2(t - 4) = 0$$

So $t = -2$ (repeated, the point P) or $t = 4$.

(iii) $t = 4$

Question 2 (OCR 4724, Jan 2009, Q6)

Worked Solution

$x = t^2 - 6t + 4,\ \ y = t - 3.$

Part (i): Find the coordinates where the curve meets the x-axis.

At the x-axis, $y = 0$:

$$t - 3 = 0 \implies t = 3$$

$$x = 9 - 18 + 4 = -5$$

(i) The curve meets the x-axis at $(-5, 0)$.

Part (ii): Find the Cartesian equation.

From $y = t - 3$: $t = y + 3$.

Substitute into x:

$$x = (y+3)^2 - 6(y+3) + 4 = y^2 + 6y + 9 - 6y - 18 + 4 = y^2 - 5$$

(ii) $x = y^2 - 5$

Part (iii): Find the equation of the tangent at $t = 2$ in the form $ax + by + c = 0$.

At $t = 2$: $x = 4 - 12 + 4 = -4,\ \ y = 2 - 3 = -1.$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2t - 6, \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = 1$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2t - 6}$$

At $t = 2$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{-2} = -\dfrac{1}{2}.$

Tangent through $(-4, -1)$ with gradient $-\frac{1}{2}$:

$$y + 1 = -\frac{1}{2}(x + 4) \implies 2y + 2 = -x - 4 \implies x + 2y + 6 = 0$$

(iii) $x + 2y + 6 = 0$

Question 3 (OCR 4724, Jun 2009, Q5)

Worked Solution

$x = 2t + t^2$, $y = 2t^2 + t^3$.

Part (i): Express $\frac{dy}{dx}$ in terms of t and find the gradient at $(3, -9)$.

$$\frac{dx}{dt} = 2 + 2t, \qquad \frac{dy}{dt} = 4t + 3t^2$$
$$\frac{dy}{dx} = \frac{4t + 3t^2}{2 + 2t} = \frac{t(4 + 3t)}{2(1 + t)}$$

At $(3, -9)$: find t. From $y = t^2(2 + t) = -9$ and $x = t(2 + t) = 3$:

From $x = t(2 + t) = 3$: $t^2 + 2t - 3 = 0 \Rightarrow (t + 3)(t - 1) = 0$, so $t = 1$ or $t = -3$.

Check y: at $t = 1$: $y = 2 + 1 = 3 \neq -9$; at $t = -3$: $y = 2(9) + (-27) = 18 - 27 = -9$. ✓

So $t = -3$:
$$\frac{dy}{dx} = \frac{(-3)(4 + 3(-3))}{2(1 + (-3))} = \frac{(-3)(-5)}{2(-2)} = \frac{15}{-4} = -\frac{15}{4}$$

(i) $\frac{dy}{dx} = \frac{t(4 + 3t)}{2(1 + t)}$; gradient at $(3, -9)$ is $-\frac{15}{4}$.

Part (ii): Find the Cartesian equation (no fractions).

Note $\frac{y}{x} = \frac{2t^2 + t^3}{2t + t^2} = \frac{t^2(2 + t)}{t(2 + t)} = t$, so $t = \frac{y}{x}$.

Substitute into $x = 2t + t^2$:

$$x = 2 \cdot \frac{y}{x} + \left(\frac{y}{x}\right)^2 = \frac{2y}{x} + \frac{y^2}{x^2}$$

Multiply through by x^2:
$$x^3 = 2xy + y^2$$

(ii) $x^3 = 2xy + y^2$

Question 4 (OCR 4724, Jun 2010, Q7)

Worked Solution

$x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

Part (i): Show that $\dfrac{dy}{dx} > 0$.

$$\frac{dx}{dt} = \frac{(t+1)\cdot 1 - (t+2)\cdot 1}{(t+1)^2} = \frac{-1}{(t+1)^2}$$

$$\frac{dy}{dt} = \frac{-2}{(t+3)^2}$$

$$\frac{dy}{dx} = \frac{-2/(t+3)^2}{-1/(t+1)^2} = \frac{2(t+1)^2}{(t+3)^2}$$

Since $(t+1)^2 \geq 0$ and $(t+3)^2 > 0$ for all valid t, and the numerator is $2(t+1)^2 \geq 0$.

When $t \neq -1$, $(t+1)^2 > 0$, so $\dfrac{dy}{dx} = \dfrac{2(t+1)^2}{(t+3)^2} > 0$. ✓

(i) $\dfrac{dy}{dx} = \dfrac{2(t+1)^2}{(t+3)^2} > 0$ since both $(t+1)^2 \geq 0$ and $(t+3)^2 > 0$, with equality only at $t = -1$ which is excluded.

Part (ii): Find the Cartesian equation (no fractions).

From $x = \dfrac{t+2}{t+1} = 1 + \dfrac{1}{t+1}$: $\quad t+1 = \dfrac{1}{x-1}$, so $t = \dfrac{1}{x-1} - 1 = \dfrac{2-x}{x-1}$.

Then $t+3 = \dfrac{2-x}{x-1} + 3 = \dfrac{2-x+3(x-1)}{x-1} = \dfrac{2x-1}{x-1}$.

Substitute into y:

$$y = \frac{2}{(2x-1)/(x-1)} = \frac{2(x-1)}{2x-1}$$

Cross-multiply: $y(2x-1) = 2(x-1) \implies 2xy - y = 2x - 2 \implies 2x + y = 2xy + 2$.

(ii) $2x + y = 2xy + 2$

Question 5 (OCR 4724, Jan 2011, Q4)

Worked Solution

$x = 2 + t^2, \;\; y = 4t.$

Part (i): Find $\dfrac{dy}{dx}$ in terms of t.

$$\frac{dx}{dt} = 2t, \qquad \frac{dy}{dt} = 4$$
$$\frac{dy}{dx} = \frac{4}{2t} = \frac{2}{t}$$

(i) $\dfrac{dy}{dx} = \dfrac{2}{t}$

Part (ii): Find the equation of the normal at $t = 4$ in the form $y = mx + c$.

At $t = 4$: $x = 2 + 16 = 18, \;\; y = 16.$

Gradient of tangent $= \dfrac{2}{4} = \dfrac{1}{2}$, so gradient of normal $= -2$.

Normal: $y - 16 = -2(x - 18) \implies y = -2x + 36 + 16 = -2x + 52.$

(ii) $y = -2x + 52$

Part (iii): Find the Cartesian equation.

From $y = 4t$: $t = \dfrac{y}{4}$.

$$x = 2 + \left(\frac{y}{4}\right)^2 = 2 + \frac{y^2}{16}$$

(iii) $x = 2 + \dfrac{y^2}{16}$, equivalently $y^2 = 16(x - 2)$.

Question 6 (OCR 4724, Jun 2011, Q8)

Worked Solution

$x = \dfrac{1}{t+1}$, $y = t - 1$. The line $y = 3x$ intersects the curve at two points.

Part (i): Show $t = -2$ at one intersection; find t at the other.

Substitute $x = \dfrac{1}{t+1}$, $y = t - 1$ into $y = 3x$:

$$t - 1 = \frac{3}{t+1} \implies (t-1)(t+1) = 3 \implies t^2 - 1 = 3 \implies t^2 = 4 \implies t = \pm 2$$

So $t = -2$ (confirmed) and $t = 2$.

(i) $t = -2$ (shown) and $t = 2$.

Part (ii): Find the equation of the normal at $t = -2$.

At $t = -2$: $x = \dfrac{1}{-1} = -1$, $y = -3$.

$$\frac{dx}{dt} = -\frac{1}{(t+1)^2}, \qquad \frac{dy}{dt} = 1$$

$$\frac{dy}{dx} = \frac{1}{-1/(t+1)^2} = -(t+1)^2$$

At $t = -2$: $\dfrac{dy}{dx} = -(-1)^2 = -1$. Gradient of normal $= 1$.

Normal: $y + 3 = 1(x + 1) \implies y = x - 2$.

(ii) $y = x - 2$

Part (iii): Find the value of t where this normal meets the curve again.

Substitute $x = \dfrac{1}{t+1}$, $y = t - 1$ into $y = x - 2$:

$$t - 1 = \frac{1}{t+1} - 2 \implies (t-1)(t+1) = 1 - 2(t+1) \implies t^2 - 1 = -2t - 1$$

$$t^2 + 2t = 0 \implies t(t+2) = 0$$

$t = 0$ or $t = -2$ (the original point). So the normal meets the curve again at $t = 0$.

(iii) $t = 0$

Part (iv): Find the Cartesian equation in the form $y = f(x)$.

From $x = \dfrac{1}{t+1}$: $t + 1 = \dfrac{1}{x}$, so $t = \dfrac{1}{x} - 1$.

$$y = t - 1 = \frac{1}{x} - 2$$

(iv) $y = \dfrac{1}{x} - 2$

Question 7 (OCR 4724, Jan 2013, Q5)

Worked Solution

$x = 2 + 3\sin\theta,\ \ y = 1 - 2\cos\theta, \quad 0 \le \theta \le \frac{1}{2}\pi.$

Part (i): Find the coordinates where the gradient is $\frac{1}{2}$.

$$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\cos\theta, \qquad \frac{\mathrm{d}y}{\mathrm{d}\theta} = 2\sin\theta$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sin\theta}{3\cos\theta} = \frac{2\tan\theta}{3} = \frac{1}{2}$$
$$\tan\theta = \frac{3}{4}$$

With $\tan\theta = \frac{3}{4}$: in a 3-4-5 triangle, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$.

$$x = 2 + 3\cdot\frac{3}{5} = 2 + \frac{9}{5} = \frac{19}{5}, \qquad y = 1 - 2\cdot\frac{4}{5} = 1 - \frac{8}{5} = -\frac{3}{5}$$

(i) Coordinates: $\left(\frac{19}{5}, -\frac{3}{5}\right)$

Part (ii): Find the Cartesian equation.

$$\sin\theta = \frac{x-2}{3}, \qquad \cos\theta = \frac{1-y}{2}$$

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{1-y}{2}\right)^2 = 1$$

(ii) $\frac{(x-2)^2}{9} + \frac{(1-y)^2}{4} = 1$
or equivalently $4(x-2)^2 + 9(1-y)^2 = 36$.

Question 8 (OCR 4724, Jun 2013, Q9)

Worked Solution

$x = \frac{1}{t} - 1, \;\; y = 2t + \frac{1}{t^2}.$

Part (i): Find $\frac{dy}{dx}$ in terms of t.

$$\frac{dx}{dt} = -\frac{1}{t^2}, \qquad \frac{dy}{dt} = 2 - \frac{2}{t^3}$$

$$\frac{dy}{dx} = \frac{2 - 2/t^3}{-1/t^2} = \frac{(2t^3 - 2)/t^3}{-1/t^2} = \frac{2t^3 - 2}{t^3} \cdot (-t^2) = -t^2 \cdot \frac{2(t^3 - 1)}{t^3} = \frac{2 - 2t^3}{t}$$

Simplifying: $\frac{dy}{dx} = -2t^2\left(1 - \frac{1}{t^3}\right) \cdot \frac{1}{1} = \frac{(2 - 2t^3)}{t}.$

More cleanly: $\frac{dy}{dt} = 2 - 2t^{-3}$, $\frac{dx}{dt} = -t^{-2}$:

$$\frac{dy}{dx} = \frac{2 - 2t^{-3}}{-t^{-2}} = \frac{2t^{-3} - 2}{t^{-2}} \cdot (-1) = -(2t^{-1} - 2t^2) = 2t^2 - \frac{2}{t}$$

> **(i)** $\frac{dy}{dx} = 2t^2 - \frac{2}{t}$

Part (ii): Find the stationary point coordinates and determine its nature.

Set $\frac{dy}{dx} = 0$:

$$2t^2 - \frac{2}{t} = 0 \implies 2t^3 = 2 \implies t = 1$$

At $t = 1$: $x = 1 - 1 = 0$, $y = 2 + 1 = 3$. Stationary point at $(0, 3)$.

Check nature by considering gradient on either side: - At $t = 0.5$: $\frac{dy}{dx} = 2(0.25) - \frac{2}{0.5} = 0.5 - 4 = -3.5 < 0$. - At $t = 2$: $\frac{dy}{dx} = 2(4) - 1 = 7 > 0$.

Gradient goes from negative to positive ⇒ **minimum**.

> **(ii)** $(0, 3)$ is a **minimum** point.

Part (iii): Find the Cartesian equation.

From $x = \frac{1}{t} - 1$: $t = \frac{1}{x+1}$.

$$y = 2t + \frac{1}{t^2} = \frac{2}{x+1} + (x+1)^2$$

(iii) $y = (x+1)^2 + \dfrac{2}{x+1}$

Question 9 (OCR 4724, Jun 2014, Q7)

Worked Solution

$x = 2\sin t,\ \ y = \cos 2t + 2\sin t, \quad -\frac{1}{2}\pi \le t \le \frac{1}{2}\pi.$

Part (i): Show $\dfrac{dy}{dx} = 1 - 2\sin t$ and find the stationary point.

$$\frac{dx}{dt} = 2\cos t, \qquad \frac{dy}{dt} = -2\sin 2t + 2\cos t$$

Using $\sin 2t = 2\sin t\cos t$:

$$\frac{dy}{dt} = -4\sin t\cos t + 2\cos t = 2\cos t(1 - 2\sin t)$$

$$\frac{dy}{dx} = \frac{2\cos t(1 - 2\sin t)}{2\cos t} = 1 - 2\sin t \quad (\text{for } \cos t \neq 0)\checkmark$$

Stationary point: $\dfrac{dy}{dx} = 0 \Rightarrow \sin t = \dfrac{1}{2} \Rightarrow t = \dfrac{\pi}{6}$.

$$x = 2\sin\frac{\pi}{6} = 1, \qquad y = \cos\frac{\pi}{3} + 2\sin\frac{\pi}{6} = \frac{1}{2} + 1 = \frac{3}{2}$$

(i) $\dfrac{dy}{dx} = 1 - 2\sin t$ (shown). Stationary point: $\left(1, \dfrac{3}{2}\right)$.

Part (ii): Find the Cartesian equation.

Use $\cos 2t = 1 - 2\sin^2 t$ and $\sin t = \dfrac{x}{2}$:

$$y = 1 - 2\sin^2 t + 2\sin t = 1 - 2\left(\frac{x}{2}\right)^2 + 2\cdot\frac{x}{2} = 1 - \frac{x^2}{2} + x$$

(ii) $y = 1 + x - \dfrac{x^2}{2}$

Part (iii): State the set of values that x can take and sketch the curve.

Since $-\frac{1}{2}\pi \le t \le \frac{1}{2}\pi$, we have $-1 \le \sin t \le 1$, so $-2 \le x \le 2$.

The Cartesian equation $y = 1 + x - \frac{x^2}{2}$ is a downward-opening parabola with vertex at $(1, \frac{3}{2})$, x-intercepts at $x = 1 \pm \sqrt{3}$, and y-intercept at $(0, 1)$.

(iii) $-2 \le x \le 2$.
The curve is a parabola $y = 1 + x - \frac{x^2}{2}$ (downward), with vertex $\left(1, \frac{3}{2}\right)$, endpoints $(-2, -3)$ and $(2, 1)$, passing through $(0, 1)$.

Question 10 (OCR 4724, Jun 2015, Q10)

Worked Solution

Part (i): Express $\frac{x+8}{x(x+2)}$ in partial fractions.

$$\frac{x+8}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$
$$x+8 = A(x+2) + Bx$$

$x=0$: $8 = 2A \Rightarrow A = 4$. $x=-2$: $6 = -2B \Rightarrow B = -3$.

(i) $\frac{x+8}{x(x+2)} = \frac{4}{x} - \frac{3}{x+2}$

Part (ii): Express $\frac{7x^2+16x+16}{x(x+2)}$ in the form $P + \frac{Q}{x} + \frac{R}{x+2}$.

Perform polynomial division: $7x^2+16x+16 \div x(x+2) = 7x^2+16x+16 \div (x^2+2x)$.

$7x^2+16x+16 = 7(x^2+2x) + (2x+16)$.

So $\frac{7x^2+16x+16}{x(x+2)} = 7 + \frac{2x+16}{x(x+2)}$.

Now decompose $\frac{2x+16}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$: $2x+16 = A(x+2)+Bx$; $x=0$: $A=8$; $x=-2$: $-6 = -2B \Rightarrow B = -3$... wait: $16-4 = -2B \Rightarrow 12 = -2B \Rightarrow B = -6$.

Check: $x=0$: $16 = 2A \Rightarrow A = 8$. $x=-2$: $12 = -2B \Rightarrow B = -6$.

(ii) $\frac{7x^2+16x+16}{x(x+2)} = 7 + \frac{8}{x} - \frac{6}{x+2}$, so $P=7$, $Q=8$, $R=-6$.

Part (iii): Given $x = \frac{2t}{1-t}$, $y = 3t + \frac{4}{t}$, show the Cartesian equation is $y = \frac{7x^2+16x+16}{x(x+2)}$.

From $x = \frac{2t}{1-t}$: $x(1-t) = 2t \Rightarrow x - xt = 2t \Rightarrow x = t(2+x) \Rightarrow t = \frac{x}{x+2}$.

Substitute into y:

$$y = 3 \cdot \frac{x}{x+2} + \frac{4(x+2)}{x} = \frac{3x^2+4(x+2)^2}{x(x+2)} = \frac{3x^2+4x^2+16x+16}{x(x+2)} = \frac{7x^2+16x+16}{x(x+2)} \checkmark$$

(iii) Shown: $y = \frac{7x^2+16x+16}{x(x+2)}$.

Part (iv): Find the area bounded by the curve, the x-axis, and lines $x=1$, $x=2$. Give answer in the form $L + M\ln 2 + N\ln 3$.

Using part (ii):

$$\int_1^2 \left(7 + \frac{8}{x} - \frac{6}{x+2}\right)\,\mathrm{d}x = \Big[7x + 8\ln x - 6\ln(x+2)\Big]_1^2$$

$$= \big(14 + 8\ln 2 - 6\ln 4\big) - \big(7 + 8\ln 1 - 6\ln 3\big)$$

$$= 7 + 8\ln 2 - 12\ln 2 + 6\ln 3 = 7 - 4\ln 2 + 6\ln 3$$

(iv) Area $= 7 - 4\ln 2 + 6\ln 3$, so $L = 7$, $M = -4$, $N = 6$.

Question 11 (OCR 4724, Jun 2016, Q9)

Worked Solution

$x = 1 - \cos t,\ \ y = \sin t \sin 2t, \quad 0 \le t \le \pi.$

Part (i): Find the coordinates where the curve meets the x-axis.

$y = 0 \Rightarrow \sin t \sin 2t = 0.$

$\sin t = 0$: $t = 0$ or $t = \pi$. $\sin 2t = 0$: $2t = 0, \pi, 2\pi$, so $t = 0, \frac{\pi}{2}, \pi$.

Values: $t = 0, \frac{\pi}{2}, \pi$.

- $t = 0$: $x = 1 - 1 = 0$ - $t = \frac{\pi}{2}$: $x = 1 - 0 = 1$ - $t = \pi$: $x = 1 - (-1) = 2$

(i) The curve meets the x-axis at $(0, 0)$, $(1, 0)$, and $(2, 0)$.

Part (ii): Show $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2t + 2\cos^2 t$, and find the stationary points.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sin t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \cos t \sin 2t + \sin t \cdot 2\cos 2t = 2\sin t \cos t \cos t + 2\sin t \cos 2t = 2\sin t \cos^2 t + 2\sin t \cos 2t$$

$$= 2\sin t(\cos^2 t + \cos 2t)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sin t(\cos^2 t + \cos 2t)}{\sin t} = 2(\cos^2 t + \cos 2t)$$

Using $\cos 2t = 2\cos^2 t - 1$:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos^2 t + 2\cos 2t \checkmark$$

Stationary points: $2\cos^2 t + 2\cos 2t = 0 \Rightarrow \cos^2 t + \cos 2t = 0.$

Using $\cos 2t = 2\cos^2 t - 1$: $3\cos^2 t - 1 = 0 \Rightarrow \cos^2 t = \frac{1}{3} \Rightarrow \cos t = \pm\frac{1}{\sqrt{3}}$ (both valid in $[0, \pi]$).

For $\cos t = \frac{1}{\sqrt{3}}$ $(0 < t < \frac{\pi}{2})$: $\sin t = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$, $x = 1 - \frac{1}{\sqrt{3}}$, $\cos 2t = \frac{2}{3} - 1 = -\frac{1}{3}$, $\sin 2t = 2 \cdot \frac{1}{\sqrt{3}} \cdot \sqrt{\frac{2}{3}} = \frac{2\sqrt{2}}{3}$. $y = \sqrt{\frac{2}{3}} \cdot \frac{2\sqrt{2}}{3} = \frac{2 \cdot \sqrt{2} \cdot \sqrt{2}}{3\sqrt{3}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$.

Similarly for $\cos t = -\frac{1}{\sqrt{3}}$ $(\frac{\pi}{2} < t < \pi)$: $x = 1 + \frac{1}{\sqrt{3}}$, $y = -\frac{4\sqrt{3}}{9}$.

(ii) Stationary points: $\left(1 - \frac{1}{\sqrt{3}}, \frac{4\sqrt{3}}{9}\right)$ and $\left(1 + \frac{1}{\sqrt{3}}, -\frac{4\sqrt{3}}{9}\right)$.

Part (iii): Find the Cartesian equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x)$ is a polynomial.

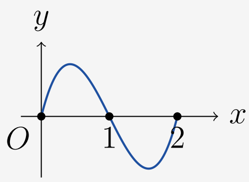
Let $c = \cos t = 1 - x$. Then $\sin^2 t = 1 - (1-x)^2 = 2x - x^2$.
$\sin 2t = 2\sin t \cos t$, $\sin t = \sqrt{2x - x^2}$ (taking positive root since $0 \le t \le \pi$).

$$y = \sin t \cdot \sin 2t = \sin t \cdot 2\sin t \cos t = 2\sin^2 t \cos t = 2(2x - x^2)(1 - x)$$
$$= 2(2x - x^2 - 2x^2 + x^3) = 2x^3 - 6x^2 + 4x$$

(iii) $y = 2x^3 - 6x^2 + 4x$

Part (iv): Sketch the curve.

The curve is a cubic $y = 2x(x^2 - 3x + 2) = 2x(x-1)(x-2)$ on $0 \le x \le 2$, with roots at $x = 0, 1, 2$ and a local maximum then minimum between these roots.



(iv) Cubic curve crossing x-axis at $x = 0, 1, 2$, with one maximum and one minimum between.

End of Worked Solutions