

Parametric Equations (From OCR 4724)

Q1, (Jun 2005, Q7)

(i) $dy/dx = (dy/dt) / (dx/dt)$ $= (-1/t^2) / 2t$ as unsimplified expression	M1 A1	(S.R.Award M1 for attempt to change to cartesian eqn & differentiate + A1 for dy/dx or dx/dy in terms of x or y)
$= -1 / 2t^3$ as simplified expression	A1 3	Not $1/-2t^3$. Not in terms of x &/or y .
(ii) $(4, -1/2) \rightarrow t = -2$ <u>only</u> Satis attempt to find equation of tgt $x - 16y = 12$ <u>only</u>	B1 M1 A1 3	Using $t = -2$ or 2 AG
(iii) $t^3 - 12t - 16 = 0$ <u>or</u> $16y^3 + 12y^2 - 1 = 0$ <u>or</u> $x^3 - 24x^2 + 144x - 256 = 0$ $t = 4$ (only) ISW giving cartesian coords	M1 A1 B2 4	For substituting $(t^2, 1/t)$ into tgt eqn <u>or</u> solving simult tgt & their cartes eqns For simplified equiv non-fract cubic S.R. Award B1 for "4 or -2". S.R. If B0, award M1 for clear indic of method of soln of correct eqn. 10

Q2, (Jan 2009, Q6)

(i) Solve $0 = t - 3$ & subst into $x = t^2 - 6t + 4$ Obtain $x = -5$ N.B. If (ii) completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)	M1 A1 (2)	(-5,0) need not be quoted
(ii) Attempt to eliminate t Simplify to $x = y^2 - 5$ ISW	M1 A1 (2)	
(iii) Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or $(-)\frac{1}{2}(x+5)^{-\frac{1}{2}}$ If $t = 2$, $x = -4$ and $y = -1$ Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn $x + 2y + 6 = 0$ AEF(without fractions) ISW	M1 A1 B1 M1 A1 (5)	Award anywhere in Que Awarded anywhere in (iii)

Q3, (Jun 2009, Q5)

(i) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ aef used M1

$= \frac{4t + 3t^2}{2 + 2t}$ A1

Attempt to find t from one/both equations M1 or diff (ii) cartesian eqn $\rightarrow$ M1

State/imply $t = -3$ is only solution of both equations A1 subst(3,-9), solve for $\frac{dy}{dx} \rightarrow$ M1

Gradient of curve $= -\frac{15}{4}$ or $\frac{-15}{4}$ or $\frac{15}{-4}$ A1 5 grad of curve $= -\frac{15}{4} \rightarrow$ A1

[SR If $t = 1$ is given as solution & not disqualified, award A0 + $\sqrt{A1}$ for grad $= -\frac{15}{4}$ & $\frac{7}{4}$;

If $t = 1$ is given/used as only solution, award A0 + $\sqrt{A1}$ for grad $= \frac{7}{4}$]

(ii) $\frac{y}{x} = t$ B1

Substitute into either parametric eqn M1

Final answer $x^3 = 2xy + y^2$ A2 4

[SR Any correct unsimplified form (involving fractions or common factors) $\rightarrow$ A1]

Q4, (Jun 2010, Q7)

- (i) Differentiate x as a quotient, $\frac{v \, du - u \, dv}{v^2}$ or $\frac{u \, dv - v \, du}{v^2}$ M1 or product clearly defined

$$\frac{dx}{dt} = -\frac{1}{(t+1)^2} \text{ or } \frac{-1}{(t+1)^2} \text{ or } -(t+1)^{-2} \quad \text{A1} \quad \text{WWW} \rightarrow 2$$

$$\frac{dy}{dt} = -\frac{2}{(t+3)^2} \text{ or } \frac{-2}{(t+3)^2} \text{ or } -2(t+3)^{-2} \quad \text{B1}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{M1} \quad \text{quoted/implicit and used}$$

$$\frac{dy}{dx} = \frac{2(t+1)^2}{(t+3)^2} \text{ or } \frac{2(t+3)^{-2}}{(t+1)^{-2}} \quad (\text{dep } 1^{\text{st}} \text{ 4 marks}) \quad * \text{A1} \quad \text{ignore ref } t = -1, t = -3$$

State squares +ve or $(t+1)^2$ & $(t+3)^2$ +ve $\therefore \frac{dy}{dx}$ +ve dep* A1 6 or $\left(\frac{t+1}{t+3}\right)^2$ +ve. Ignore ≥ 0

- (ii) Attempt to obtain t from either the x or y equation M1 No accuracy required

$$t = \frac{2-x}{x-1} \quad \text{AEF} \quad \text{or} \quad t = \frac{2}{y} - 3 \quad \text{AEF} \quad \text{A1}$$

Substitute in the equation not yet used in this part M1 or equate the 2 values of t

Use correct meth to eliminate ('double-decker') fractions M1

Obtain $2x + y = 2xy + 2$ ISW AEF A1 5 but not involving fractions

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Q5, (Jan 2011, Q4)

- (i) Attempt to use $\frac{dy}{dx}$ or $\frac{dy}{dt} \cdot \frac{dt}{dx}$ M1 Not just quote formula

$$\frac{4}{2t} \text{ or } \frac{2}{t} \quad \text{A1} \quad 2$$

- (ii) Subst $t = 4$ into their (i), invert & change sign M1

Subst $t = 4$ into (x,y) & use num grad for tgt/normal M1

$$y = -2x + 52 \quad \text{AEF} \quad \text{CAO (no f.t.)} \quad \text{A1} \quad 3 \quad \text{Only the eqn of normal accepted}$$

- (iii) Attempt to eliminate t from the 2 given equations M1

$$x = 2 + \frac{y^2}{16} \text{ or } y^2 = 16(x-2) \quad \text{AEF} \quad \text{ISW} \quad \text{A1} \quad 2 \quad \text{Mark at earliest acceptable form.}$$

Q6, (Jun 2011, Q8)

Cartesian equation may be used in parts (i) - (iii) and corresponding marks awarded

- (i) Sub parametric eqns into $y = 3x$ & produce $t = -2$

OR sub $t = -2$ into para eqs, obtain $(-1, -3)$ & state $y = 3x$

OR other similar methods producing (or verifying) $t = -2$ B1

Value of t at other point is 2

B1 2

$t = \pm 2$ is sufficient for B1+B1

- (ii) Use (not just quote) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

M1

$$= -(t+1)^2$$

A1

or $\frac{-1}{x^2}$ or $\frac{-(2+y)}{x}$

Attempt to use $-\frac{1}{\frac{dy}{dx}}$ for gradient of normal

M1

Gradient normal = 1 cao

A1

Subst $t = -2$ into the parametric eqns.

M1

to find pt at which normal is drawn

Produce $y = x - 2$ as equation of the normal WWW

A1 6

'A' marks in (ii) are dep on prev 'A'

- (iii) Substitute the parametric values into their eqn of normal M1

Produce $t = 0$ as final answer cao

A1 2

This is dep on final A1 in (ii)

N.B. If $y = x - 2$ is found fortuitously in (ii) (& $\therefore$ given A0 in (ii)), you must award A0 here in (iii).

- (iv) Attempt to eliminate t from the parametric equations M1

Produce any correct equation

A1

e.g. $x = \frac{1}{y+2}$

Produce $y = \frac{1}{x} - 2$ or $y = \frac{1-2x}{x}$ ISW

A1 3

Must be seen in (iv)

{N.B. Candidate producing only $y = \frac{1}{x} - 2$ is awarded both A1 marks.}

(i)	their $\frac{dy}{d\theta} / \frac{dx}{d\theta}$ $\frac{dy}{dx} = \frac{2 \sin \theta}{3 \cos \theta}$ their $\frac{dy}{dx} = \frac{1}{2}$ $\tan \theta = \frac{3}{4}$ $(3.8, -0.6)$ or $\left(\frac{19}{5}, -\frac{3}{5}\right)$ or $x = 3.8, y = -0.6$	M1 A1 M1 A1 A1 [5]	If $\tan \theta = \frac{3}{4}$ not seen, award this A1 only if coords are correct	
(ii)	Manipulating equations into form $\sin \theta = f(x)$ and $\cos \theta = g(y)$ and then using $\sin^2 \theta + \cos^2 \theta = 1$ $\frac{(x-2)^2}{9} + \frac{(1-y)^2}{4} = 1$ oe www ISW Accept e.g. $\left(\frac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	M1 A1 [2]	If part (ii) is attempted first, and then part (i), allow B1 for obtaining $\frac{dy}{dx} = \frac{4(x-2)}{9(y-1)}$ M1 for equating their $\frac{dy}{dx}$ to $\frac{1}{2}$ A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn... A1 for $(3.8, -0.6)$	<u>the following marks in part (i):-</u> and their Cartesian equation

Q8, (Jun 2013, Q9)

(i)	$\frac{dy}{dt} = 2(+)-\frac{2}{t^3}; \quad \frac{dx}{dt} = -\frac{1}{t^2} \quad \text{oe soi ISW}$ $\frac{2}{t} - 2t^2 \text{ or } -2\left(t^2 - \frac{1}{t}\right), \frac{2t^3 - 2}{-t}, -t^2\left(2 - \frac{2}{t^3}\right) \text{ oe}$	B1, B1 B1 [3]	ISW. Must not involve (implied) 'triple-deckers' e.g. fractions with neg powers...	... e.g. $\frac{2 - 2t^{-3}}{-t^2}$
(ii)	(Any of their expressions for $\frac{dy}{dx}$) = 0 or their $\frac{dy}{dt} = 0$ $t = 1 \rightarrow$ (stationary point) = (0, 3) Consider values of x on each side of their critical value of x which lead to finite values of $\frac{dy}{dx}$ Hence (0, 3) is a minimum point www	M1 A1 M1 A1 [4]	Not awarded if $\frac{dy}{dx}$ is wrong in (i) and used here BUT allow recovery if only explicitly considering $\frac{dy}{dt} = 0$ Totally satis; values of x must be close to 0 & not going below or equal to $x = -1$	
(iii)	Attempt to find t from $x = \frac{1}{t} - 1$ and substitute into the equation for y $y = \frac{2}{x+1} + (x+1)^2$ oe (can be unsimplified) ISW	M1 A1 [2]		

Q9, (Jun 2014, Q7)

(i)	$\frac{dy}{dt} = -2 \sin 2t + 2 \cos t \text{ soi}$ $\frac{dy}{dx} = \text{their } \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \text{ oe}$ $\frac{-2 \sin 2t + 2 \cos t}{2 \cos t} \text{ soi}$ $\frac{-4 \sin t \cos t + 2 \cos t}{2 \cos t} \text{ or } \frac{2 \cos t(-2 \sin t + 1)}{2 \cos t} \text{ and}$ completion to $1 - 2 \sin t$ www $(1, 1\frac{1}{2})$	B1 M1 A1 A1 B1 [5]	NB $\frac{dx}{dt} = 2 \cos t$ or equivalent intermediate step NB $t = \frac{\pi}{6}$	if B0M0A0 SC3 for $\frac{dy}{dx} = 1 - x$ from correct Cartesian equation seen in part (i) or part (ii) B1 for substitution of $x = 2 \sin t$ from $1 - 2 \sin t = 0$
	$(y =) 1 - 2 \sin^2 t + 2 \sin t$ substitution of $\sin t = \frac{1}{2}x$ to eliminate t $y = 1 + x - \frac{1}{2}x^2$ oe isw	B1 M1 A1 [3]	may be awarded after correct substitution for x eg $(y =) 1 - \frac{x^2}{4} - \sin^2 t + 2 \sin t$ or B3 www	or $(y =) x + \cos 2t$ substitution of $t = \sin^{-1}(\frac{x}{2})$ to eliminate t $y = x + \cos 2(\sin^{-1}(\frac{x}{2}))$ oe isw

(iii)	$-2 \leq x \leq 2$ or $x \geq -2$ (and) $x \leq 2$ or $ x \leq 2$ sketch of negative quadratic with endpoints in 1 st and 3 rd quadrants positive y-intercept and one distinguishing feature is w	B1 M1 A1 [3]	cao RH point must be to the right of the maximum one from: endpoints $(-2, -3)$ and $(2, 1)$, vertex at $(1, 1\frac{1}{2})$, y-intercept is $(0, 1)$, x-intercept is $(1 - \sqrt{3}, 0)$	
Q10, (Jun 2015, Q10)				
(i)	$\frac{A}{x} + \frac{B}{x+2}$ $x + 8 = A(x+2) + Bx$ so $A = 4$ and $B = -3$	B1 M1 A1 [3]	allow one sign error	award if only implied by answer clearing fractions successfully if M0 , B1 for each value www
(ii)	quotient (P) is 7 $2x + 16$ seen $7 + \frac{8}{x} - \frac{6}{x+2}$	B1 B1 B1 [3]	if B0 , B1 for $Q=8$ and B1 for $R=-6$ www	eg as remainder or in division chunking or allow $P=7, Q=8, R=-6$

(iii)	$t = f(x)$ $t = \frac{x}{x+2}$ $y = 3 \times \text{their } \frac{x}{x+2} + \frac{4}{\text{their } \frac{x}{x+2}}$ eg $\frac{3x^2 + (8+4x)(x+2)}{x(x+2)}$ and completion to $y = \frac{7x^2 + 16x + 16}{x(x+2)}$ www AG	M1* from $x = \frac{2t}{1-t}$; M0 for $t = g(y)$ A1 or B2 if unsupported M1dep* A1	at least one correct, constructive, intermediate step shown if M0M0 , SC2 for substitution of $x = \frac{2t}{1-t}$ in RHS of given equation and completion with at least two correct, constructive intermediate steps to $y = 3t + \frac{4}{t}$ www
(iv)	$\int \text{their } (P + \frac{Q}{x} + \frac{R}{x+2}) [dx]$ $F[x] = 7x + 8\ln x - 6\ln(x+2)$ $F[2] - F[1]$ $7 - 4\ln 2 + 6\ln 3$	[4] M1* where P , Q and R are constants obtained in (ii) A1FT allow recovery from omission of brackets in subsequent working M1dep* A1 [4]	allow omission of dx if M0 , SC1 for $Px + Q\ln x + R\ln(x+2)$ where constants are unspecified or arbitrary

Q11. (Jun 2016, Q9)

(i)	$\sin t \sin 2t = 0$ oe seen $(0, 0)$ $(1, 0)$ and $(2, 0)$ or $x = 0, x = 1, x = 2$ cao	M1 A2 [3]	A1 for two of three correct	NB $t = 0, \frac{1}{2}\pi, \pi$ deduct 1 mark if all three correct plus extra values if A0 , allow SC1 for $t = 0, \frac{1}{2}\pi, \pi$ if unsupported, full marks for all three values correct
(ii)	$\left[\frac{dy}{dt} \right] = 2 \sin t \cos 2t + \cos t \sin 2t$ $\frac{(2 \sin t \cos 2t + \cos t \sin 2t)}{\sin t}$ or $\frac{(4 \sin t \cos^2 t - 2 \sin^3 t)}{\sin t}$ substitution of $\sin 2t = 2 \sin t \cos t$ in their $\frac{(2 \sin t \cos 2t + \cos t \sin 2t)}{\sin t}$ and completion to $2 \cos 2t + 2 \cos^2 t$ www NB AG eg $2(2 \cos^2 t - 1) + 2 \cos^2 t = 0$ or $2 \cos 2t + 2 \times \frac{1}{2}(1 + \cos 2t) = 0$ $(1 + \frac{1}{\sqrt{3}}, \frac{-4}{3\sqrt{3}})$ oe isw $(1 - \frac{1}{\sqrt{3}}, \frac{4}{3\sqrt{3}})$ oe isw	B1 M1 M1 A1 M1 A1 A1 [7]	or $4 \sin t \cos^2 t - 2 \sin^3 t$ allow sign errors and/or one incorrect coefficient may be seen before differentiation at least one correct intermediate step needed use of double angle formula to obtain quadratic equation in eg $\cos t$ or linear equation in $\cos 2t$; may be seen before differentiation if A0 , A0 , allow A1 for both x values correct	mark intent: allow sign error, bracket error, omission of one coefficient eg $(\frac{\sqrt{3} + 3}{3}, -\frac{4\sqrt{3}}{9})$

(iii)	$y = 2(1 - \cos^2 t) \cos t$ oe may be implicit equation, may be implied by partial substitution for $\cos t$ eg $(1 - x)^2 + \frac{y}{2 \cos t} = 1$ $y = 2(1 - (1 - x)^2)(1 - x)$ $y = 2x^3 - 6x^2 + 4x$ or $y = 2x(x^2 - 3x + 2)$ or $y = 2x(x - 1)(x - 2)$ oe cao	M1	or $\frac{dy}{dx} = 6 \cos^2 t - 2$	use of double angle formula (and Pythagoras) to obtain expression for y or $\frac{dy}{dx}$ in terms of $\cos t$ only;
		M1	or $\frac{dy}{dx} = 6(1 - x)^2 - 2$	substitution of $\cos t = \pm 1 \pm x$ to obtain expression in terms of x only
		A1	integration and substitution of eg $(0, 0)$ to obtain correct answer must see $y =$ at some stage for A1	allow sign errors, bracket errors or minor slips in arithmetic eg omission of 2 for these method marks
		[3]		
(iv)	cubic with two turning points and of correct orientation through $(0, 0)$ x-intercepts correct and only for $0 \leq x \leq 2$	M1		
		A1		
		[2]		