



Parametric Equations and Differentiation (Sheet 2)

Q1.

The curve C has parametric equations

$$x = 3t - 4, y = 5 - \frac{6}{t}, \quad t > 0$$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P . Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$$y = \frac{ax + b}{x + 4}, \quad x > -4$$

where a and b are integers to be determined.

(3)

(Total for question = 8 marks)

Q2.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(4\sqrt{3}, \frac{15}{2}\right)$.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P .

Give your answer as a simplified surd.

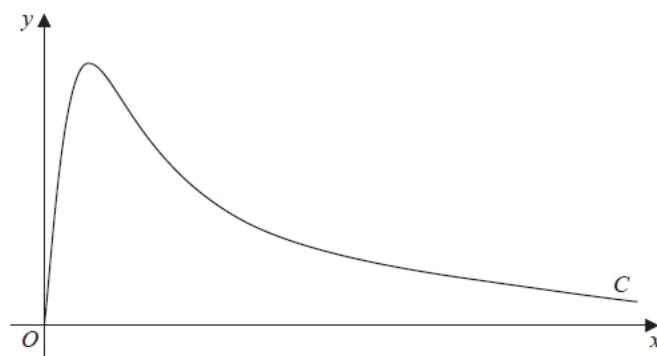


Figure 2

(4)

The point Q lies on the curve C , where $\frac{dy}{dx} = 0$

(b) Find the exact coordinates of the point Q .

(2)

(Total for question = 6 marks)



Q3.

The curve C has parametric equations

$$x = 2 \cos t, \quad y = \sqrt{3} \cos 2t, \quad 0 \leq t \leq \pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t .

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P .

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q .

(c) Find the exact coordinates of Q .

You must show clearly how you obtained your answers.

(6)

(Total for question = 13 marks)

Q4.

A curve has parametric equations

$$x = \tan^2 t, \quad y = \sin t, \quad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . You need not simplify your answer.

(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.

(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

(Total 12 marks)



Q5.

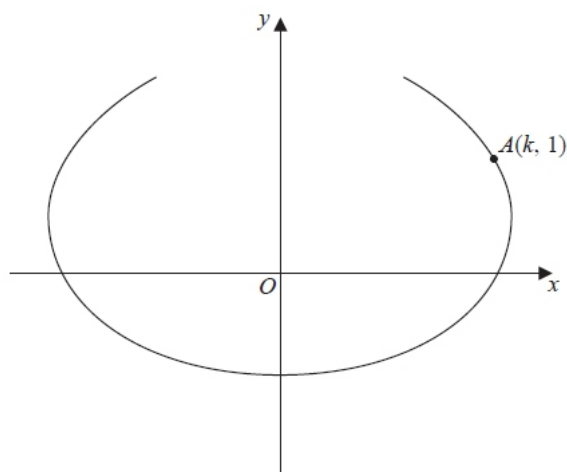


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = t - 4 \sin t, \quad y = 1 - 2 \cos t, \quad -\frac{2\pi}{3} \leq t \leq \frac{2\pi}{3}$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$

(a) find the exact value of k ,

(2)

(b) find the gradient of the curve at the point A.

(4)

There is one point on the curve where the gradient is equal to $-\frac{1}{2}$

(c) Find the value of t at this point, showing each step in your working and giving your answer to 4 decimal places.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(6)

(Total 12 marks)

Q6.

The curve C has parametric equations

$$x = \sin 2\theta \quad y = \operatorname{cosec}^3 \theta \quad 0 < \theta < \frac{\pi}{2}$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of θ

(3)

(b) Hence find the exact value of the gradient of the tangent to C at the point where $y = 8$

(3)

(Total for question = 6 marks)



Q7.

A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

- (a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

- (b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

(Total for question = 6 marks)