

Question 1

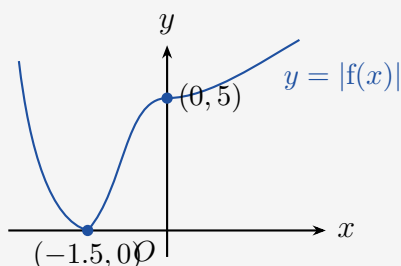
Worked Solution

The curve $y = f(x)$ passes through $P(-1.5, 0)$ and $Q(0, 5)$.

(a) $y = |f(x)|$

This reflects any part of $y = f(x)$ that lies below the x -axis upward, leaving parts above (or on) the axis unchanged.

Key points: $P(-1.5, 0)$ stays at $(-1.5, 0)$; $Q(0, 5)$ stays at $(0, 5)$.

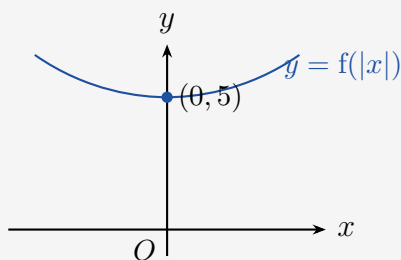


Shape (including cusp) in quadrants 1 and 2. Points: $(-1.5, 0)$ and $(0, 5)$.

(b) $y = f(|x|)$

Replace x with $|x|$: keep the portion of the graph for $x \geq 0$ and reflect it in the y -axis. The result is symmetric about the y -axis.

Key point: $(0, 5)$ stays at $(0, 5)$.

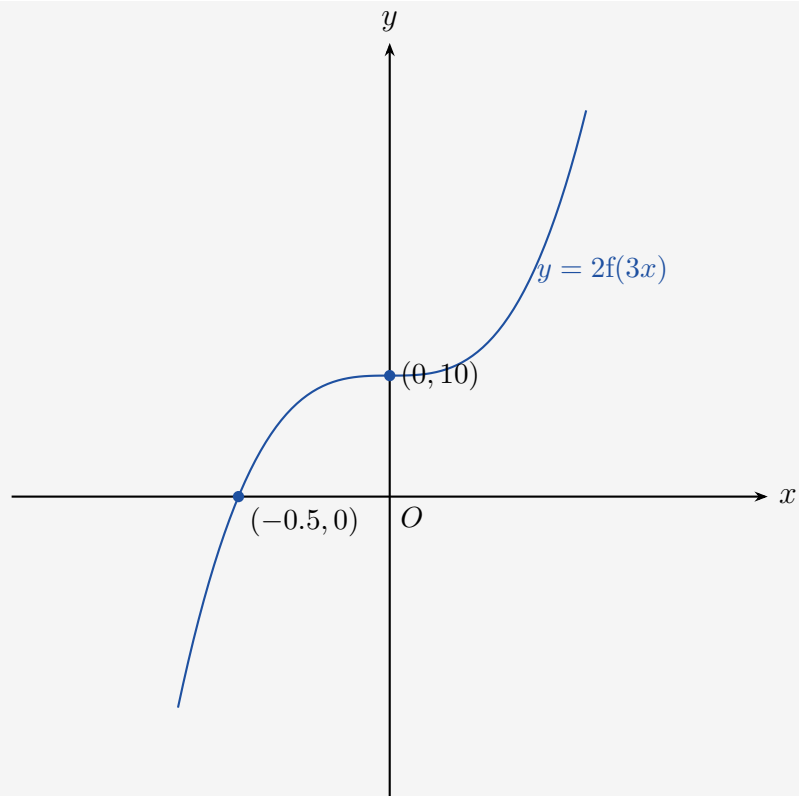


U-shaped curve, symmetric about y -axis, passing through $(0, 5)$.

(c) $y = 2f(3x)$

Horizontal stretch by $\frac{1}{3}$, then vertical stretch by 2.

$P(-1.5, 0) \rightarrow (-\frac{1.5}{3}, 0) = (-0.5, 0)$; $Q(0, 5) \rightarrow (0, 10)$.



Same shape as $f(x)$ but narrower and taller. Points: $(-0.5, 0)$ and $(0, 10)$.

Question 2

Worked Solution

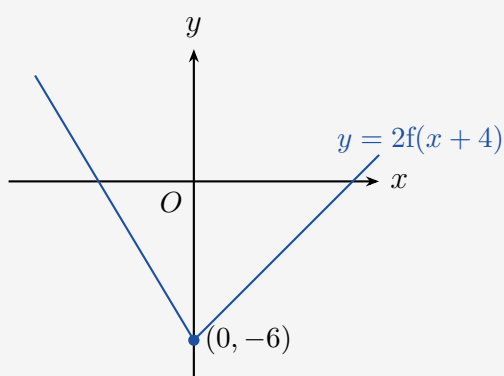
$y = f(x)$ consists of two line segments meeting at $R(4, -3)$.

(a) $y = 2f(x + 4)$

Translate left by 4, then stretch vertically by 2.

$$R(4, -3) \rightarrow (4 - 4, 2 \times -3) = (0, -6).$$

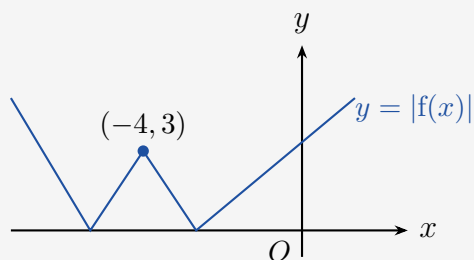
The graph is a V-shape with vertex on the y -axis at $(0, -6)$. Both branches cross the x -axis.



V shape with vertex at $(0, -6)$; both branches cross the x -axis. R maps to $(0, -6)$.

(b) $y = |f(x)|$

The section of $f(x)$ below the x -axis (near the vertex $R(4, -3)$) is reflected upward, creating a W-shape. $R(4, -3) \rightarrow (4, 3)$.



W shape. Point corresponding to R is $(4, 3)$. Graph is in quadrants 1 and 2.

Question 3

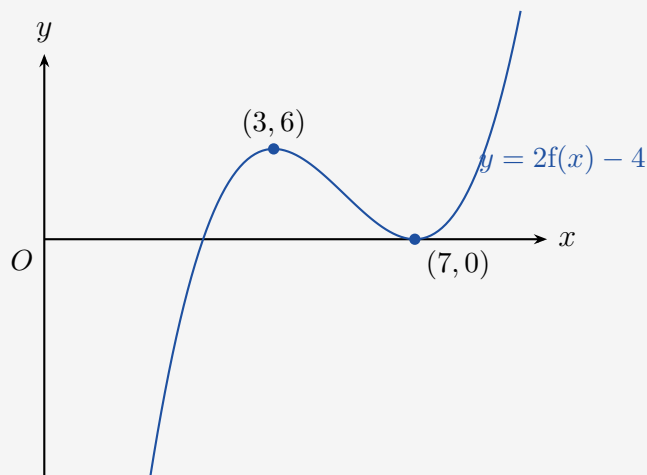
Worked Solution

$y = f(x)$ has turning points $T(3, 5)$ and $S(7, 2)$, domain $1 < x < 9$.

(a) $y = 2f(x) - 4$

Vertical stretch by S.F. 2, then translate down 4: $y \rightarrow 2y - 4$.

$T(3, 5) \rightarrow (3, 2(5) - 4) = (3, 6)$; $S(7, 2) \rightarrow (7, 2(2) - 4) = (7, 0)$.

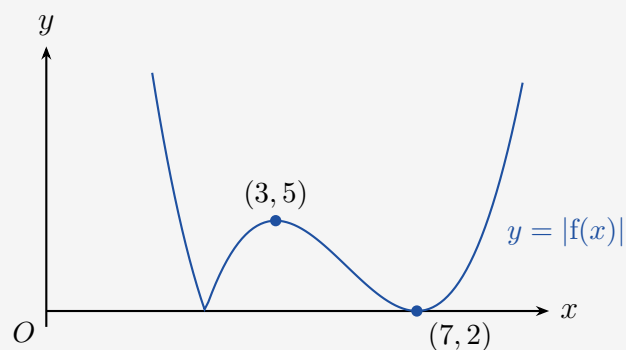


Shape similar to $f(x)$. Turning points: $T \rightarrow (3, 6)$ and $S \rightarrow (7, 0)$.

(b) $y = |f(x)|$

Both turning points $T(3, 5)$ and $S(7, 2)$ are already positive, so they are unchanged.

Any part of $f(x)$ that is negative (for x near 1) is reflected upward.



Turning points unchanged at $(3, 5)$ and $(7, 2)$; any section below x -axis reflected upward.

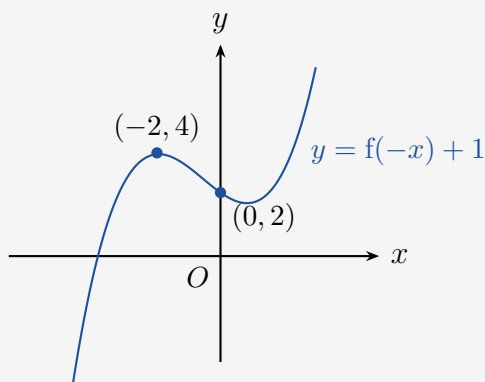
Question 4

Worked Solution

$y = f(x)$ passes through $(0, 1)$ with maximum turning point $A(2, 3)$.

(i) $y = f(-x) + 1$

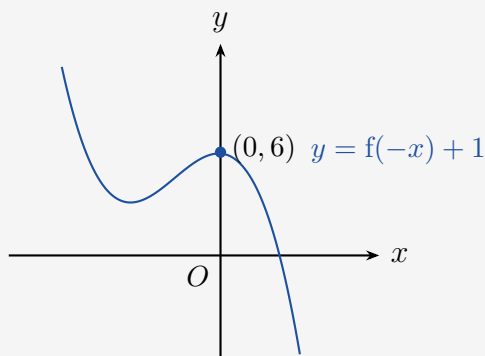
Reflect in y -axis ($A \rightarrow (-2, 3)$, $(0, 1) \rightarrow (0, 1)$), then translate up 1 ($A' \rightarrow (-2, 4)$, y -int $\rightarrow (0, 2)$).



y -intercept: $(0, 2)$; A maps to $A'(-2, 4)$.

(ii) $y = f(x + 2) + 3$

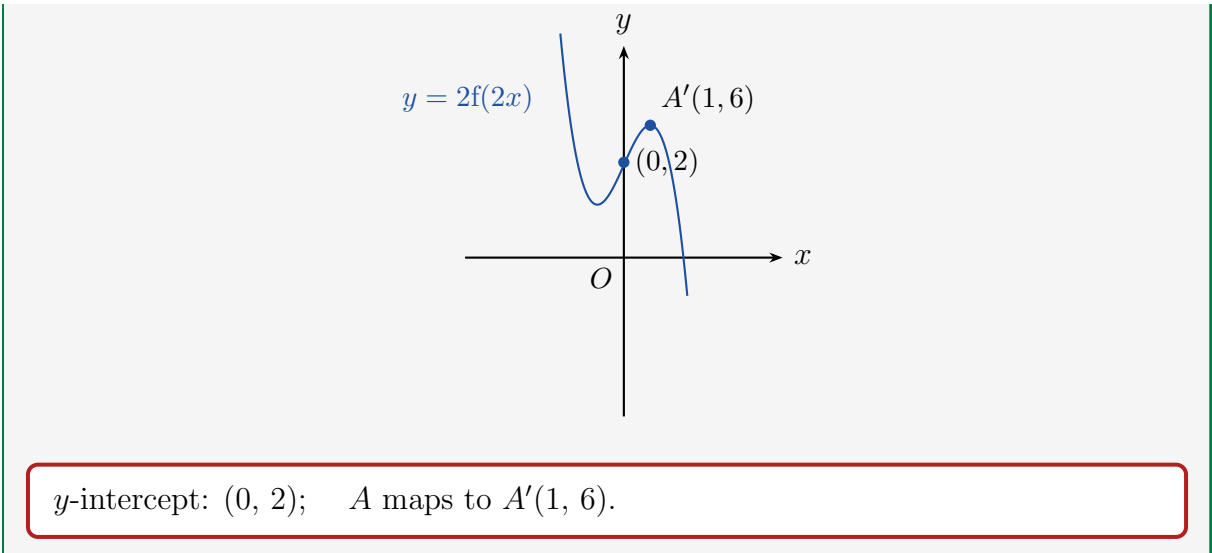
Translate left 2 ($A \rightarrow (0, 3)$, $(0, 1) \rightarrow (-2, 1)$), then up 3 ($A' \rightarrow (0, 6)$, y -int $\rightarrow (0, 6)$).



y -intercept: $(0, 6)$; A maps to $A'(0, 6)$.

(iii) $y = 2f(2x)$

Horizontal stretch by $\frac{1}{2}$ ($A \rightarrow (1, 3)$), then vertical stretch by 2 ($A' \rightarrow (1, 6)$, y -int $\rightarrow (0, 2)$).



Question 5

Worked Solution

$y = f(x)$ has turning point $A(3, -4)$ and passes through $(0, 5)$.

(a)(i) Image of A under $y = |f(x)|$:

$y = -4 < 0$, so reflect: $A \rightarrow (3, 4)$.

$(3, 4)$

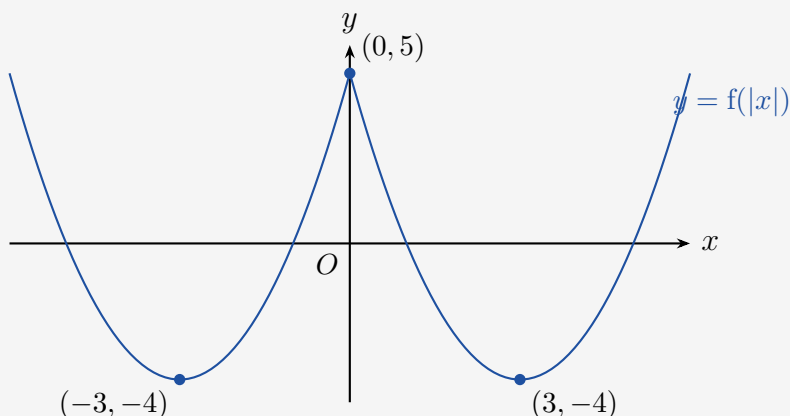
(a)(ii) Image of A under $y = 2f\left(\frac{1}{2}x\right)$:

Horizontal stretch by 2: $x \rightarrow 6$. Vertical stretch by 2: $y \rightarrow -8$. So $A \rightarrow (6, -8)$.

$(6, -8)$

(b) Sketch of $y = f(|x|)$

Keep $x \geq 0$ portion (minimum at $(3, -4)$, passes through $(0, 5)$) and reflect in y -axis. Result is symmetric about y -axis with minima at $(\pm 3, -4)$ and y -intercept $(0, 5)$.



W-shaped curve, symmetric about y -axis. y -intercept $(0, 5)$; minima at $(\pm 3, -4)$.

(c) Find $f(x)$.

$f(x)$ is a translation of $y = x^2$ with vertex (minimum) at $(3, -4)$:

$$f(x) = (x - 3)^2 - 4$$

$f(x) = (x - 3)^2 - 4$ (equivalently $x^2 - 6x + 5$)

(d) Why does f not have an inverse?

f is many-to-one (e.g. $f(0) = f(6) = 5$), so it is not one-to-one and therefore does not have an inverse.

Question 6

Worked Solution

Point $P(-2, -5)$ lies on $y = f(x)$.

(a) Image of P under $y = f(x) + 2$:

$y \rightarrow -5 + 2 = -3$; x unchanged.

$(-2, -3)$

(b) Image of P under $y = |f(x)|$:

$y \rightarrow |-5| = 5$; x unchanged.

$(-2, 5)$

(c) Image of P under $y = 3f(x - 2) + 2$:

The transformation $x \rightarrow x - 2$ means we need the original point with x -value satisfying $x_{\text{new}} - 2 = -2$, i.e. $x_{\text{new}} = 0$.

So $x : -2 \rightarrow 0$. Then $y : -5 \rightarrow 3(-5) + 2 = -13$.

$(0, -13)$

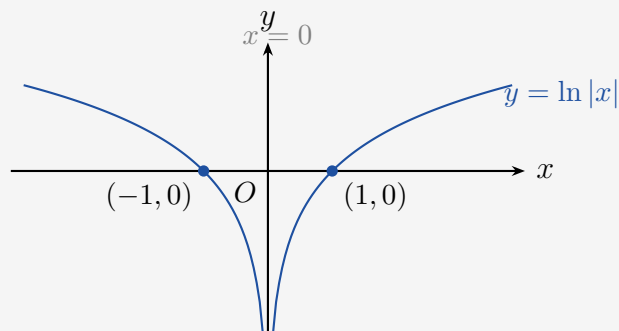
Question 7

Worked Solution

Sketch $y = \ln|x|$, stating any axis intercepts.

For $x > 0$: $y = \ln x$ (standard log). For $x < 0$: $y = \ln(-x)$ (reflection of $\ln x$ in the y -axis).

x -intercepts: $|x| = 1 \Rightarrow x = \pm 1$, so $(\pm 1, 0)$. No y -intercept (vertical asymptote $x = 0$).



x -intercepts: $(-1, 0)$ and $(1, 0)$. Vertical asymptote: $x = 0$.

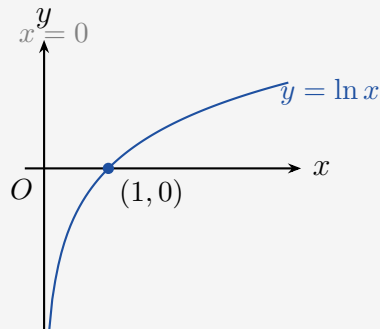
Question 8

Worked Solution

$$f(x) = \ln x, x > 0.$$

(i) $y = \ln x$

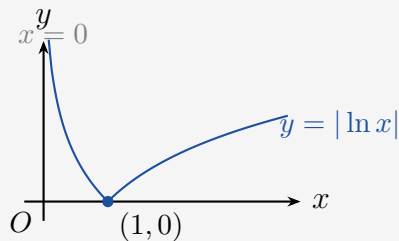
Standard logarithm. Crosses x -axis at $(1, 0)$. Asymptote: $x = 0$.



Increasing curve through $(1, 0)$; asymptote $x = 0$.

(ii) $y = |\ln x|$

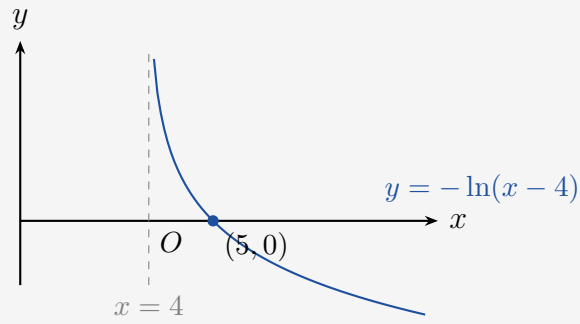
For $0 < x < 1$: $\ln x < 0$, so $|\ln x| = -\ln x$ (reflected upward). For $x \geq 1$: unchanged. Cusp at $(1, 0)$.



V/cusp shape touching x -axis at $(1, 0)$; asymptote $x = 0$.

(iii) $y = -\ln(x - 4)$

Translate right 4, reflect in x -axis. Crosses x -axis at $(5, 0)$. Asymptote: $x = 4$.



Decreasing curve crossing x -axis at $(5, 0)$; asymptote $x = 4$.

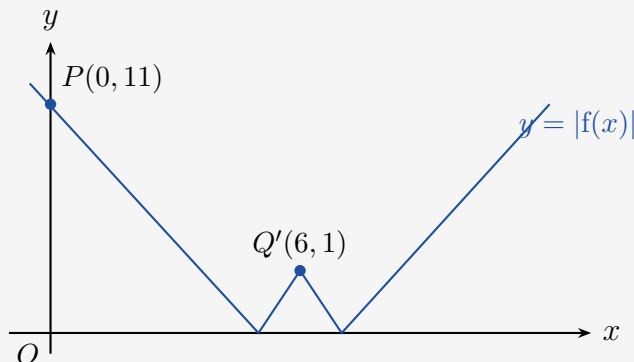
Question 9

Worked Solution

$y = f(x)$ consists of two line segments meeting at $Q(6, -1)$; crosses y -axis at $P(0, 11)$.

(a) $y = |f(x)|$

$Q(6, -1)$ reflected to $Q'(6, 1)$; $P(0, 11)$ stays. The negative portion creates a W-shape.



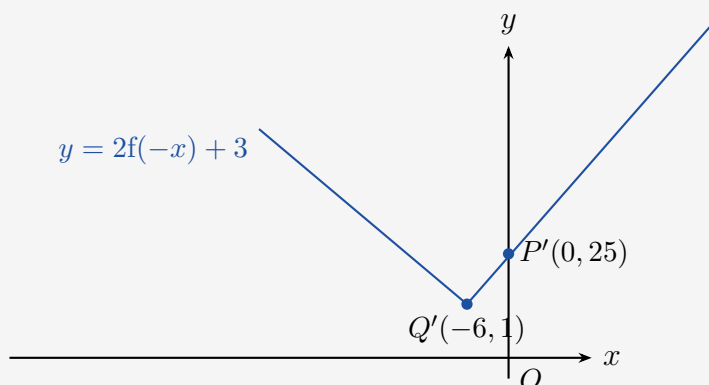
W shape. P corresponds to $(0, 11)$; Q corresponds to $(6, 1)$.

(b) $y = 2f(-x) + 3$

Step 1 – replace x by $-x$ (reflect in y -axis): $Q(6, -1) \rightarrow (-6, -1)$; $P(0, 11) \rightarrow (0, 11)$.

Step 2 – multiply by 2: $Q \rightarrow (-6, -2)$; $P \rightarrow (0, 22)$.

Step 3 – add 3: $Q' = (-6, 1)$; $P' = (0, 25)$.



V shape. P corresponds to $(0, 25)$; Q corresponds to $(-6, 1)$.

(c) **Given** $f(x) = a|x - b| - 1$, **find** a **and** b .

Minimum of $f(x)$ is at $Q(6, -1)$, so vertex is at $x = b = 6$.

At $P(0, 11)$: $f(0) = a|0 - 6| - 1 = 6a - 1 = 11 \Rightarrow 6a = 12 \Rightarrow a = 2$.

$$a = 2, \quad b = 6$$

End of Worked Solutions