Modelling With Trigonometric Functions Exam Questions (Suitable For All Exam Boards)

Q1

(a) Express $3 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.
State the value of R and give the value of α to 2 decimal places.

(3)

The temperature in a greenhouse, G °C, is modelled by the equation

$$G = 17 + 3 \sin (15t)° - 4 \cos (15t)°, \quad 0 \leq t \leq 17,$$

where t is the time in hours after 5 a.m.

(b) Find, according to this model,

(i) the maximum temperature in the greenhouse,

(1)

(ii) the time, after midday, when the temperature in the greenhouse is 20 °C.
Give your answer to the nearest minute.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

Q2

The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2 \sin (30t)°, \quad 0 \leq t < 24,$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 a.m. and it takes 2 hours to load its cargo. The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

Q3

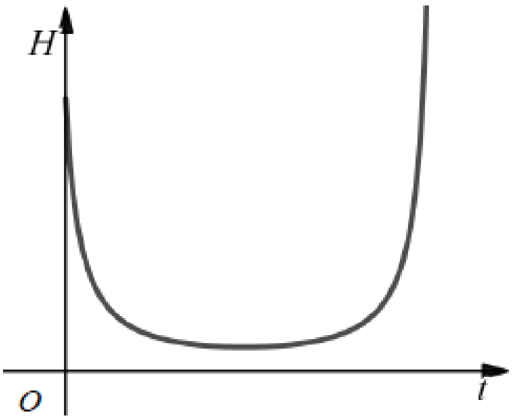

Figure 4

A scientist is studying the flight of seabirds in a colony.

She models the height above sea level, H metres, of one of the birds in the colony by the equation

$$H = \frac{140}{A + 45\sin 2t° - 28\cos 2t°}, \quad 0 \leq t \leq T,$$

where t seconds is the time after the bird leaves its nest and A and T are constants.

Figure 4 is a sketch showing the graph of H against t.

Given that this seabird's nest is 20 m above sea level,

(a) find a complete equation for H.

(3)

Given that

$$45\sin 2t° - 28\cos 2t° \equiv 53\sin(2t - \alpha)°, \qquad 0 < \alpha < 90,$$

(b) find the value of α to one decimal place.

(2)

Find, according to this model,

(c) the minimum height of the sea bird above sea level giving your answer to the nearest cm,

(2)

(d) the limitation on the value of T.

(2)

Q4

(a) Express $1.5\sin\theta - 1.2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of R and the value of α to 3 decimal places.

(3)

The height, H metres, of sea water at the entrance to a harbour on a particular day, is modelled by the equation

$$H = 3 + 1.5\sin\left(\frac{\pi t}{6}\right) - 1.2\cos\left(\frac{\pi t}{6}\right), \quad 0 \leqslant t < 12$$

where t is the number of hours after midday.

(b) Using your answer to part (a), calculate the minimum value of H predicted by this model and the value of t, to 2 decimal places, when this minimum occurs.

(4)

(c) Find, to the nearest minute, the times when the height of sea water at the entrance to the harbour is predicted by this model to be 4 metres.

(6)

Q5

(a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α, in radians, to 3 significant figures.

(3)

In a town in Norway, a student records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leqslant t \leqslant 365$$

where t is the number of days since he began recording.

(b) Using your answer to part (a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answers to 3 significant figures.

(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

Q6

(a) Write $2\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to one decimal place.
(3)

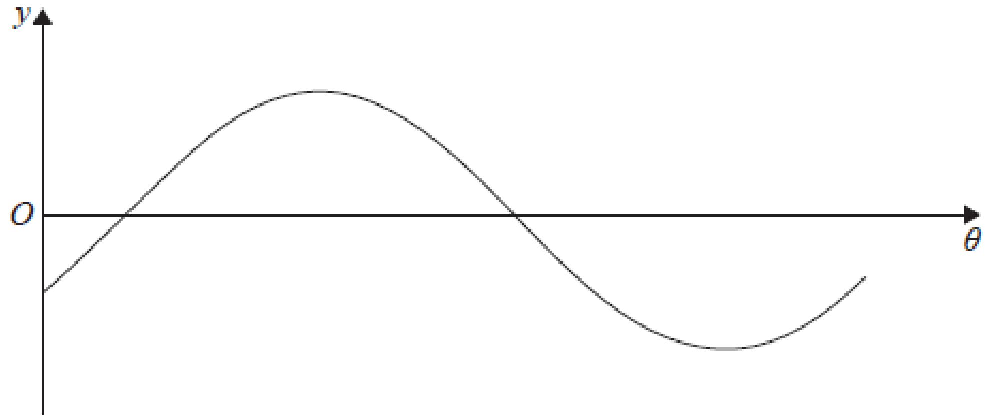

Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta$, $0 \leqslant \theta < 360°$

(b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \qquad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.
(3)

The temperature of a warehouse is modelled by the equation

$$f(t) = 5 + |2\sin(15t)° - \cos(15t)°|, \qquad 0 \leqslant t < 24$$

where f(t) is the temperature of the warehouse in degrees Celsius and t is the time measured in hours from midnight.

State

(c) (i) the maximum value of f(t),

(ii) the largest value of t, for $0 \leqslant t < 24$, at which this maximum value occurs. Give your answer to one decimal place.
(3)