Modelling With Trigonometric Functions Exam Questions (Mark Scheme)

Q1

(a)	$\tan\alpha = \frac{4}{3}$ o.e.	M1	1.1b
	Either $R = 5$ or $\alpha =$ awrt 53.13	B1	1.1b
	$5\sin(\theta - 53.13°)$	A1	1.1b
		(3)	
(b)(i)	$G_{max} = 17 + "5" = 22$ (°C)	B1ft	3.4
		(1)	
(b)(ii)	$G = 17 + 3\sin(15t)° - 4\cos(15t)°;\ 0 \leqslant t \leqslant 17$		
	$20 = 17 + "5"\sin(15t - "53.13")$	M1	3.4
	$\sin(15t - "53.13") = \frac{3}{"5"}$ or $\sin(\theta - "53.13") = \frac{3}{"5"}$	M1	1.1b
	After midday solution $\Rightarrow 15t - "53.13" = 180 - 36.86989...$ $\Rightarrow t = \frac{143.1301... + "53.13"}{15}$	M1	3.1b
	$\Rightarrow t = 13.0840... \Rightarrow$ Time = 6:05 p.m. or 18:05	A1	3.2a
		(4)	
			(8 marks)

Q2

(a)	$D = 5 + 2\sin(30 \times 6.5)° =$ awrt 4.48m with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

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Q3

Question	Scheme	Marks	AOs
(a)	Substitutes $t = 0, H = 20$ into $H = \dfrac{140}{A + 45\sin 2t - 28\cos 2t}$	M1	3.1b
	Full method to find A $\quad 20 = \dfrac{140}{A - 28} \Rightarrow A = \ldots$	dM1	1.1b
	$H = \dfrac{140}{35 + 45\sin 2t - 28\cos 2t}$	A1	3.3
		(3)	
(b)	$\tan\alpha = \dfrac{28}{45}$	M1	1.1b
	$\alpha = 31.9$	A1	1.1b
		(2)	
	$H = \dfrac{140}{35 + 53\sin(2t - 31.9)}$		
(c)	Obtains $H_{\min} = \dfrac{140}{"A" + 53}$	M1	3.4
	1.59 metres or 159 cm	A1	1.1b
		(2)	
(d)	Sets $35 + 53\sin(2T_{\max} - 31.9) = 0$	M1	3.4
	The model is only valid for $(0 \,,\,) \; T < 126.6$ s	A1	3.5b
		(2)	
			(9 marks)

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Q4

Question Number	Scheme	Marks
(a)	$(R=\sqrt{1.5^2+1.2^2})=$ awrt 1.921 - accept e.g. $\sqrt{3.69}$ or $\frac{3\sqrt{41}}{10}$	B1
	$\tan\alpha=\frac{1.2}{1.5}\Rightarrow\alpha=0.675$ or 0.215π	M1A1
		(3)
(b)	$H=3+1.921\sin\left(\frac{\pi t}{6}-0.675\right)$	
	$H_{\min}=3-'1.921'=$ awrt 1.08	M1A1
	$\left(\frac{\pi t}{6}-"0.675"\right)=\frac{3\pi}{2}\Rightarrow t=10.29$	M1A1
		(4)
(c)	$4=3+1.921\sin\left(\frac{\pi t}{6}-0.675\right)\Rightarrow\sin\left(\frac{\pi t}{6}-0.675\right)=\frac{1}{1.921}$	M1
	$\frac{\pi t}{6}-0.675=0.548\Rightarrow t=$ awrt 2.33 or 2.34	dM1A1
	$\frac{\pi t}{6}-0.675=\pi-0.548=2.594\Rightarrow t=$ awrt 6.24 or 6.25	ddM1A1
	Times are 2:20pm and 6:15pm or 6.14pm (14:20 and 18:15 or 18:14) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes	A1
	Extra values in the range – lose final A mark.	**(6)**
		(13 marks)

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Q5

(a)	$R = \sqrt{4+16} = \sqrt{20}$ *or* $2\sqrt{5}$	B1
	$\tan\alpha = \frac{4}{2}$	M1
	$\Rightarrow \alpha = 1.11$ (awrt)	A1
		(3)
(b)	Maximum is $12+2R$ or minimum is $12-2R$	M1
	maximum = 20.9 (hours) (20h 57m) and minimum = 3.06 (hours) (3 hours 3 m)	A1 A1
		(3)
(c)	$17 = 12 + k"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$	M1
	$\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) = ...$	dM1
	For proceeding to one value for t from $17 = 12 + 2"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$	M1
	$t = 99$ (days) **or** 212 or 213 (days)	A1
	For finding two values for t	dM1
	$t = 99$ (days) **and** 212 or 213 (days)	A1
		(6)
		(12 marks)

Q6

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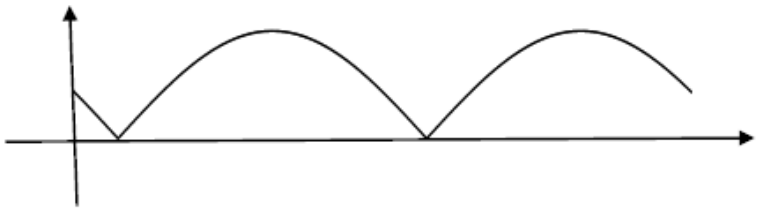
Question Number	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow(\alpha=)26.6°$	M1,A1
		(3)
(b)	 Shape	B1
	$(0,1)$	B1
	$("26.6",0)$ and $("206.6",0)$ (Allow in radians i.e. their α and π+α	B1ft
		(3)
(c)(i)	$5+'R'=5+\sqrt{5}$	B1ft
(c)(ii)	$15t-'26.6'=270\Rightarrow t=19.8$	M1,A1
		(3)
		(9 marks)

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