Mixed Sequences Exam Questions MS (from OCR 4722)

Jun 2006, Q6

(i)	(a) $100+239\times5=£1295$	M1 A1	 2	For relevant use of $a + (n - 1)d$ For correct value 1295
	(b) $\frac{1}{2}\times240\times(100+1295)=£167400$	M1 A1	 2	For relevant use of $\frac{1}{2}n(a+l)$ or equivalent For correct value 167400
(ii)	$100r^{239}=1500\Rightarrow r=1.01139...$ Hence total is $\dfrac{100(1.01139^{240}-1)}{1.01139-1}=£124359$	B1 M1 A1 M1 A1	 5	For correct statement of $100r^{239}=1500$ Attempt to find r For correct value 1.01 For relevant use of GP sum formula For correct value 124359 (3 s.f. or better)
			9	

Jun 2007, Q7

(a)	$S_{70}=\frac{70}{2}\{(2\times12)+(70-1)d\}$	M1		Attempt S_{70}
		A1		Obtain correct unsimplified expression
	$35(24+69d)=12915$	M1		Equate attempt at S_{70} to 12915, and attempt to find d
	$d=5$	A1	4	Obtain $d=5$
R				
	$\frac{70}{2}\{12+l\}=12915$	M1		Attempt to find d by first equating $^{n}/_{2}(a+l)$ to 12915
	$l=357$	A1		Obtain $l=357$
	$12+69d=357$	M1		Equate u_{70} to l
	$d=5$	A1		Obtain $d=5$
(b)	$ar=-4$	B1		Correct statement for second term
	$\frac{a}{1-r}=9$	B1		Correct statement for sum to infinity
	$\frac{-4}{r}=9-9r$ or $a=9-\left(9\times\frac{-4}{a}\right)$	M1		Attempt to eliminate either a or r
	$9r^2-9r-4=0$ $\quad a^2-9a-36=0$	A1		Obtain correct equation (no algebraic denominators/brackets)
	$(3r-4)(3r+1)=0$ $\quad (a+3)(a-12)=0$	M1		Attempt solution of three term quadratic equation
	$r=\frac{4}{3}$, $r=-\frac{1}{3}$ $\quad a=-3$, $a=12$	A1		Obtain at least $r=-\frac{1}{3}$ (from correct working only)
	Hence $r=-\frac{1}{3}$	A1	7	Obtain $r=-\frac{1}{3}$ only (from correct working only) SR: answer only / T&I is B2 only
			11	

Jun 2008, Q10

(i)	$u_{15} = 2 + 14 \times 0.5$ $= 9$ km	**M1** **A1** 2	Attempt use of $a + (n-1)d$ Obtain 9 km
(ii)	$u_{20} = 2 \times 1.1^{19} = 12.2$ $u_{19} = 2 \times 1.1^{18} = 11.1$	**B1** **M1** **A1**	State, or imply, $r = 1.1$ Attempt u_{20}, using ar^{n-1} Obtain $u_{20} = 12.2$, and obtain $u_{19} = 11.1$
	OR		
		B1 **M1** **A1** 3	State, or imply, $r = 1.1$ Attempt to solve $ar^{n-1} = 12$ Obtain $n = 20$ (allow $n \geq 20$)
(iii)	$\frac{2(1.1^n - 1)}{(1.1-1)} > 200$ $1.1^n > 11$ $n > \frac{\log 11}{\log 1.1}$ $n > 25.2$ ie Day 26	**B1** **M1** **A1** **A1** 4	State or imply $S_N = \frac{2(1.1^n - 1)}{(1.1-1)}$ Link (any sign) their attempt at S_N (of a GP) to 200 and attempt to solve Obtain 26, or 25.2 or better Conclude $n = 26$ only, or equiv eg Day 26
(iv)	swum $= 2 \times 30 = 60$ km run $= \frac{1}{2} \times 30 \times (4 + 29 \times 0.5)$ $= 277.5$ km cycle $= \frac{2(1.1^{30} - 1)}{(1.1-1)}$ $= 329.0$ km total $= 666$ km	**B1** **M1** **M1** **A1** 4	Obtain 60 km, or 2 x 30km Attempt sum of AP, $d = 0.5$, $a = 2$, $n = 30$ Attempt sum of GP, $r = 1.1$, $a = 2$, $n = 30$ Obtain 666 or 667 km

Jun 2010 , Q4

4 (i)	$u_1 = 6$, $u_2 = 11$, $u_3 = 16$	**B1**	**1**	State 6, 11, 16
(ii)	$S_{40} = {}^{40}/_{2}\,(2 \times 6 + 39 \times 5)$ $= 4140$	**M1** **M1** **A1**	 **3**	Show intention to sum the first 40 terms of a sequence Attempt sum of their AP from (i), with $n = 40$, $a =$ their u_1 and $d =$ their $u_2 - u_1$ Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p-1) \times 5 = 56$ $p = 11$	**B1** **M1** **A1** 7	 **3**	State or imply $w_3 = 56$ Attempt to solve $u_p = k$ Obtain $p = 11$

Jun 2010, Q9

(i)	$ar = a + d,\ \ ar^3 = a + 2d$ $2ar - ar^3 = a$ $ar^3 - 2ar + a = 0$ $r^3 - 2r + 1 = 0$ **A.G.**	**M1**		Attempt to link terms of AP and GP, implicitly or explicitly.
		M1		Attempt to eliminate d, implicitly or explicitly, to show given equation.
		A1	**3**	Show $r^3 - 2r + 1 = 0$ convincingly
(ii)	$f(r) = (r-1)(r^2 + r - 1)$	**B1**		Identify $(r-1)$ as factor or $r = 1$ as root
		M1*		Attempt to find quadratic factor
	$r = \dfrac{-1 \pm \sqrt{5}}{2}$ Hence $r = \dfrac{-1+\sqrt{5}}{2}$	**A1**		Obtain $r^2 + r - 1$
		M1d*		Attempt to solve quadratic
		A1	**5**	Obtain $r = \dfrac{-1+\sqrt{5}}{2}$ only
(iii)	$\dfrac{a}{1-r} = 3 + \sqrt{5}$	**M1**		Equate S_∞ to $3+\sqrt{5}$
	$a = (\frac{3}{2} - \frac{\sqrt{5}}{2})(3 + \sqrt{5})$	**A1**		Obtain $\dfrac{a}{1-\left(\frac{-1+\sqrt{5}}{2}\right)} = 3 + \sqrt{5}$
	$a = {}^9/_2 - {}^5/_2$ $a = 2$	**M1**		Attempt to find a
		A1	**4**	Obtain $a = 2$
			12	

Jan 2012, Q6

(i)	$u_1 = 80$	**B1**		State 80
	$u_2 = 75,\ u_3 = 70$	**B1** **[2]**		State 75 and 70
(ii)	$S_{20} = \frac{20}{2}(2 \times 80 + 19 \times -5)$ $= 650$	**M1**		Show intention to sum 1st 20 terms of an arithmetic sequence
		M1		Attempt use of correct sum formula for an AP, with $n = 20$, $a = 80$, $d = \pm 5$
		A1 **[3]**		Obtain 650

(iii)		$r = \dfrac{60}{80} = 0.75$ $u_p = 80 \times 0.75^2 = 45$ $85 - 5p = 45$ $p = 8$	**M1*** **A1** **M1d*** **A1** **[4]**	Attempt to find u_p Obtain 45 Attempt to solve $85 - 5p = k$ Obtain $p = 8$
(iv)		$S_\infty = \dfrac{80}{1 - 0.75}$ $= 320$	**M1** **A1** **[2]**	Use correct formula for sum to infinity Obtain 320

Jan 2013, Q6

(i)		$(x+4)-2x=(2x-7)-(x+4)$ **OR** $2x+d=x+4 \quad 2x+2d=2x-7$ $2x=15$ $x=7.5$	M1 A1 A1 **[3]**	Attempt to eliminate d to obtain equation in x only Obtain correct equation in just x Obtain $x=7.5$
(ii)	**(a)**	terms are 16, 12, 9 $^{12}/_{16}=0.75$, $^{9}/_{12}=0.75$ common ratio of 0.75 so GP $S_\infty = {}^{16}/_{1-0.75}$ $=64$	B1 B1 M1 A1 **[4]**	List 3 terms Convincing explanation of why it is a GP Attempt use of $^{a}/_{1-r}$ Obtain 64

(ii)	**(b)**	$(2x-7)/(x+4) = (x+4)/2x$ $4x^2 - 14x = x^2 + 8x + 16$ **OR** $2xr = x + 4 \quad 2xr^2 = 2x - 7$ $3x^2 - 22x - 16 = 0$ $(3x + 2)(x - 8) = 0$ $x = -2/3, x = 8$	M1* A1 M1d* A1 **[4]**	Attempt to eliminate r to obtain equation in x only Obtain $3x^2 - 22x - 16 = 0$ Attempt to solve quadratic Obtain $x = -2/3$

Jun 2013

(i)		$S_{30} = 30/2\ (2 \times 6 + 29 \times 1.8)$	M1	Use $d = 1.8$ in AP formula
			A1	Correct unsimplified S_{30}
		$= 963$	A1 **[3]**	Obtain 963
(ii)		$r = 7.8/6 = 1.3$	M1	Use $r = 1.3$ in GP formula
		$\frac{6(1-1.3^N)}{1-1.3} \leq 1800$	A1	Correct unsimplified S_N
		$1 - 1.3^N \geq -90$	M1	Link sum of GP to 1800 and attempt to rearrange to $1.3^N \leq k$
		$1.3^N \leq 91$ **AG**	A1	Obtain given inequality