

Integration of Rational Functions Exam Questions MS

Q1, (OCR 4724, Jun 2006, Q3)

<p>(i) $\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$</p> <p>$\frac{1}{x}$</p> <p>$-\frac{1}{3-x}$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct format + suitable method</p> <p>seen in (i) or (ii)</p> <p>3 ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately</p>
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<p>(ii) $\int \frac{1}{x} (dx) = \ln x$ or $\ln x$</p> <p>$\int \frac{1}{3-x} (dx) = -\ln(3-x)$ or $-\ln 3-x$</p> <p>Correct method idea of substitution of limits $\ln 2 (+ \ln 1 - \ln 1) - \ln 2 = 0$</p> <p>Alternative Method If ignoring PFs, $\ln x(3-x)$ immediately As before</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B2</p> <p>M1,A1 (4)</p>	<p>Check sign carefully; do not allow $\ln(x-3)$</p> <p>Dep on an attempt at integrating</p> <p>4 Clearly seen; WWW AG</p> <p>$\ln x(x-3) \rightarrow 0$</p>
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<p>(iii) Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)</p>	<p>B1</p>	<p>1</p>

Q2, (OCR 4724, Jan 2007, Q6)

<p>(i) $2x + 1 = / \equiv A(x-3) + B$</p> <p>$A = 2$</p> <p>$B = 7$</p>	<p>M1</p> <p>A1</p> <p>A/B 1</p>	<p>3 Cover-up rule acceptable for B1</p>
<p>(ii) $\int \frac{1}{x-3} (dx) = \ln(x-3)$ or $\ln x-3$</p> <p>$\int \frac{1}{(x-3)^2} (dx) = -\frac{1}{x-3}$</p> <p>$6 + 2 \ln 7$ Follow-through $\frac{6}{7}B + A \ln 7$</p>	<p>B1</p> <p>B1</p> <p>√B2</p>	<p>Accept A or $\frac{1}{A}$ as a multiplier</p> <p>Accept B or $\frac{1}{B}$ as a multiplier</p> <p>4</p>

Q3, (OCR 4724, Jun 2009, Q6)

- (i) $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$ M1
 $A = 5$ A1 'cover-up' rule, award B1
 $B = -5$ A1
 $C = -6$ A1 4 'cover-up' rule, award B1
 Cands adopting other alg. manip. may be awarded M1 for a full satis method + 3 @ A1

(ii) $\int \frac{A}{x-5} dx = A \ln(5-x)$ or $A \ln|5-x|$ or $A \ln|x-5|$ $\sqrt{B1}$ but not $A \ln(x-5)$

$\int \frac{B}{x-3} dx = B \ln(3-x)$ or $B \ln|3-x|$ or $B \ln|x-3|$ $\sqrt{B1}$ but not $B \ln(x-3)$

If candidate is awarded B0,B0, then award **SR** $\sqrt{B1}$ for $A \ln(x-5)$ and $B \ln(x-3)$

$\int \frac{C}{(x-3)^2} dx = -\frac{C}{x-3}$ $\sqrt{B1}$

$5 \ln \frac{3}{4} + 5 \ln 2$ aef, isw $\sqrt{A \ln \frac{3}{4} - B \ln 2}$ $\sqrt{B1}$ Allow if **SR** B1 awarded

-3 $\sqrt{\frac{1}{2}C}$ $\sqrt{B1}$ 5

[Mark at earliest correct stage & isw; no ln 1] 9

Q4, (OCR 4724, Jun 2007, Q7)

(i) Leading term in quotient = $2x$ <u>Suff evidence</u> of division or identity process Quotient = $2x + 3$ Remainder = x	B1 M1 A1 A1	Stated or in relevant position in division 4 Accept $\frac{x}{x^2 + 4}$ as remainder
(ii) their quotient + $\frac{\text{their remainder}}{x^2 + 4}$	$\sqrt{B1}$	1 $2x + 3 + \frac{x}{x^2 + 4}$
(iii) <u>Working with their expression in part (ii)</u> their $Ax + B$ integrated as $\frac{1}{2}Ax^2 + Bx$	$\sqrt{B1}$	Ignore any integration of $\frac{D}{x^2 + 4}$
their $\frac{Cx}{x^2 + 4}$ integrated as $k \ln(x^2 + 4)$	M1	
$k = \frac{1}{2}C$	$\sqrt{A1}$	
Limits used correctly throughout	M1	
$14 + \frac{1}{2} \ln \frac{13}{5}$	A1	5 logs need not be combined.