Integration Involving Trigonometric Functions

Q1, (OCR 4724, Jun 2006, Q8i)

(i) Show that $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24}\sin 12x + c$. [3]

Q32, (OCR 4724, Jun 2012, Q7)

Find the exact value of $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)^2 \, dx$. **[7]**

Q3, (OCR 4724, Jun 2016, Q2)

Use integration to find the exact value of $\int_{\frac{1}{16}\pi}^{\frac{1}{8}\pi} (9 - 6\cos^2 4x) \, dx$. **[5]**

Q4, (OCR 4724, Jun 2013, Q5)

(i) Show that $\frac{1}{1-\tan x} - \frac{1}{1+\tan x} \equiv \tan 2x$. **[2]**

(ii) Hence evaluate $\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \left(\frac{1}{1-\tan x} - \frac{1}{1+\tan x} \right) dx$, giving your answer in the form $a \ln b$. **[5]**

Q5, (OCR 4724, Jan 2010, Q3)

By expressing $\cos 2x$ in terms of $\cos x$, find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \frac{\cos 2x}{\cos^2 x} \, dx$. **[5]**

Q6, (OCR 4724, Jan 2011, Q3)

(i) Show that the derivative of $\sec x$ can be written as $\sec x \tan x$. **[4]**

(ii) Find $\int \frac{\tan x}{\sqrt{1+\cos 2x}} \, dx$. **[4]**

Q7, (OCR 4724, Jun 2014, Q4)

Show that $\int_0^{\frac{1}{4}\pi} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x} \, dx = \frac{1}{2}\ln 2$. **[5]**

Q8, (OCR 4724, Jun 2015, Q6)

(i) Use the quotient rule to show that the derivative of $\frac{\cos x}{\sin x}$ is $\frac{-1}{\sin^2 x}$. **[2]**

(ii) Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{\sqrt{1+\cos 2x}}{\sin x \sin 2x} \, dx = \frac{1}{2}(\sqrt{6} - \sqrt{2})$. **[6]**

Q9, (OCR 4724, Jan 2008, Q7)

(i) Given that

$$A(\sin\theta + \cos\theta) + B(\cos\theta - \sin\theta) \equiv 4\sin\theta,$$

find the values of the constants A and B. [3]

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{4\sin\theta}{\sin\theta + \cos\theta}\,\mathrm{d}\theta,$$

giving your answer in the form $a\pi - \ln b$. [5]
