

Integration Involving Trigonometric Functions MS

Q1, (OCR 4724, Jun 2006, Q8i)

Integration method

Attempt to change $\cos^2 6x$ into $f(\cos 12x)$ **M1**
 $\cos^2 6x = \frac{1}{2}(1 + \cos 12x)$ **A1** with $\cos^2 6x$ as the subject of the formula
 $\int = \frac{1}{2}x + \frac{1}{24} \sin 12x + c$ **A1** **AG** Accept $\frac{1}{2}(x + \frac{1}{12} \sin 12x)$

Differentiation method

Differentiate RHS producing $\frac{1}{2} + \frac{1}{2} \cos 12x$ ---(E) **B1**
 Attempt to change $\cos 12x$ into $f(\cos 6x)$ **M1** Accept $+/- 2 \cos^2 6x + +/- 1$
 Simplify (E) WWW to $\cos^2 6x +$ satis finish **A1** **3**

Q32, (OCR 4724, Jun 2012, Q7)

Satisfactory start method eg attempt square of $(1 - \sin 3x)$

[N.B. The squaring process might include a term $\sin^2 9x$]
 The next 2 marks are awarded for integrating $-2\sin 3x$

Obtain $\int -2 \sin 3x \, dx = \frac{2}{3} \cos 3x$

Obtain $-\frac{2}{3}$ or $(\dots + 0 \dots) - (\dots + \frac{2}{3} \dots)$

The next 3 marks are awarded for integrating $\sin^2 3x$

Use $\sin^2 3x = k(+/-1 +/- \cos 6x)$

Correct version = $\frac{1}{2}(1 - \cos 6x)$

$\int \cos 6x \, dx = \frac{1}{6} \sin 6x$, seen anywhere, indep

Final answer = $\frac{1}{4} \pi + \text{their} - \frac{2}{3}$

M1	Not e.g. $\frac{(1 - \sin 3x)^3}{3}$.
*A1	
A1dep*	
M1	or for <u>integrating</u> $\sin^2 ax$ where $a = 6$ or 9 only
A1	$\sin^2 ax = k(+/-1 +/- \cos 2ax)$
B1	Correct = $\frac{1}{2}(1 - \cos 2ax)$
A1	or $\int \cos 2ax \, dx = \frac{1}{2a} \sin 2ax$
[7]	Check that the $\frac{1}{4} \pi$ is from $\left[\frac{3}{2}x - \frac{1}{12} \sin 6x \right]_0^{\frac{1}{6}\pi}$

Q3, (OCR 4724, Jun 2016, Q2)

$\cos 8x$ seen in integrand

$F[x] = Ax + B \sin 8x$ oe

$F[x] = 6x - \frac{3}{8} \sin 8x$

$F[\frac{1}{8}\pi] - F[\frac{1}{16}\pi]$

$\frac{3}{8}\pi + \frac{3}{8}$ oe

M1	
M1*	A and B are non-zero constants
A1	
M1*dep	
A1	
[5]	

Q4, (OCR 4724, Jun 2013, Q5)

(i)	$\frac{(1 + \tan x) - (1 - \tan x)}{(1 - \tan x)(1 + \tan x)}$ $= \frac{2 \tan x}{1 - \tan^2 x} = \tan 2x$ <p style="text-align: right;">Answer Given</p>	M1	Combine (or write as 2 separate fractions) using common denominator
		A1	$\frac{2 \tan x}{1 - \tan^2 x}$ essential stage
			N.B. If $\tan x$ changed into $\frac{\sin x}{\cos x}$ before manipulation, apply same principles
		[2]	
(ii)	$\int \tan 2x \, dx = \lambda \ln(\sec 2x) \text{ or } \mu \ln(\cos 2x) \quad [= F(x)]$ $\lambda = \frac{1}{2} \quad \text{or} \quad \mu = -\frac{1}{2}$ <p>their $F[\frac{\pi}{6}] - \text{their } F[\frac{\pi}{12}]$</p> $\frac{1}{2} \ln 2 - \frac{1}{2} \ln \frac{2}{\sqrt{3}} \quad \text{oe}$ $\frac{1}{2} \ln \sqrt{3} \quad \text{or} \quad \frac{1}{4} \ln 3 \quad \text{or} \quad \ln 3^{1/4} \quad \text{or} \quad \frac{1}{2} \ln \frac{6}{2\sqrt{3}} \quad \text{oe ISW}$	M1	
		A1	
		M1	dependent on attempt at integration.....
		A1	i.e. any correct but probably unsimplified numerical version
		+A1	i.e. any correct version in the form $a \ln b$
		[5]	

Q5, (OCR 4724, Jan 2010, Q3)

Use $\cos 2x = a \cos^2 x + b / \pm \cos^2 x - \sin^2 x / 1 - 2\sin^2 x$ *M1

Obtain $\lambda + \mu \sec^2 x$ dep*M1

using 'reasonable' Pythag attempt

$\int \lambda + \mu \sec^2 x \, dx = \lambda x + \mu \tan x$ A1

(λ or μ may be 0 here/prev line)

Obtain correct result $2x - \tan x$ A1

no follow-through

$\frac{1}{6} \pi - \sqrt{3} + 1$ ISW A1

exact answer required

5

Q6, (OCR 4724, Jan 2011, Q3)

(i) State/implicitly $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$ or $\frac{d}{dx}(\cos x)^{-1}$	B1	Not just $\sec x = \frac{1}{\cos x}$
Attempt quotient rule or chain rule to power -1	M1	Allow $\frac{u dv - v du}{v^2}$ & wrong trig signs
Obtain $\frac{\sin x}{\cos^2 x}$ or $-(\sin x)(\cos x)^{-2}$	A1	No inaccuracy allowed here
Simplify with suff evid to AG e.g. $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$	A1 4	Or vice versa. <u>Not</u> just = $\sec x \cdot \tan x$
(ii) Use $\cos 2x = +/ - 1 + / - 2 \cos^2 x$ or $+ / - 1 + / - 2 \sin^2 x$	M1	or $\pm(\cos^2 x - \sin^2 x)$
Correct denominator = $\sqrt{2 \cos^2 x}$	A1	$\sqrt{2 - 2 \sin^2 x}$ needs simplifying
Evidence that $\frac{\tan x}{\cos x} = \sec x \tan x$ or $\int \frac{\tan x}{\cos x} dx = \sec x$	B1	irrespective of any const multiples
$\frac{1}{\sqrt{2}} \sec x$ (+ c)	A1 4	Condone θ for x except final line

Q7, (OCR 4724, Jun 2014, Q4)

$\int \frac{\cos 2x}{1 + \sin 2x} (dx)$	B1*	$\cos 2x = 1 - 2\sin^2 x$ or $(1 +) \sin 2x = (1 +) 2\sin x \cos x$ seen	if B0B0M0A0, SC4 for $F[x] = \frac{1}{2} \ln(1 + 2\sin x \cos x)$ or $\frac{1}{2} \ln(1 + \sin 2x)$ final mark may still be awarded
$F[x] = k \ln(1 + \sin 2x)$ soi	B1*	numerator and denominator both correct in the integral soi	
$k = \frac{1}{2}$	M1 dep*	or $k \ln(1 + u)$ or $k \ln(u)$ following their substitution www	
$\frac{1}{2} \ln(1 + \sin(\pi/2)) - \frac{1}{2} \ln(1 + 0)$	A1	correct k for their substitution	
$= \frac{1}{2} \ln 2$	A1 AG	correct use of limits www	minimum working: $\frac{1}{2} \ln 2 - \frac{1}{2} \ln 1$ or $\frac{1}{2} \ln(1 + 1)$ oe
	[5]		

Q8, (OCR 4724, Jun 2015, Q6)

<p>(i)</p>	$\frac{\sin x \times -\sin x - \cos x \times \cos x}{\sin^2 x}$ <p>may be implied by $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$</p> <p>eg</p> $= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$ <p>and completion to</p> $\frac{-1}{\sin^2 x} \text{ AG}$	<p>M1</p> <p>or $-\sin x \times \frac{1}{\sin x} + \cos x \times -(\sin x)^{-2} \times \cos x$ oe</p> <p>eg</p> <p>A1</p> $= \frac{-\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$ <p>oe and completion to</p> $\frac{-1}{\sin^2 x}$ <p>[2]</p>	<p>allow sign errors only if M0, SC1 for just</p> $\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$ <p>need to see at least two correct, constructive steps and statement of given answer for A1 NB $\sin^2 x + \cos^2 x = 1$ seen may be a constructive intermediate step</p>
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(ii)	<p>$\cos 2x = 2\cos^2 x - 1$ substituted in numerator</p> <p>$\sin 2x = 2\sin x \cos x$ substituted in denominator</p> $\frac{\sqrt{2} \cos x}{2 \sin^2 x \cos x}$ <p>$F[x] = \pm k \frac{\cos x}{\sin x}$</p> $F\left[\frac{1}{4}\pi\right] - F\left[\frac{1}{6}\pi\right]$ $= \frac{1}{2}(\sqrt{6} - \sqrt{2}) \text{ www AG}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>or alternative form of double angle formula plus Pythagoras leading to no term in $\sin^2 x$ in numerator</p> <p>k must not be obtained from square rooting a negative number</p> <p>eg $\frac{-\cos \pi/4}{\sqrt{2} \times \sin \pi/4} - \frac{-\cos \pi/6}{\sqrt{2} \times \sin \pi/6}$</p>	<p>may be awarded if not seen as part of fraction</p> $\text{NB } \int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \frac{1}{\sqrt{2} \sin^2 x} dx$ $\text{NB } -\frac{\cos x}{\sqrt{2} \sin x}$ $\text{eg } \frac{-1/\sqrt{2}}{\sqrt{2} \times 1/\sqrt{2}} - \frac{-\sqrt{3}/2}{\sqrt{2} \times 1/2}$ <p>at least one correct intermediate step following substitution needed as well as statement of given result</p> $\text{eg } -\frac{\sqrt{2}}{2}(1 - \sqrt{3})$
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Q9, (OCR 4724, Jan 2008, Q7)

<p>(i) Perform an operation to produce an equation connecting A and B (or possibly in A or in B) $A = 2$ $B = -2$</p>	<p>M1 A1 A1</p>	<p>Probably substituting value of θ, or comparing coefficients of $\sin x$, and/or $\cos x$ 3 WW scores 3</p>
<p>(ii) Write $4 \sin \theta$ as $A(\sin \theta + \cos \theta) + B(\cos \theta - \sin \theta)$ and re-write integrand as $A + \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta}$ $\int A d\theta = A\theta$ $\int \frac{B(\cos \theta - \sin \theta)}{\sin \theta + \cos \theta} d\theta = B \ln(\sin \theta + \cos \theta)$ Produce $\frac{1}{4} A\pi + B \ln \sqrt{2}$ f.t. with their A, B</p>	<p>M1 √B1 √A2 √A1</p>	<p>A and B need not be numerical – but, if they are, they should be the values found in (i). general or numerical general or numerical 5 Expect $\frac{1}{2}\pi - \ln 2$ (Numerical answer only)</p>