



Integration By Substitution Exam Questions

Q1, (Jun 2005, Q4)

(i) Show that the substitution $x = \tan \theta$ transforms $\int \frac{1}{(1+x^2)^2} dx$ to $\int \cos^2 \theta d\theta$. [3]

(ii) Hence find the exact value of $\int_0^1 \frac{1}{(1+x^2)^2} dx$. [4]

Q2, (Jan 2006, Q6)

(i) Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\frac{x}{1-x}} dx$ to $\int 2 \sin^2 \theta d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}} dx$. [5]

Q3, (Jan 2008, Q10i)

(i) Use the substitution $x = \sin \theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}} dx. \quad [6]$$

Q4, (Jun 2008, Q8)

(i) Given that $\frac{2t}{(t+1)^2}$ can be expressed in the form $\frac{A}{t+1} + \frac{B}{(t+1)^2}$, find the values of the constants A and B . [3]

(ii) Show that the substitution $t = \sqrt{2x-1}$ transforms $\int \frac{1}{x+\sqrt{2x-1}} dx$ to $\int \frac{2t}{(t+1)^2} dt$. [4]

(iii) Hence find the exact value of $\int_1^5 \frac{1}{x+\sqrt{2x-1}} dx$. [4]

Q5, (Jan 2009, Q5)

(i) Show that the substitution $u = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})} dx$ to $\int \frac{2}{u(1+u)} du$. [3]

(ii) Hence find the exact value of $\int_1^9 \frac{1}{x(1+\sqrt{x})} dx$. [5]

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Q6, (Jan 2010, Q4)

Use the substitution $u = 2 + \ln t$ to find the exact value of

$$\int_1^e \frac{1}{t(2 + \ln t)^2} dt. \quad [6]$$

Q7, (Jan 2011, Q5)

In this question, I denotes the definite integral $\int_2^5 \frac{5-x}{2+\sqrt{x-1}} dx$. The value of I is to be found using two different methods.

(i) Show that the substitution $u = \sqrt{x-1}$ transforms I to $\int_1^2 (4u - 2u^2) du$ and hence find the exact value of I . [5]

(ii) (a) Simplify $(2 + \sqrt{x-1})(2 - \sqrt{x-1})$. [1]

(b) By first multiplying the numerator and denominator of $\frac{5-x}{2+\sqrt{x-1}}$ by $2 - \sqrt{x-1}$, find the exact value of I . [3]

Q8, (Jan 2013, Q6)

Use the substitution $u = 2x + 1$ to evaluate $\int_0^{\frac{1}{2}} \frac{4x-1}{(2x+1)^5} dx$. [7]

Q9, (Jun 2016, Q6)

Use the substitution $u = x^2 - 2$ to find $\int \frac{6x^3 + 4x}{\sqrt{x^2 - 2}} dx$. [6]

Q10, (Jun 2017, Q9)

Use the substitution $u = 1 + \ln x + x$ to find $\int \frac{3(x+1)(1 - \ln x - x)}{x(1 + \ln x + x)} dx$. [6]