Integration By Substitution (Sheet 2)

Q1.

Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x+1)}}\,dx$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

(Total 8 marks)

Q2.

Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1)$$

(6)

(Total 6 marks)

Q3.

(i) Given that $y > 0$, find

$$\int \frac{3y - 4}{y(3y + 2)}\,dy$$

(6)

(ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta \, d\theta$$

where λ is a constant to be determined.

(5)

(b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4 - x}\right)}\,dx$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total for question = 15 marks)

Q4.

(a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = \int \frac{2}{u(2u-1)}\,\mathrm{d}u$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

(Total 10 marks)

Q5.

Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

(Total for question = 7 marks)

Q6.

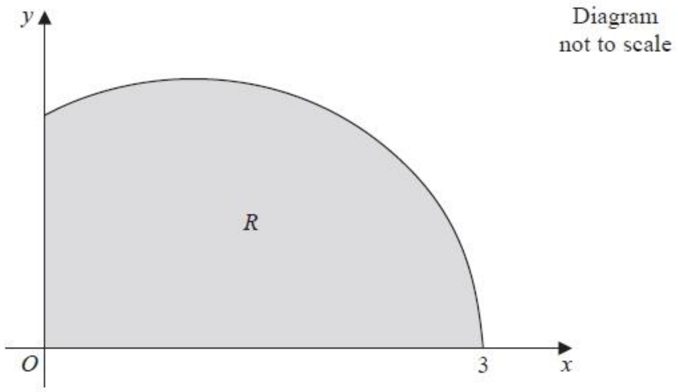

Figure 2

Figure 2 shows a sketch of the curve with equation
$y = \sqrt{(3-x)(x+1)}\,,\quad 0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,\mathrm{d}\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

(Total for question = 8 marks)

Q7. [Note: Parts (a) and (b) of this question involve use of the trapezium rule. Leave these pars out if you have not yet covered these topics.]

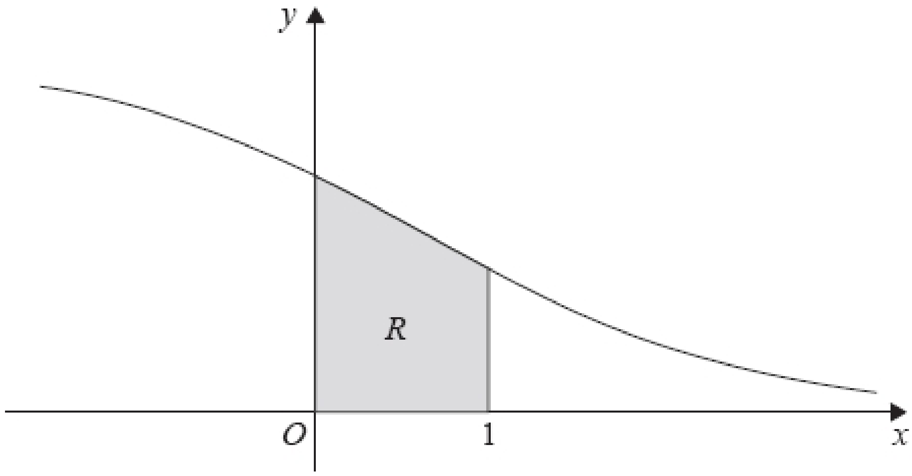

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \frac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(a) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(c) Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,du$$

where a and b are constants to be determined.

(2)

(d) Hence use calculus to find the exact area of R.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(Total for question = 12 marks)

Q8. [Note: Parts (a) and (b) of this question involve use of the trapezium rule. Leave these pars out if you have not yet covered these topics.]

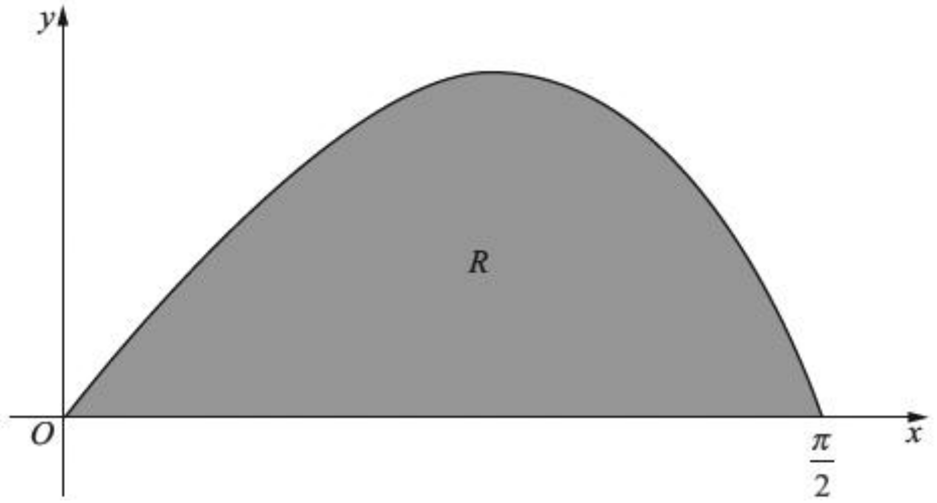

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving
your answer to 4 decimal places.

(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.

(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.

(3)

(Total 12 marks)

Q9. [Note: Parts (a) and (b) of this question involve use of the trapezium rule. Leave these pars out if you have not yet covered these topics.]

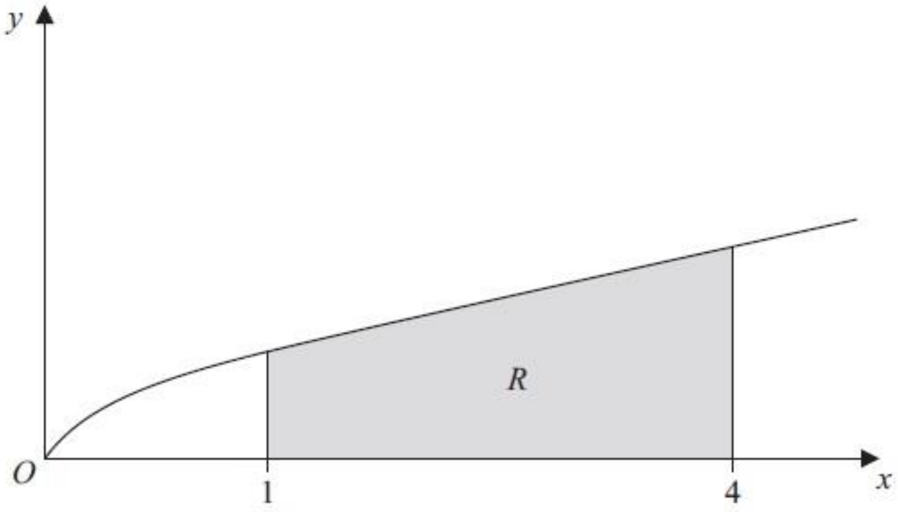

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

(Total 12 marks)

Q10. [Note: Parts (a) and (b) of this question involve use of the trapezium rule. Leave these pars out if you have not yet covered these topics.]

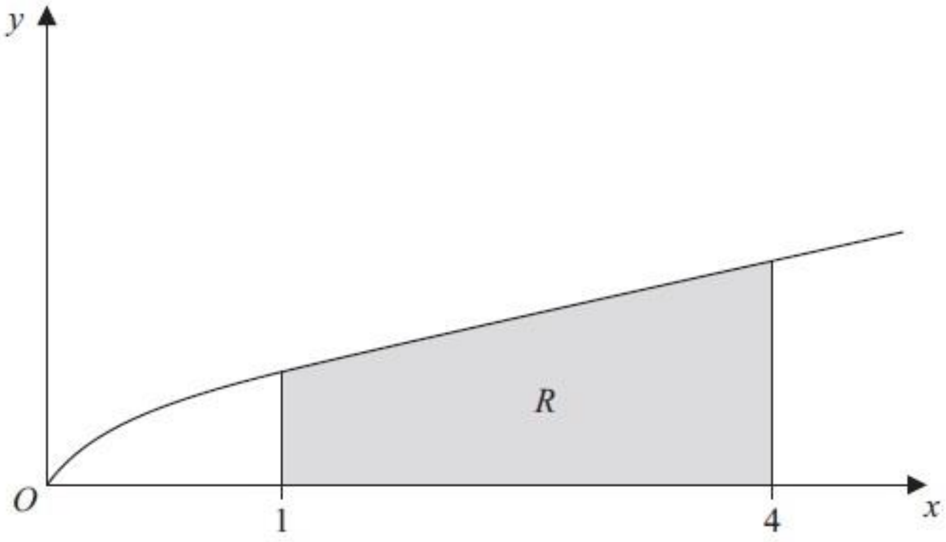

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

(Total 12 marks)