

Integration By Parts MS (From OCR 4724)

Q1, (Jun 2005, Q2)

$$x \sin x - \int \sin x \, dx$$

$$= x \sin x + \cos x$$

Answer = $\frac{1}{2} \pi - 1$

M1		For attempt at parts going correct way ($u = x, dv = \cos x$ and $f(x) +/ - \int g(x) (dx)$)
A1		For both terms correct
B1		Indic anywhere that $\int \sin x \, dx = -\cos x$
M1		For correct method of limits
A1		For correct exact answer ISW 5

Q2, (Jan 2006, Q4)

(i) Parts using correct split of $u = x, \frac{dv}{dx} = \sec^2 x$ M1 1st stage result of form

$$f(x) +/ - \int g(x) \, dx$$

$$x \tan x - \int \tan x \, dx$$

A1 Correct 1st stage

$$\int \tan x \, dx = -\ln \cos x \text{ or } \ln \sec x$$

B1

$$x \tan x + \ln \cos x + c \text{ or } x \tan x - \ln \sec x + c$$

A1 **4**

(ii) $\tan^2 x = +/ - \sec^2 x +/ - 1$ M1 or $\sec^2 x = +/ - 1 +/ - \tan^2 x$

$$\int x \sec^2 x \, dx - \int x \, dx \text{ s.o.i.}$$

A1

Correct 1st stage

$$x \tan x + \ln \cos x - \frac{1}{2} x^2 + c$$

A1✓ **3** f.t. their answer to part (i) $-\frac{1}{2} x^2$

Q3, (Jan 2007, Q2)

Use parts with $u = \ln x, dv = x$

$$\text{Obtain } \frac{1}{2} x^2 \ln x - \int \frac{1}{x} \cdot \frac{1}{2} x^2 (dx)$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 (+c)$$

Use limits correctly

$$\text{Exact answer } 2 \ln 2 - \frac{3}{4}$$

M1		& give 1 st stage in form $f(x) +/ - \int g(x)(dx)$
A1		or $\frac{1}{2} x^2 \ln x - \int \frac{1}{2} x (dx)$
A1		
M1		
A1		5 AEF ISW

Q4, (Jun 2007, Q2)

Use parts with $u = x^2, dv = e^x$

$$\text{Obtain } x^2 e^x - \int 2x e^x (dx)$$

Attempt parts again with $u = (-)(2)x, dv = e^x$

$$\text{Final } = (x^2 - 2x + 2) e^x \text{ AEF incl brackets}$$

Use limits correctly throughout

$$e^{(1)} - 2 \text{ ISW Exact answer only}$$

*M1		obtaining a result $f(x) +/ - \int g(x)(dx)$
A1		
M1		
A1		s.o.i. eg $e + (-2x + 2) e^x$
dep*M1		Tolerate (their value for $x = 1$) (-0)
A1		6 Allow 0.718 \rightarrow M1

Q5, (Jan 2010, Q8)

(i) $-\sin x e^{\cos x}$	B1	1
<hr/>		
(ii) $\int \sin x e^{\cos x} dx = -e^{\cos x}$	B1	anywhere in part (ii)
Parts with split $u = \cos x, dv = \sin x e^{\cos x}$	M1	result $f(x) +/ - \int g(x) dx$
Indef Integ, 1st stage $-\cos x e^{\cos x} - \int \sin x e^{\cos x} dx$	A1	accept ... $-\int -e^{\cos x} \cdot -\sin x dx$
Second stage = $-\cos x e^{\cos x} + e^{\cos x}$	*A1	
Final answer = 1	dep*A2	6
		7

Q6, (Jun 2010, Q9i)

Attempt to multiply out $(x + \cos 2x)^2$	M1	Min of 2 correct terms
<u>Finding</u> $\int 2x \cos 2x dx$		
Use $u = 2x, dv = \cos 2x$	M1	1 st stage $f(x) +/ - \int g(x) dx$
1 st stage $x \sin 2x - \int \sin 2x dx$	A1	
$\therefore \int 2x \cos 2x dx = x \sin 2x + \frac{1}{2} \cos 2x$	A1	
<u>Finding</u> $\int \cos^2 2x dx$		
Change to $k \int +/ - 1 +/ - \cos 4x dx$	M1	where $k = \frac{1}{2}, 2$ or 1
Correct version $\frac{1}{2} \int 1 + \cos 4x dx$	A1	
$\int \cos 4x dx = \frac{1}{4} \sin 4x$	B1	seen anywhere in this part
Result = $\frac{1}{2} x + \frac{1}{8} \sin 4x$	A1	
(i) ans = $\frac{1}{3} x^3 + x \sin 2x + \frac{1}{2} \cos 2x + \frac{1}{2} x + \frac{1}{8} \sin 4x (+ c)$	A1 9	Fully correct

Q7, (Jun 2013, Q2)

$u = \ln 3x$ and dv or $\frac{dv}{dx} = x^8$
 $\frac{d}{dx}(\ln 3x) = \frac{1}{x}$ or $\frac{3}{3x}$
 $\frac{x^9}{9} \ln 3x - \int \frac{x^9}{9} \text{their } \frac{du}{dx} (dx)$ FT
 Indication that $\int kx^8 dx$ is required
 $\frac{x^9}{9} \ln 3x - \frac{x^9}{81}$ or $\frac{1}{9} x^9 \left(\ln 3x - \frac{1}{9} \right)$ ISW (+c) cao

If candidate manipulates $\ln(3x)$ first of all

$\ln(3x) = \ln 3 + \ln x$
 $u = \ln x$ and $dv = x^8$
 $\frac{x^9}{9} \ln x - \int \frac{x^9}{9} \cdot \frac{1}{x} (dx)$ or better
 $\frac{x^9}{9} \ln x - \frac{x^9}{81}$

Their $\int x^8 \ln x dx + \frac{x^9}{9} \ln 3$ (+c) FT ISW

M1

integ by parts as far as $f(x)+/- \int g(x)(dx)$

If difficult to assess, x^8 must be integrated, so look for term in x^9

B1

stated or clearly used

√A1

i.e. correct understanding of 'by parts'...

..even if $\ln(3x)$ incorrectly differentiated

M1

i.e. before integrating, product of terms must be taken

The product may already have been indicated on the previous line

A1

$\frac{1}{9} \frac{x^9}{9}$ to be simplif to $\frac{x^9}{81}$; $\frac{3x^9}{243}$ satis

[5]

B1

If, however, $\ln(3x)$ is said to be $\ln 3 \cdot \ln x$, then B0 followed by possible M1 A1 A1 in line with alternative solution on LHS, where the 'M' mark is for dealing with

M1

In order to find $\int x^8 \ln x dx$:

A1

$\int x^8 \ln x dx$ 'by parts' in the right order and the 2 @ A1 are for correct results.

A1

√A1

Q8, (Jun 2014, Q8)

<p>(i)</p>	<p>t^2 in quotient and $t^3 + 2t^2$ seen</p> <p>$-2t$ in quotient and $-2t^2 - (-2t^2 - 4t) = 4t$ seen</p> <p>completion to obtain correct quotient and remainder identified www</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>or $\frac{t(t^2 - 4) + 4t}{(t + 2)}$</p> <p>$\frac{t(t + 2)(t - 2)}{(t + 2)} + \frac{4t}{t + 2}$</p> <p>$t(t - 2) + \frac{4(t + 2) - 8}{t + 2}$</p>	<p>or $\frac{(t + 2)^3 - 6t^2 - 12t - 8}{(t + 2)}$</p> <p>$\frac{(t + 2)^3}{(t + 2)} - \frac{6((t + 2)^2 - 4t - 4) + 12t + 8}{(t + 2)}$ oe</p> <p>$(t + 2)^2 - 6(t + 2) + \frac{12t + 16}{t + 2}$ oe</p> <p>$= t^2 + 4t + 4 - 6t - 12 + \frac{12(t + 2) - 8}{t + 2}$ oe</p> <p>both steps needed for final B1</p>
<p>(i)</p>	<p>alternatively $\frac{t^3}{t + 2} \equiv At^2 + Bt + C + \frac{D}{(t + 2)}$</p> <p>equate coefficients to obtain correctly $A = 1, 0 = 2A + B$ and $B = -2$ www</p> <p>$0 = 2B + C$ and $0 = 2C + D$ obtained and solved correctly www</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>or $t^3 \equiv (At^2 + Bt + C)(t + 2) + D$</p>	<p>or B1 for $\frac{t^2(t + 2) - 2t^2}{(t + 2)}$</p> <p>B1 for $t^2 + \frac{-2t(t + 2) + 4t}{(t + 2)}$</p> <p>B1 for $t^2 - 2t + \frac{4(t + 2) - 8}{(t + 2)}$</p>

<p>(ii)</p>	<p>integration by parts with $u = \ln(t + 2)$ and $dv = 6t^2$ to obtain $f(t) \pm \int g(t)(dt)$</p> $2t^3 \ln(t + 2) - \int \frac{2t^3}{t + 2} (dt) \text{ cao}$ <p>result from part (i) seen in integrand; must follow award of at least first M1</p> $F[t] = 2t^3 \ln(t + 2) \pm \frac{2t^3}{3} \pm 2t^2 \pm 8t \pm 16 \ln(t + 2)$ <p>their $F[2] - F[1]$</p> $-6^{2/3} - 18 \ln 3 + 32 \ln 4 \text{ oe cao}$	<p>M1*</p> <p>A1</p> <p>M1*</p> <p>A1</p> <p>M1dep*</p> <p>A1</p> <p>[6]</p>	<p>$f(t)$ must include t^3 and $g(t)$ must not include a logarithm</p> <p>no integration required for this mark</p> $2t^3 \ln(t + 2) - \frac{2t^3}{3} + 2t^2 - 8t + 16 \ln(t + 2)$ <p>at least one of their terms correctly integrated</p>	<p>ignore spurious dx etc</p> <p>alternatively, following $u = t + 2$</p> $\int 2(u^2 - 6u + 12 - \frac{8}{u}) du \text{ oe}$ $\frac{2u^3}{3} - 6u^2 + 24u - 16 \ln u \text{ and}$ $2t^3 \ln(t + 2)$ <p>NB limits following substitution are $u = 4$ and $u = 3$</p>
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Q9, (Jun 2016, Q4)

$Ax^{\frac{2}{3}} \ln x - \int Bx^{\frac{2}{3}} \times \frac{1}{x} dx \text{ oe}$ $\frac{3}{2} x^{\frac{2}{3}} \ln x - \int \frac{3}{2} x^{\frac{2}{3}} \times \frac{1}{x} dx$ $F[x] = \frac{3}{2} x^{\frac{2}{3}} \ln x - \frac{3/2}{2/3} x^{\frac{2}{3}}$ <p>$F[8] - F[1]$</p> $18 \ln 2 - \frac{27}{4} \text{ cao}$	<p>M1* A and B are non-zero constants;</p> <p>A1 ignore $+ c$</p> <p>A1 ignore limits for first three marks</p> <p>M1*dep and also dependent on integration of their $\frac{3}{2} x^{\frac{1}{3}}$</p> <p>A1</p> <p>[5]</p>	<p>NB $\frac{3}{2} x^{\frac{2}{3}} \ln x - \int \frac{3}{2} x^{\frac{1}{3}} dx$</p> <p>Allow both marks if dx omitted</p> <p>NB A0 for $6 \ln 8 - \frac{27}{4}$</p>
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