Integration By Parts (Sheet 2)

Q1.

(a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

(Total 6 marks)

Q2.

Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

(6)

(Total 6 marks)

Q3.

(a) Use integration by parts to find $\int x e^x \, dx$.

(3)

(b) Hence find $\int x^2 e^x \, dx$.

(3)

(Total 6 marks)

Q4.

$$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(a) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(b) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta f(\theta) \, d\theta$.

(7)

(Total 10 marks)

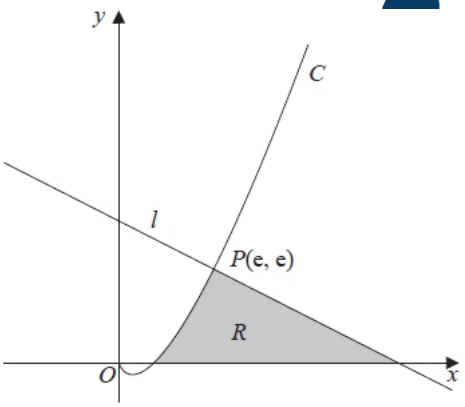
Q5.

The diagram shows a sketch of part of the curve C with equation $y = x \ln x, \quad x > 0$

The line l is the normal to C at the point $P(e, e)$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $Ae^2 + B$ where A and B are rational numbers to be found.



(10)

(Total for question = 10 marks)

Q6.

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}, \ x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of ln2, the x coordinate of the point A.

(2)

(b) Find

$$\int xe^{\frac{1}{2}x} dx$$

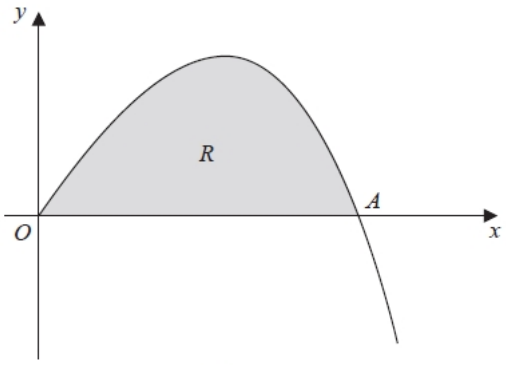


Figure 1

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x}, \ x \geqslant 0$$

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of ln2

(3)

(Total for question = 8 marks)

Q7.

(a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx$$

(2)

(Total 7 marks)

Q8.

(a) Find $\int x \cos 2x \, dx$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$.

(3)

(Total 7 marks)

Q9.

(a) Find $\int \sqrt{(5-x)}\, dx$.

(2)

Figure 3 shows a sketch of the curve with equation

$$y = (x-1)\sqrt{(5-x)}, \quad 1 \leqslant x \leqslant 5$$

(b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{(5-x)}\, dx$$

(4)

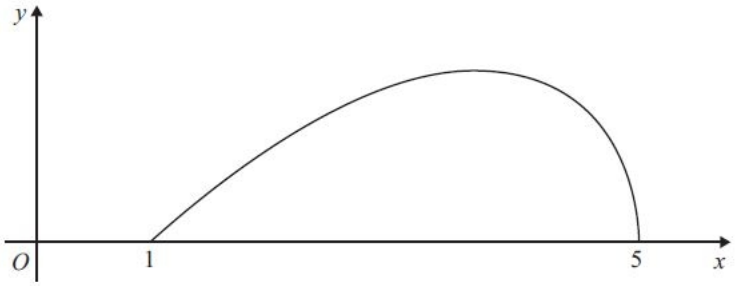


Figure 3

(b) (ii) Hence find $\int_1^5 (x-1)\sqrt{(5-x)}\, dx$.

(2)

(Total 8 marks)